

**TESTING THE PREDICTIVE POWER OF VARIOUS EXCHANGE RATE  
MODELS IN FORECASTING THE VOLATILITY OF EXCHANGE RATE**

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IN FORECASTING THE VOLATILITY OF EXCHANGE RATE

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## **Dedication**

I dedicate this thesis to my dad and mum Mr. Aboagye Kodom and Mrs. Dorothy Asante.

## **Abstract**

This Thesis tests the predictive power of ARCH, GARCH and EGARCH models in forecasting exchange rate volatility of Canadian dollar, Euro, British Pound, Swiss Franc and Japanese Yen using the US dollar as the base currency. We investigate both in-sample and out-of-sample performance of the volatility models using loss functions. The study further examines if the best model for the in-sample forecast will emerge as the best model for the out-of-sample forecast. The study finds that the GARCH(1,1) model outperforms all the other volatility models during the in-sample period. However in terms of the out-of-sample performance of the volatility models, the results are inconclusive, even though the ARCH model performed better most of the time than the complex models. The study concludes that the simple models should be given special consideration in terms of forecasting. Our results are robust to research on exchange rate volatility forecasting.

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## CHAPTER ONE

### 1.1 Introduction

Issues related to foreign exchange rate have always been the interest of researchers in modern financial theory because exchange rate, which is the price of one currency in terms of another currency, has a great impact on the volume of foreign trade and investment. Analysing the Forex market by volume shows that global daily Foreign Exchange transactions exceeded \$4 trillion in 2010 which is bigger than the annual value of global trade (Bank for International Settlement, 2010). It is therefore not surprising that stable exchange rate has served as a catalyst for economic growth for most developed countries (countries which produce for export) whereas a highly volatile exchange rate is a major problem for most developing countries (developing countries who are import dependent).

The huge volume of transactions in the forex market makes it difficult for one to examine global interaction among countries with little or no understanding of the driving forces of exchange rates. In addition to that, the forex market is also an extremely nonlinear vibrant system which performance is influenced by a number of factors, namely inflation rates, interest rates, economic atmosphere, political issues, and many other factors. Hence forecasting the exchange rate index is challenging and a tricky issue for both for investors and academics (Sutheebanjard and Premchaiswadi, 2010).

The post Bretton Woods exchange system has seen substantial and growing literature on testing the predictive power of models used for forecasting foreign exchange rates movements. Reasons put forward for the intense research in this topical area are that,

accurate forecasts of exchange rate volatility can signal an early warning to a government of an upcoming economic crisis, and thereby aids policy makers to design and implement appropriate exchange rate policies to mitigate the anticipated economic turmoil. Similarly, accurate exchange rates volatility forecast is an important decision instrument that guides the choice of an exchange rate regime that is best for a particular country (Hernandez and Montiel, 2001) and can signal the optimality of a monetary union (Wyplosz, 2002). Ogawa (2002) posits that forecasting high exchange rate volatility in countries with flexible exchange rate regimes can influence the countries to enter a common exchange regime system as it is said to promote economic stability.

According to Antonakakis (2007), increase knowledge of the behaviour of the exchange rate does not only benefit the governments of countries but also the Central banks. His study asserts that, the Central banks rely on internal forecasts of exchange rate volatility to ascertain whether an exchange rate will fluctuate within or outside a target zone. Thus, exchange rate forecast in excess of the target rate will necessitate the intervention of the central bank, some of which include inflation targeting, interest rate targeting and other controls. The research by Antonakakis (2007) also posits that accurate knowledge of exchange rate forecasting is important for international traders' export and import decisions because forecasting excessive exchange rate volatility in a country, *ceteris paribus*, will have a different impact on traders as it will make risk-averse traders reduce the volume of their transactions with such a country because of the uncertainty of their profits. However, risk-lover traders who trade on risk could benefit from seeking out hedging opportunities.

Accurate volatility forecasting of exchange rate also guides the decisions of investors, especially international investors who require portfolio diversification beyond national borders, or risk managers using internal models such as value-at-risk applications. International investors and risk managers have certain levels of risks that they can bear, so an accurate forecast of exchange rate volatility over the investment holding period serves as a good starting point for assessing their investment risk. As far as a risk-averse investor is concerned, uncertainty is the most important factor in pricing any financial asset. Hence most asset pricing theories try to model this uncertainty or risk by using the covariance between asset return and the market portfolio. Although it has been recognized for quite some time that the uncertainty of speculative prices, as measured by the variance and covariance, is changing through time, it is not until recently that researchers have started to explicitly model this time variation in second or higher order moments. This breakthrough in the area of volatility modeling confirms that accurate forecast of large exchange rate volatility has greatly affected the decision making process of highly risk-averse as opposed to risk-neutral investors. In addition, risk managers who want to minimize risks and are aware of accurate forecasts of large exchange rate volatility in a country, can still hold assets denominated in that country's currency by investing in a second country's currency, where exchange rate volatility is equally large but negatively correlated with that in the former country. This negative correlation could help offset risks. Accurate volatility forecasts therefore act as an input in their portfolio diversification and risk management (Antonakakis, N. 2007). In conclusion, a continuous improvement in forecasting accuracy will no doubt have substantial benefits for the various groups of agents since their decision making will be better informed. In contrast,

the costs of generating poor forecasts are substantial, as their decision making will be flawed and have unanticipated effect on the global economy.

Notwithstanding the importance of forecasting exchange rate volatility, theories of forecasting volatility of exchange rate has been in existence for many centuries with different models yielding different forecasting results either in-sample or out of sample data. The traditional financial economics research on exchange rate volatility has focused on the mean of exchange rate returns but in recent times the emphasis on mean return of exchange rate has shifted to the volatility of the exchange rate. Also the international stock market crash of 1987 has further increased the focus of regulators, practitioners and researchers on forecasting the volatility of exchange rate (Brailsford, 1995). As a result, some researchers have employed multivariate regression approach to study and predict the exchange rate volatility based on macroeconomic variables. Several others have also relied on structural models to predict the volatility of exchange rate, but these approaches have limitations in the sense that there are generally weak relationships between exchange rate and virtually any macroeconomic variable a situation commonly known as the “exchange rate disconnect puzzle” (Obstfeld et al., 2000). Also Flood et al., (1995), observes that nominal exchange rates are much more volatile (at low frequencies) than the macroeconomic variables which determines exchange rate theoretically hence this excess volatility associated with exchange rate makes structural models unsuccessful in forecasting exchange rate volatility. Cheung, Chinn and Pascual (2002), provides a survey of the literature on forecasting exchange (conditional mean) based on macro fundamentals. The study fails to identify papers with good predictive power that are able to overcome the random walk model. Also following the seminal paper of Meese and

Rogoff (1983), many tests have been done in search of macroeconomic models that have good power for forecasting movements but prove futile. These limitations associated with macroeconomic variables and structural models in predicting exchange rate volatility have led some researchers to ask the question that “why not exchange rate explains itself?”. That is with the little information on exchange rate, it should be able to forecast its current as well as future volatility by using robust time series or technical model (Onasanya *et al.*, 2013). Nevertheless, the uncertainty of the exchange rate continues to present a challenge for economic agents to determine the directionality of the actual or future volatility of exchange rate. This is because the more forecast errors economic behaviors make in predicting the directionality of exchange rate volatility, the higher the trends in the uncertainty of the exchange rate are shown (Yoon et al, 2008).

Forecasting volatility of financial assets continues to occupy a center stage in research (Longmore and Robinson 2004). However, variance or standard deviation which is the traditional measure for volatility is unconditional and does not recognize the patterns in asset volatility such as time-varying and clustering properties. Researchers have therefore introduced various models to explain and predict these patterns in volatility. Engle (1982) is the first to introduce the autoregressive conditional heteroscedasticity (ARCH) to model volatility. The ARCH, models the heteroscedasticity by relating the conditional variance of the disturbance term to the linear combination of the squared disturbances in the recent past. Bollerslev (1986) generalizes the ARCH model by modeling the conditional variance to depend on its lagged values as well as squared lagged values of disturbance, and he calls this model generalized autoregressive conditional heteroskedasticity (GARCH). The ARCH/GARCH model has made a great impact on the

development of financial econometrics in the past two decades. The GARCH model is useful for interpreting the volatility-clustering effect, which is an important facet of asset prices (Mandelbrot, 1963). Volatility clustering means that large changes of an asset's price are followed by large changes in either sign. The model is also useful for many financial applications, such as measuring volatility of financial markets, pricing options, and computing Value at Risk (VAR) (Engle, 2002). Engle (1982) notices that although OLS (Ordinary Least Square) maintains its optimality properties when it is used to forecast exchange rate volatility, however, the parameters of ARCH models provides a better estimate when maximum likelihood method is employed. Similarly, Lastrapes (1989) complements the work of Engle when he observes that ARCH provides a better explanation of exchange rate process and it is consistent with exchange rates behaviour. Zivot (2009) also provides an in-depth empirical analysis of GARCH models for financial time series with emphasis on practical issues associated with model specification, estimation, diagnostics, and forecasting and finds the GARCH model to provide better description of financial data compared to the OLS. According to Bala et al., (2013) the GARCH model has dominated the literature on volatility since the early 1980s because the model allows for persistence in conditional variance by imposing an autoregressive structure on squared errors of the process. This indicates that the superiority of the ARCH/GARCH models over other models in forecasting exchange rate volatility has contributed to its entrenched position and success in the forecasting literature.

Since the work of Engle (1982) and Bollerslev (1986), various forms of GARCH model have also been developed to model volatility. Some of the models include Integrated

GARCH (IGARCH) originally proposed by Engle and Bollerslev (1986), GARCH in-Mean (GARCH-M) model introduced by Engle, Lilien and Robins (1987), the standard deviation GARCH model introduced by Taylor (1986) and Schwert (1989), the Exponential GARCH model (EGARCH) proposed by Nelson (1991), Threshold ARCH (TARCH), and the Threshold GARCH have been introduced independently by Zakoïan (1994) and Glosten, Jaganathan, and Runkle (1993), the Power ARCH model generalised by Ding, Zhuanxin, C. W. J. Granger, and R. F. Engle (1993) etc.

Recently, the observance of volatility breaks in time series data has paved way for volatility models which allow for volatility breaks, a typical example is the Markov-switching models of conditional heteroscedasticity (see Lange and Rahbek, 2008). In view of that, Hammoudeh and Li (2008) have analysed sudden changes in volatility for five Gulf area stock markets and find that accounting for the large shifts in volatility in the GARCH (1,1) models reduces the estimated persistence of the volatility of the Gulf stock markets significantly.

Notwithstanding the vast aforementioned models in forecasting the volatility of exchange rate, other research work indicate that the behavior of dealers and other market participants can influence fluctuations in exchange rates (Lyons and Evans, 2002; Covrig and Melvin, 1998). Inventory adjustments and bid-ask spread reactions to informative incoming order flows are some of ways by which dealer behavior affects exchange rate determination and volatility. Also different market participants depend on both private and public information sets; hence it is natural to assume that equilibrium exchange rate expectations are formed based on a combination of macroeconomic fundamentals and market microstructure variables (Goldberg and Tenorio, 1997). Moreover, it is most

likely that the macroeconomic information and order flow information are processed in a nonlinear fashion. In line with this thinking, Gradojevic et al. (2006) apply the (Artificial Neural Network) ANN model to forecast high-frequency Canadian/US dollar exchange rate.

## **1.2 Thesis Objectives**

Despite the different models that have evolved to forecast exchange rate volatility, large swings in price movements has become more prevalent and observers have damned institutional changes for this increase in volatility. These concerns have led researchers to analyse the level and stationarity of volatility over time. Specifically, research have been directed toward examining the accuracy of volatility forecasts obtained from various econometric models including the autoregressive conditional heteroscedasticity (ARCH) family of models. Interestingly, research conducted in the area of comparing volatility models are very limited. Also empirical research aim at comparing volatility models using at most three currencies and most of the time ignore the vehicle currencies in the world.

However, given the important role that exchange rate plays in each economy and the challenge in explicitly rating one volatility model over the other, this thesis examines the predictive power of various exchange rate models for six major trading currencies (Canadian dollar, US dollars, British pounds, Euro, the Japanese Yen, and Swiss franc).

Specifically, the study will

1. Provide extensive development of each model
2. Test the in-sample fit of the ARCH, GARCH and EGARCH models



3. Test the out-of-sample performance of the ARCH, GARCH and EGARCH model.

Various models have been used to forecast the volatility of exchange rate. However, researchers have not reached a consensus as to which model is better than the other in terms of exchange rate volatility forecasting. It is against this background that the purpose of this research is unique as it seeks to test the predictive power of various ARCH/GARCH (Generalised Autoregressive Conditional Heteroscedasticity) type models which include ARCH(p,q), GARCH(p,q) and EGARCH(p,q) to forecast exchange rate volatility in the context of six major trading currencies in the world (Euro, Canadian dollar, the Great British Pound, the Japanese Yen and the Swiss franc). The US dollar is used as the base currency against which all the other currencies are compared.

Adamu (2005) for example examines the impact of exchange rate volatility on private investment using GARCH type models and confirms an adverse effect of exchange rate volatility on private investment. Mordi (2006) employing GARCH model argues, that failure to properly manage exchange rates can induce distortions in consumption and production patterns. The study concludes that; excessive currency volatility creates risks with destabilizing effects on the economy. Hence a study that compares the predictive power of volatility models will contribute immensely to the smooth running of the global economy.

The study compares the ARCH family model which is at the core of volatility modelling with other extensions of volatility models in order to ascertain whether the basic models should be repudiated during forecasting exchange rate volatility since available literature advocate for complex models in forecasting volatility.

### **1.3 Thesis Contribution**

Forecasting volatility and determinants of exchange rate have been well established in the literature. Efforts have been made to compare the performance of volatility models in forecasting exchange rate at various forecasting horizons. Also noted in the literature is the evaluation of the forecasting performance of volatility models in a particular family. Almost all the empirical research are either limited in the number of currencies used or focus on estimating the forecasting performance of volatility models at various forecasting horizons. An area which has witnessed less quantitative research is the evaluation of the forecasting performance of various exchange rate models from different family of volatility models. Hence the main objective of the thesis is to assess the forecasting performance of various exchange rate models in forecasting the volatility of exchange rate. This will help assess whether the simple models like the ARCH are able to perform better than the complex models or not. Most of the time research are directed towards assessing the performance of complex exchange rate volatility models with little or no reference to the simple models for forecasting exchange rate volatility. However as noted by Dimson and Marsh (1990), the simple models cannot be overlooked in forecasting volatility of exchange.

The thesis compares the forecasting performance of volatility models across six vehicle currencies in the world (Euro, Canadian dollar, the Great British Pound, the Japanese Yen and the Swiss franc). Previous studies have investigated only a limited range of exchange rates volatility between different currencies. Hence the use of six major currencies serves as an extension of previous studies. This also makes study particular of interest against the background that these six currencies are the world's most heavily traded currencies, a

fact which cannot be explained by economic size alone. Furthermore, these currencies are characterised by a relatively high degree of speculative activity and a significant and highly variable amount of interventions by their monetary authorities. This study therefore assesses the performance of the volatility models notwithstanding the monetary control of the respective countries.

The study contributes to available literature by employing current data set and extensively developing the volatility models used in the study. With the passage of time the evolution of volatility may change, hence the use of current data set which spans the period where the world has witnessed the greatest financial crisis of the time will be literature enriching. Unlike other empirical studies which just state the volatility model to be compared, this study extensively develop each model to be used for forecasting which contributes to clear understanding of the subject matter of the thesis.

#### **1.4 Thesis Organisation**

This thesis is organized as follows: Chapter 2 provides a detailed discussion of empirical studies on exchange rate forecasting exchange rate volatility. Chapter 3 provides extensive development of the forecasting models. Chapter 4 provides estimation and forecasting with the various models, distributional assumptions of the volatility models and how the data are used for forecasting both in-sample and out of sample. It also presents how the models for comparison are going to be selected and also discusses the forecast evaluation of each model. Chapter 5 presents data and discusses the empirical results while summary, recommendation and concluding remarks are presented in Chapter 6. Appendix A describes the realized volatility for the various exchange rate

series. Appendix B contains the sample autocorrelation for the various exchange rate series and Appendix C describes the selection of the models for forecasting using AIC.

## CHAPTER TWO

### 2.1 Literature Review

Predicting exchange rate volatility is difficult task. Despite the large body of research on exchange rate modeling, a key popular fact in international finance is that the best prediction for tomorrow's exchange rate is today's rate (known as the "random-walk forecast"). This result is first discovered by Meese and Rogoff (1983a, b) and after 25 years later, only few models could do better than random walk. A related result, also found in the literature by Hansen et al. (1980) indicates that the forward rate does not provide the best prediction for tomorrow's exchange rate. Clarida et al., (2003) reaffirms the proposition of Hansen et al., (1980) as he notes that "from the early 1980s onwards, exchange rate forecasting is a difficult task because of the uncertainty associated with its forecast. The uncertainty associated with exchange rate forecast has made forecasting exchange rate increasingly come to be seen as a hazardous occupation and hence using the random walk model is the best specification for modeling exchange rate volatility.

A good review of the theories that have dilated on the exchange rate variability can be found in the research work by Cheung, Chinn and Pascual (2002). These authors use different approaches to study exchange rate volatility and they test the predictive power of four exchange rate models for the exchange rates series of Canada, Britain, Germany, Switzerland and Japan using the U.S. dollars as a base currency. The models tested in their works include stick-price monetary model, in line with Dornbush-Frankel, Balassa-Samuelson, which considers productivity differentials among countries, Behavioral Equilibrium Exchange Rate (BEER), which incorporates features of a wide range of models; and Unconvered Interest Rate Parity. Their studies find that, none of the tested

models outperforms the random walk model in forecasting the exchange rate volatility using the criterion of mean squared error. However, their studies conclude that on a longer horizon, structural models on the average provides a higher forecasting performance than the random walk model.

Since the seminal works of Mandelbrot (1963a, 1963b, 1967) and Fama (1965), researchers have found that the characteristics of the foreign exchange returns are non-linear, exhibits temporal dependence, and the distribution of exchange rate returns are leptokurtic (Friedman and Vandersteel (1982), Bollerslev (1987), Diebold (1988), Hsieh (1988, 1989a, 1989b) Diebold and Erllove (1989), Baillie and Bollerslev (1989)). Their researches also find that both large and small changes in returns are 'clustered' together over time, and the return distribution of the data used are bell-shaped, symmetric and fat-tailed. Data with such characteristics are captured well by Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982) and the Generalised ARCH (GARCH) model developed by Bollerslev (1986).

However, the ARCH model is first used to model the currency exchange rate by Hsieh (1989a). The study investigates whether daily changes in five major foreign exchange rates contain any nonlinearity. The research concludes that although the data contains no linear correlation, however there is the presence of substantial nonlinearity in a multiplicative rather than additive form. The study then suggested that the generalized ARCH (GARCH) model can offer explanation to a large part of the nonlinearities for all five exchange rates. Since then, applications of these models to currency exchange rates have increased tremendously, with studies like Hsieh (1989b), Bollerslev (1990), Pesaran et al. (1993), Copeland et al. (1994), Takezawa (1995), Episcopos et al. (1995), Brooks

(1997), Hopper (1997), Cheung et al. (1997), Laopodis (1997), Lobo et al. (1998) and Duan et al. (1999). In most of the applications of the ARCH model, it is found that a very high-order ARCH model is required to model the changing variance. The alternative and more flexible lag structure is the Generalised ARCH (GARCH) introduced by Bollerslev (1986). Bollerslev et al., (1992) indicates that the squared returns of not only exchange rate data, but all speculative price series, typically exhibit autocorrelation and that large and small error tend to cluster together in continuous time periods in what has come to be known as volatility clustering.

Amidst the difficulty in forecasting the exchange behavior, Lam et al., (2008) compares five structural models (the PPP (Purchasing Power Parity) model, the UIP (Uncovered Interest Parity) model, the SP(semiparametric) model, the BMA (Bayesian Moving Average) model) and the composite (specification incorporating the above four models) model to the random walk model in predicting exchange rate behaviour in Hong Kong. The research uses the mean- square errors to determine the predictive power of the models and concludes that, the structural models are able to outperform the random walk model in certain time horizons but the combined forecast model has the best predictive power as it dominates all the models compared.

Similarly, Dan Bianco et al (2012) construct fundamentals-based econometric model for the weekly changes in the euro-dollar rate between 1999-2010 to forecast both the short run and the long run exchange rate between the euro and the dollar. The research further tests the predictive power of this model against the random walk model using mean square prediction error and a non-parametric test of predictive performance developed by Pesaran and Timmermann (1992). The research finds statistically significant

improvements upon the hard-to-beat random walk model using traditional statistical measures of forecasting error at all horizons. Also the study employs Direction of Change (DCH) measure, which evaluates out-of-sample forecasts by comparing the sign of the forecasts with the sign of the true observation. This test also complements the predictive power of the economic model against the hard to beat random walk model.

In a similar vein, Groen (2005) tests the a long-run link between monetary fundamentals, the Euro, exchange rates of Canada, Japan, and the United States and finds out that there exist a strong link between the monetary fundamentals and the long run exchange rate of the countries involved. The research further tests the out-of-sample performance of the model with the random walk model and finds out that, the model is superior to the random walk using the cointegrated VAR (Vector Autoregressive) model-based forecasts, especially at horizons of 2 to 4 years.

Gradojevic et al. (2006) applies the ANN (Artificial Neural Network model) model to forecast high-frequency Canadian/US dollar exchange rate. This is the first time microstructure variable is applied to forecast Canada/US exchange rate. The research concludes that the introduction of the microstructure variable improved both the linear and nonlinear models of exchange rate. The research further employs the Root mean square error prediction to the test the predictive power of the model and concludes that the ANN model is consistently better than the random walk and other linear models for the various out-of-sample set sizes.

Andersen and Bollerslev (1998a) have examined the DM/USD (Deutsh mark/US dollar) intraday volatility based on a year sample of five minutes returns with emphasis on



activity patterns, macroeconomic announcement and calendar effects. The study reveals that market activity is correlated with price variability. Hansen and Lunde (2005) compare 330 ARCH-type models in terms of their ability to describe the conditional variance, and finds no evidence that a GARCH (1,1) model is outperformed by more sophisticated models in their analysis of exchange rates. However, Teräsvirta (2009) reviews several univariate models of conditional heteroscedasticity and reports that GARCH models tend to exaggerate volatility persistence.

Kasman et al. (2011) investigates the relationship between interest rate and exchange rate changes on Turkish bank's stock returns and finds significant negative association. Their results further reveal that interest and exchange rate volatility are the major determinants of conditional bank stock return volatility.

In assessing the performance of volatility models in the in-sample, Taylor (1987) and more recently West and Chow (1995) examine the forecast ability of exchange rate volatility using a number of models including ARCH. The study considers five U.S. bilateral exchange rate series. They find that generalised ARCH (GARCH) models are preferable at a one-week horizon, whilst for less frequent data, no clear victor is evident. Some other studies which tested the forecasting ability of volatility models in the in-sample include Meese and Rose (1991), McKenzie (1997), Christian (1998), Longmore and Wayne Robinson (2004), Yang (2006) Yoon and Lee (2008) among others.

It is quite surprising that the available literature concentrates on in-sample forecast of exchange rate volatility with little emphasis on out-of-sample forecast. The few papers

that have tested the forecasting ability of ARCH/GARCH models in the out-of-sample have reached mixed results. Akgiray (1989), finds that a GARCH (1,1) specification exhibits superior forecasting ability of weekly US stock market volatility compared to more traditional models. However, Tse (1991) and Tse and Tung (1992) reach a different conclusion on the superiority of the GARCH model in the Japanese and Singaporean markets, respectively. Their studies find evidence strongly in favour of an exponentially weighted moving average (EWMA) model. In an examination of the UK equity market, Dimson and Marsh (1990) conclude that simple models provide more accurate forecasts, and hence the study recommends the exponential smoothing and simple regression models. However, Dimson and Marsh did not subject ARCH models to examination. Nevertheless, the conclusions of Dimson and Marsh have important implications for forecasts obtained from the relatively complex GARCH model as their research finally concludes that with the increase interest in using complicated econometric techniques for volatility forecasting, their research bet to differ from the status quo. That implies that those who are interested in forecasts with reasonable predictive accuracy, the best forecasting models may well be the simplest ones (Dimson and Marsh, 1990).

Also Hansen et al. (2005) compare 330 ARCH-type models in terms of their ability to describe the conditional variance. The models are compared based on their out-of-sample performance using DM/USD (Deutsche Mark/US Dollars) exchange rate data and IBM return data. The study finds no evidence that a GARCH(1,1) is outperformed by more sophisticated models in the analysis of exchange rates. However, the GARCH(1,1) model is found inferior to models that can accommodate a leverage effect in the analysis of IBM returns. The study also employs the test for superior predictive ability (SPA) and the

reality check for data snooping (RC) to compare the models' predictive power. The empirical results show that the RC lacks power to an extent that makes it unable to distinguish 'good' from 'bad' models in their analysis.

In forecasting exchange rate volatility between Naira and US Dollar, the Statistics Unit of Usmanu Danfodiyo University (2014), investigates the volatility modeling of daily Dollar/Naira exchange rate using GARCH, GJR-GARCH, TGARCH and TS-GARCH() models and daily data over the period June 2000 to July 2011. The results show that the GJR-GARCH and TGARCH models exhibit the existence of statistically significant asymmetry effect. The research further examines the forecasting ability of the models using the symmetric loss functions which are the Mean Absolute Error (MAE), Root Mean Absolute Error (RMAE), Mean Absolute Percentage Error (MAPE) and Theil inequality Coefficient. The results show that TGARCH model provide the most accurate forecasts because it captures all the necessary stylize facts (common features) of financial data, such as persistent, volatility clustering and asymmetric effects.

Bonilla et al (2003) apply the Hinich portmanteau bicorrelation test to find out the suitability of using GARCH (Generalized Autoregressive Conditional Heteroscedasticity) as the data-generating process to model conditional volatility of stock market index rates of return in 13 emerging economies. The study finds that a GARCH formulation or any of its forms fail to provide an adequate description for the underlying process of the 13 emerging stock market indices. The study also examines the existence of ARCH effects, over windows of 200, 400 and 800 observations, using Engle's LM (Lagrange Multiplier) test, and find no evidence to support the existence of ARCH effects over long periods of time. The results of the study also hints that policymakers should exercise caution when

using autoregressive models for policy analysis and forecast because of the failure of the GARCH models to efficiently model the data generating process of the stock indices. The failure of the GARCH to model the stock indices has strong implications for the pricing of stock index options, portfolio selection and risk management. The study finally concludes that in analysing spillover effects and output volatility the GARCH model will not be efficient and hence cannot be used to evaluate economic policy in this direction. Will the same conclusion be reached when the GARCH model is used to forecast exchange rate volatility?

## CHAPTER THREE

### 3.1 Volatility Models

The volatility models that is estimated in this study include ARCH(p), GARCH(p,q) and EGARCH(p,q). According to Tsay (2005), differences in volatility models are due to the way variance evolves over time. Conditional heteroscedastic models can therefore be classified into two groups. The first group uses exact functions to govern evolution of variance( $\sigma_t^2$ ) , while the second category uses stochastic equations to describe  $\sigma_t^2$ . ARCH/GARCH models belong to the first category whereas stochastic volatility models belong to the second category (Tsay, 2005). This study therefore adopts a family in the first group where an estimated volatility model is used to examine volatilities in the six exchange rate series under investigation. We choose the ARCH/GARCH models because of the limitation of the ordinary least square.

A major assumption behind Ordinary least square is that the expected value of all errors terms when squared is the same at any given point or constant variance (a property of OLS commonly known as homoscedastic). If this condition is violated, then the Ordinary least square estimates for the coefficients will not have minimum variance even though the coefficients will still be unbiased estimates. However, the standard error and confidence intervals calculated in this case become too narrow, giving a false sense of precision. The ARCH/GARCH models handle this problem by modeling volatility to depend on previous volatility in the regression equation and thereby correcting the deficiencies of least squares model. The ability of the ARCH model to capture volatility clustering and the fact that it is mean reverting makes it a preferred model over other models which fail to capture these important features of time series data.

The GARCH model is a generalization of the ARCH model and it contains the ARCH model as a special case. The GARCH model has an added lagged conditional variance  $\sigma_t^2$  to prevent adding many lagged squared returns as is the case of the ARCH model. The GARCH(p,q) can be explained in a context where a currency trader predicts this period's variance by forming a weighted average of a long term average (the constant), the forecasted variance from last period (the GARCH term), and information about volatility observed in the previous period (the ARCH term). If the asset return is unexpectedly large in either upward or downward direction, then the trader will increase/decrease his estimate of next period's variance. There are some limitations of the GARCH(p,q) model since the model places restriction on the coefficients. The non-negativity conditions on can be violated by the estimation method, since the coefficients of the model can be negative. This limitation of the GARCH model influenced the consideration of another volatility model called the Exponential GARCH (EGARCH) model proposed by Nelson (1991). The EGARCH model eliminates the restriction that the GARCH model imposes on its parameters. The EGARCH model also allows for asymmetric effects between positive and negative asset returns and is a slight modification of the GARCH(p,q) model. This model has been proposed to solve the GARCH model's failure to capture the asymmetric effects of positive and negative asset returns.

### 3.1.1 ARCH(p)

Let  $r_t$  be the return on exchange rate. In a standard linear regression, the ARCH model according to Engle, (1982) and Tsay, (2005), is characterized by mean and volatility equations, which are specified as:

$$r_t = E(r_t | I_{t-1}) + \varepsilon_t = \mu_t |_{t-1} + \varepsilon_t \quad (1)$$

where  $I_{t-1}$  is the information set up to  $t-1$  and  $\mu_{t|t-1} = E(r_t | I_{t-1})$

$\mu_{t|t-1}$  is a conditional mean of  $r_t$ , which is a function of the information set containing all relevant information up to time  $t - 1$ . The conditional variance at time  $t$ , that is  $\sigma_{t|t-1}^2$  is defined as the expectation of the squared process in deviation from its mean:

$$\sigma_{t|t-1}^2 = E \left[ (r_t - \mu_{t|t-1})^2 | I_{t-1} \right] = E(\varepsilon_t^2 | I_{t-1}) \quad (2)$$

If the conditional mean  $\mu_{t|t-1}$  is zero which is a common feature of the financial returns, then the conditional variance becomes  $\sigma_{t|t-1}^2 = E(r_t^2 | I_{t-1})$ . To model the variance efficiently, an important characteristic of the error term  $\varepsilon_t$  and the conditional variance must be noted. The error term  $\varepsilon_t$  is conditionally heteroscedastic because the conditional variance is time varying. We assume that the error term is multiplicative as in  $\varepsilon_t = \sigma_{t|t-1} z_t$  with  $z_t$  being an independent and identically distributed innovation with  $E(z_t) = 0$  and  $var(z_t) = 1$ . The assumption on  $z_t$  is consistent with the definition of the conditional variance. Hence by taking conditional expectation of  $\varepsilon_t^2$  following an ARCH process of order  $p$ , (ARCH(p)) is given by:

$$E(\varepsilon_t^2 | I_{t-1}) \equiv E \left[ (r_t - \mu_{t|t-1})^2 | I_{t-1} \right] = E(\sigma_{t|t-1}^2 z_t^2 | I_{t-1}) = \sigma_{t|t-1}^2 E(z_t^2 | I_{t-1}) = \sigma_{t|t-1}^2 \quad (3)$$

$$= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \dots + \alpha_p \varepsilon_{t-p}^2 \quad (4)$$

where we have used the law of iterative expectation in equation (4). From equation (4), the dependent variable is  $\varepsilon_t^2$  and there are  $p$  autoregressive terms  $\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots, \varepsilon_{t-p}^2$ . From this specification, it is seen that the conditional variance of  $r_t$  is a function of the

magnitude of the previous surprises  $\varepsilon_{t-i}$  for  $i= 1, 2 \dots p$ . Because  $\varepsilon_{t-i}$  is squared, the sign of the surprise is irrelevant; only the magnitude. This specification makes sense if we think that “surprises” are the source of uncertainty, which is at the heart of volatility. To obtain the conditional variance of equation (3), the values of  $\alpha_i$ s need to be estimated. For the conditional variance to be positive,  $\alpha_0 > 0$ ,  $\alpha_i \geq 0 \forall i$ . Also we assume that the conditional mean  $\mu_{t|t-1}$  is constant over time hence we concentrate on modeling of the conditional variance of the stochastic process that is equation (4). To be able to fit the model to the data, the order of  $p$  needs to be specified. The order of  $p$  in the ARCH model can be determined by using either the AIC or the Sample Partial Autocorrelation function (PACF) of the squared returns where the Sample PACF is not significantly different from zero for lags greater than  $p$ . Research has shown that the ARCH model requires a high order to be able to accurately model conditional variance (see Hansen and Lunde (2005) and Bollerslev et al., (1992).

### **3.1.2 GARCH(p,q)**

A generalization of the ARCH model is the GARCH parameterization introduced by Bollerslev (1986). The GARCH models the conditional variance to depend on the weighted average of past squared residuals and it assigns a declining weight to past squared residuals which does not go completely to zero. The most commonly used GARCH specification states that, the best predictor of the variance in the next period is a weighted average of the long run average variance. There are several extensions of GARCH models, however this study follows the general GARCH(p,q) model proposed by Bollerslev (1986). In the traditional GARCH(p,q) model, using equation(1) where



$$r_t = \mu_{t|t-1} + \varepsilon_t \text{ and } \varepsilon_t = r_t - \mu_{t|t-1}$$

then conditional variance is specified as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i (r_{t-i} - \mu_{t|t-1})^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (5)$$

where  $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0$  and  $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$

Equation (6) can also be written as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \quad (6)$$

Equation (6) shows that the GARCH model is similar to ARCH model with a certain structure for the value of the parameters of the lagged returns. In equation (6), the conditional variance does not only depend on past innovation but also on the most recent past level of volatility.  $\sigma_t^2$  is a function of three terms: the mean  $\alpha_0$ , the ARCH ( $\sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2$ ) term and the GARCH term ( $\sum_{j=1}^q \beta_j \sigma_{t-j}^2$ ). The coefficients for the GARCH model are assumed to be positive in order to keep the conditional variance ( $\sigma_t^2$ ) positive. According to Zivot (2009), a GARCH (1,1) model with only three parameters in the conditional variance equation is enough to obtain a good fit for financial time series.

In considering the GARCH(1,1) the conditional variance can be obtained by using a continuous recursive substitution

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (7)$$

where  $\sigma_{t-1}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-2}^2$  hence by replacing

$$= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 (\alpha_0 + \alpha_1 \varepsilon_{t-2}^2 + \beta_1 \sigma_{t-2}^2)$$

$$\begin{aligned}
&= \alpha_0(1 + \beta_1) + \alpha_1(\varepsilon_{t-1}^2 + \beta_1\varepsilon_{t-2}^2) + \beta_1^2\sigma_{t-2}^2 \\
&= \alpha_0(1 + \beta_1 + \beta_1^2) + \alpha_1(\varepsilon_{t-1}^2 + \beta_1\varepsilon_{t-2}^2 + \beta_1^2\varepsilon_{t-3}^2) + \beta_1^3\sigma_{t-3}^2 \\
\sigma_t^2 &= \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{i=0}^{\infty} \varepsilon_{t-1-i}^2 \beta_1^i \tag{8}
\end{aligned}$$

From equation (8) it is seen that the GARCH (1,1) model looks like ARCH( $\infty$ ) model with a variation in the structure of the value of the parameters and the lagged returns  $\varepsilon_{t-1}^2$ .

As already noted the GARCH model cannot account for leverage effects and does not allow for any direct feedback between the conditional variance and the conditional mean. For these reasons the study considers an asymmetric GARCH model called EGARCH model created by Nelson (1991) to measure the volatility of exchange rate.

### 3.1.3 The Exponential GARCH

The EGARCH (p,q) is specified as

$$\varepsilon_t = z_t \sqrt{\sigma_t} \text{ then conditional variance is given by} \tag{9}$$

where  $\varepsilon_t \sim N(0, \sigma^2)$  and the function  $g(\cdot)$  is specified as

$$\log(\sigma_t^2) = \alpha_0 + \sum_{i=1}^p \{ \alpha_i \varepsilon_{t-i} + \gamma_i [ |\varepsilon_{t-i}| - E(|\varepsilon_{t-i}|) ] \} + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) \tag{10}$$

From equation (10) the asymmetrical effect of positive and negative asset returns is very clear. Positive shocks have an impact  $(\alpha_i + \gamma_i)$  on the logarithm of the conditional variance while negative shocks have an impact  $(\alpha_i - \gamma_i)$ . Normally,  $\alpha_i < 0, 0 \leq \gamma_i < 0$  and  $\beta_i + \gamma_i < 1$ . This conformation means that negative shocks have larger impact

than positive shocks which conforms to the empirical evidence by the so called leverage effect. The EGARCH model does not impose any restriction on the parameters to ensure that the variance is nonnegative. The EGARCH model has the unique ability to model volatility persistence, mean reversion as well as asymmetrical effect. The ability of the EGARCH model to split the impact of positive shocks and negative shocks on volatility makes it differ from the GARCH model.

## CHAPTER FOUR

### 4.1 Estimation and forecasting

This chapter discusses the various techniques used in estimating the parameters of the volatility models. Once the models and their respective order has been then we employ the maximum likelihood estimation fit the models to the data by estimating the parameters of the models.

#### 4.1.1 Maximum Likelihood Estimation

The likelihood function is a joint probability density function which estimates the parameters of a statistical model with a given data.  $L(\Theta | \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$  where  $\Theta$  is the set of parameters to be estimated. If the returns are independent of each other the joint density function will simply be defined as the product of the marginal densities. In the ARCH/GARCH models returns are not independent however, the joint density function can be decomposed as the products of the conditional density and the marginal density functions and it can be written as

$$f(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n | \Theta) = f(\varepsilon_n | I_{n-1})f(\varepsilon_{n-1} | I_{n-2}) \dots f(\varepsilon_2) \dots f(\varepsilon_1) \quad (11)$$

The likelihood function is maximized with respect to the unknown parameters and is defined as

$$L(\Theta | I_{n-1}) = \prod_{t=1}^n f(\varepsilon_t | I_{t-1}) \quad (12)$$

Where  $I_t$  represents the information available at time  $t$  and  $f$  is the density function of  $\varepsilon_t$ . The exact form of the likelihood function is dependent on the parametric form of the distribution of the innovations. This means that the distribution for the error,  $\varepsilon_t$ , needs to

be determined to be able to fully specify each model. The error terms are assumed to be identically distributed and independent with zero mean and unit variance. This study will therefore consider two types of error distributions, the standard normal distribution  $\varepsilon_t \sim N(0,1)$  and the heavier tailed student's t-distribution  $\sqrt{v/(v-2)}\varepsilon_t \sim t_v$ . The study considers the student's t-distribution because the analysis of the exchange rate return data exhibit heavier tail most of the time. The scale factor of the error term when the distribution is assumed to be a student-t distribution is introduced to make the variance of  $\varepsilon_t$  equal to 1.

### Normal Distribution

If  $\varepsilon_t$  is assumed to follow a normal distribution with mean  $\mu_t$  and variance  $\sigma_t^2$  then the density function of the distribution is defined as

$$f(\varepsilon_t) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{(\varepsilon_t - \mu_t)^2}{2\sigma_t^2}}, \quad -\infty < \varepsilon_t < \infty \quad (13)$$

### Student t-distribution

On the other hand, if  $\varepsilon_t$  is assumed to follow student t distribution with  $\nu$  degrees of freedom then the density function is given by

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\varepsilon_t^2}{\nu}\right)^{-\left(\frac{\nu+1}{2}\right)}, \quad -\infty < \varepsilon_t < \infty \quad (14)$$

Where  $\nu$  represents the degree of freedom and  $\Gamma$  denotes the gamma function,  $\Gamma(x) = \int_0^\infty y^{x-1}e^{-y}dy$ . To obtain the standardized distribution the density function is scaled

with  $\sqrt{\frac{v-2}{v}}$ . Given the distributional assumptions for  $\varepsilon_t$ , it follows that the corresponding log-likelihood is given by

$$\log[L(\theta | I_{n-1})] = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^n \log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^n \frac{\varepsilon_t^2}{\sigma_t^2} \quad (15)$$

When  $\varepsilon_t \sim N(\mu_t, \sigma_t^2)$  and

$$\log[L(\theta | I_{n-1})] = n \log \left[ \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)}\Gamma(\frac{v}{2})} \right] - \frac{1}{2}\sum_{t=1}^n \log(\sigma^2) - \frac{v+1}{2}\sum_{t=1}^n \log \left[ 1 + \frac{\varepsilon_t^2}{\sigma_t^2(v-2)} \right] \quad (16)$$

When  $\varepsilon_t \sim \text{student } t$  distribution with  $v$  degrees of freedom.

Note that the variance  $\sigma_t^2$  is substituted recursively with the specified conditional variance model discussed in section 3. For a given volatility model, by maximizing either equation (18) or (19) will yield the estimates of the parameters. However, for this study, the maximization of the log-likelihood functions is done using E-view software. The log-likelihood function is maximized with respect to the parameters to yield the optimal in-sample fit for each model. These parameters will then be used to construct volatility forecasts for the respective models.

## 4.2 Forecasting

In forecasting, it is most appropriate to select the loss function to be used. However, in Economics and Finance the use of a quadratic loss function is common because it is often more mathematically tractable than other loss functions. Also the quadratic loss function is symmetric which means that an error above a set target causes the same loss as the same magnitude of error below the set target. In quadratic loss function, the loss is

expressed in a quadratic form as the deviations of the variables of interest from their desired values; this approach is tractable because it results in linear first-order conditions. The functional form for a quadratic loss function is given by  $L(\varepsilon_{t,h}) = \alpha \varepsilon_{t,h}^2$  for  $\alpha > 0$  and  $h$  is the forecast horizon. In the context of stochastic control, the expected value of the quadratic form is used to obtain the optimal forecast. To construct the expected value of the loss is to minimize the expected loss function.

$$E(L(\varepsilon_{t,h}) | I_t) = \alpha E(\varepsilon_{t,h}^2) = \alpha E(y_{t+h} - f_{t,h}^* | I_t)^2 \quad (17)$$

$$\frac{\partial E(L(\varepsilon_{t,h}) | I_t)}{\partial f_{t,h}} = -2\alpha E(y_{t+h} | I_{t-1}) + 2\alpha f_{t,h}^* = 0 \quad (18)$$

which implies;

$$f_{t,h}^* = E(y_{t+h} | I_t) \quad (19)$$

Where  $f_{t,h}^*$  is the optimal forecast at time  $t$  of the random variable  $y_{t+h}$ . If we assume that the conditional density is a normal density with conditional mean. The uncertainty of the forecast which is defined as the variance of the forecast error, is given by,

$$\sigma_{t+h|t}^2 \equiv \text{var}(y_{t+h} | I_t) = E(y_{t+h} - f_{t,h} | I_t)^2$$

#### 4.2.1 Forecasting with ARCH (p) model

In obtaining the optimal forecast of the stochastic process, using equation (3) will necessitate the use of the recursive process and we also assume a quadratic loss function that is the mean squared errors. With information set up to time  $t$ , the 1-step-ahead variance forecast is given by:

$$\sigma_{t+1|t}^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 \quad (20)$$

If  $h = 2$  given the information set up to  $t$ , the innovation  $\varepsilon_{t+1}$  is unknown at time  $t$ , hence conditional expectation of the squared innovation is used in the forecasting process. Thus the two step-ahead forecasts is:

$$\sigma_{t+2|t}^2 = \alpha_0 + \alpha_1 E(\varepsilon_{t+1}^2 | I_t) = \alpha_0 + \alpha_1 \sigma_{t+1|t}^2 \quad (21)$$

Reasoning along the same lines the three-step-ahead forecast is given by:

$$\sigma_{t+3|t}^2 = \alpha_0 + \alpha_1 E(\varepsilon_{t+2}^2 | I_t) = \alpha_0 + \alpha_1 \sigma_{t+2|t}^2 \quad (22)$$

By substituting the previous expression for  $\alpha_1 \sigma_{t+2|t}^2$  from equation (22) we obtain

$$\sigma_{t+3|t}^2 = \alpha_0(1 + \alpha_1) + \alpha_1^2 \sigma_{t+1|t}^2 \quad (23)$$

Hence for any  $h > 1$ , using backward substitution we find the  $h$ -step-ahead forecast of the conditional variance is

$$\sigma_{t+h|t}^2 = \alpha_0(1 + \alpha_1 + \alpha_1^2 + \alpha_1^3 + \dots + \alpha_1^{h-2}) + \alpha_1^{h-1} \sigma_{t+1|t}^2 \quad (24)$$

$$= \frac{\alpha_0}{1 - \alpha_1} + \alpha_1^{h-1} \sigma_{t+1|t}^2 \text{ as } h \rightarrow \infty \quad (25)$$

Equation (28) follows by taking into account the sum of geometric progression with  $\alpha_1 < 1$ , which is equal to  $1/(1 - \alpha_1) = 1 + \alpha_1 + \alpha_1^2 + \alpha_1^3 + \dots$ . In addition, the forecast converges to a constant when the forecast horizon is large, that is  $\sigma_{t+h|t}^2 = \frac{\alpha_0}{1 - \alpha_1}$  for  $h \rightarrow \infty$ .



#### 4.2.2 Forecasting with GARCH (p,q) model

To obtain the optimal forecast of the GARCH(1,1) model, as in the ARCH processes we assume a quadratic loss function that is a mean squared error loss. With an information set up to time  $t$ , the 1-step-ahead variance forecast is given as

$$\sigma_{t+1|t}^2 = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 \sigma_{t|t-1}^2 \quad (26)$$

When the forecast horizon is more than 1-step-ahead that is  $h > 1$  the recursive procedure is used. For instance, when  $h=2$ , with information up to time  $t$ , the 2-step-ahead forecast is

$$\sigma_{t+2|t}^2 = \alpha_0 + \alpha_1 E(\varepsilon_{t+1}^2 | I_t) + \beta_1 \sigma_{t+1|t}^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+1|t}^2 \quad (27)$$

and the 3-step-ahead forecast is:

$$\sigma_{t+3|t}^2 = \alpha_0 + \alpha_1 E(\varepsilon_{t+2}^2 | I_t) + \beta_1 \sigma_{t+2|t}^2 = \alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+2|t}^2 \quad (28)$$

$$= \alpha_0 (1 + \alpha_1 + \beta_1) + (\alpha_1 + \beta_1)^2 \sigma_{t+1|t}^2 \quad (29)$$

The general  $j$ -step ahead forecast for  $\sigma_{t+j|t}^2$  at the forecast origin  $t$  using recursive substitution is given by

$$\sigma_{t+j|t}^2 = \alpha_0 + \alpha_1 E(\varepsilon_{t+j-1}^2 | I_t) + \beta_1 \sigma_{t+j-1|t}^2 \quad (30)$$

$$= \alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+j-1|t}^2$$

$$= \alpha_0 + (\alpha_1 + \beta_1) [\alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+j-2|t}^2]$$

$$= [\alpha_0 + (\alpha_1 + \beta_1) \alpha_0] + (\alpha_1 + \beta_1)^2 [\alpha_0 + (\alpha_1 + \beta_1) \sigma_{t+j-3|t}^2]$$

$$\begin{aligned}
&= [\alpha_0 + (\alpha_1 + \beta_1)\alpha_0 + (\alpha_1 + \beta_1)^2\alpha_0] + (\alpha_1 + \beta_1)^3[\alpha_0 + (\alpha_1 + \beta_1)\sigma_{t+j-4|t}^2] \\
&\quad \cdot \\
&\quad \cdot \\
&\quad \cdot \\
&= \alpha_0 \sum_{i=0}^{j-1} (\alpha_1 + \beta_1)^i + (\alpha_1 + \beta_1)^{j-1} \sigma_{t+1|t}^2
\end{aligned} \tag{31}$$

Using the sum of geometric progression formula where  $\sum_{i=1}^{j-1} r^i = \frac{(1-r^{j-1})}{(1-r)}$  the last equation becomes

$$\sigma_{t+j|t}^2 = \alpha_0 \frac{1-(\alpha_1+\beta_1)^{j-1}}{1-(\alpha_1+\beta_1)} + (\alpha_1 + \beta_1)^{j-1} \sigma_{t+1|t}^2 \tag{32}$$

#### 4.2.3 Forecasting with the EGARCH(p,q) model

To obtain the forecast of the EGARCH(1,1) which is reported in this study, at the forecast origin  $t$ , the 1-step-ahead forecast of  $\sigma_{t+1}^2$  is

$$\log(\sigma_{t+1}^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \gamma_1 (|\varepsilon_{t-1}| - E(|\varepsilon_{t-1}|)) + \beta_1 \log(\sigma_{t-1}^2) \tag{33}$$

#### 4.3 Forecasting using the Data

For the in-sample period the data used are weekly exchange rate of the five currencies. The study also uses the sum of the daily return of exchange rate to compute the proxy for the latent weekly volatility (realized volatility). The realized volatility is used as one of the variables to calculate for both the in-sample and the out-of-sample performance of the volatility model. The study considers both in-sample-fit and out-of-sample forecast because in forecasting, it is not necessarily the case that the model that provides the best in-sample fit becomes the model that produces the best out-of-sample volatility forecast Sharmiri et al (2009). Hansen and Lunde (2001) assert that, it is very important to use

the out-of-sample forecast to guide the selection of the model which possesses the best predictive power. The evaluation of the out-of-sample forecast performance of the various volatility models use a different data set from the one that is used to fit the model for the in-sample-forecast. Typically, the data is divided into two subsets. One subset is for fitting the parameters of the model and the other subset is used to evaluate the forecasting performance of the models. That is if the data consist of  $n$  number of data points  $\{p_1, p_2, p_3, \dots, p_n\}$  then it can be divided into the subset  $\{p_1, p_2, p_3, \dots, p_t\}$  and  $\{p_{t+1}, p_{t+2}, p_{t+3}, \dots, p_n\}$  where  $t$  is the initial forecast origin. According to Tsay (2008), a reasonable choice is  $= \frac{2n}{3}$  for the in-sample fit and the rest is for forecasting the out-of-sample. if  $h$  is the maximum forecast horizon of interest, then the out-of-sample forecasting evaluation process using the recursive scheme following the work of Wennstrom (2014) work as follows

1. If  $t$  is the initial forecast origin. Then we fit each of the models using the data  $\{p_1, p_2, p_3, \dots, p_t\}$ . After that we calculate the 1-step to  $h$ -step ahead forecast from the forecast origin  $t$  using the fitted models.
2. The forecast errors can be estimated for each of the 1-  $h$ -step ahead forecast for the models. That is the difference between the forecast volatility and the actual volatility.
3. As we advance the origin by 1, that is  $t + 1$  and start over from step 1. Repeat this process until the forecast origin  $t$  is equal to the last data point  $n$

After all the forecast and their forecast errors for the models have been estimated then the loss functions are used to evaluate the 1-step-  $h$ -step ahead forecasts for each of the

models. The forecasting scheme described above clearly shows that the estimation sample expands as the forecast origin is increased. The advantage of this type of forecasting scheme is that at each step all forecast is based on all past information and the forecasting scheme is most useful when the model is stable over time but everything breaks down when there is structural break in the data. However, this study makes use of fixed forecasting environment where each model is fitted to the data only once using the estimation sample that contains information up to the forecasting origin. Then the forecast can be estimated. To compute the forecast for the next data point will necessitate the use of the same parameters but when forecasting from the origin, the observation until that week is available, that is at each observation only the data is updated with new information. This study sets the forecast horizon to 1 hence the forecast that is computed at each data point is 1-week-ahead forecasts. This is in line with the objective of this study which focuses on comparing the forecasting performance of volatility models and not comparing forecasting horizons.

In analyzing financial time series most researchers study the return time series rather than raw price data. Campbell, Lo and MacKinlay (1997) provide two main reasons for this. First, the return of an asset is a complete, scale free summary of that particular investment opportunity. Secondly, the return series are much easier to analyze than the raw price series since returns are uncorrelated, have time-varying volatility and a large (small) movements tends to be followed by large (small) movements. Another interesting future worth noting about returns is that, it is not normally distributed because it has fat tails. There are several definitions of returns (the absolute return, squared return and log of return). However, this paper uses the log of the returns to replace returns. The

variables of interest are weekly log returns,  $r_t$ , defined by the intraweek difference of the natural logarithm of the weekly exchange rate,  $x_t$ . Log of the returns is used because it possesses all the properties which are required for successful implementation of our model (that is the log of the returns are stationary, have serial correlation and volatile as well). Also volatility clustering maybe more apparent in the log of the return than using just the absolute return. The weekly exchange rate returns are thus defined by

$$r_t = [\log(x_t/x_{t-i})] = [\log(x_t) - \log(x_{t-i})] \quad (34)$$

and  $i = i^{th}$  series of the exchange rate

#### 4.4 Model Selection

To select the models to be used for comparison, each volatility model with the appropriate order has to be determined. This appropriate order selection of volatility models can be done by fitting each model to the data. The general evaluation methods that are commonly used to compare any model's fit to data is an information criterion test. The basic idea of information criteria tests are that the maximum likelihood for each of the model is subjected to a penalty for its complexity, usually the numbers of parameters. The calculation of the penalty differs from one information criterion test to the other. This study employs one of the most well used criteria, Akaike information criterion (AIC) since it suitable for weekly financial time series data as noted in literature.

The Akaike information criterion is given by

$$AIC = -2 \log L(\theta) + 2k \quad (36)$$

Where  $\log L(\theta)$  is the maximized log-likelihood function for a model with  $k$  number of parameters.

To compare the in-sample fit of models using the AIC information criterion tests, the smaller the value of the criterion, the better. A model with a smaller AIC provides the best in-sample fit taking into account the numbers of parameters needed.

#### **4.5 Forecast Evaluation**

When the conditional variance models have been fully specified with their respective order and error distribution, the models have been fitted to the in-sample data and the 1 week-ahead forecasts for each of the week in the out-of-sample period has been computed, the out-of-sample evaluation is the final step which is performed in the study. However, volatility is unobserved hence to achieve the objective of examining the in-sample forecast and out-of-sample performance of the various models requires the construction of realized volatility to serve as a proxy for the actual volatility. In addition to the realized volatility is the choice of statistical measure for the evaluation of the volatility models and as pointed out by Bollerslev et al., (1994), the estimation of the realized volatility and the choice of the statistical measure is a difficult task.

Realized volatility used in this study is given by

$Vol = \sum_{t=1}^n R_t^2$  where the  $R_t =$  continuously compounded daily returns as calculated by

$$R_t = \ln\left(\frac{r_t}{r_{t-i}}\right)$$

$n$  =number of trading days in the week and  $t$  =counter representing each day. The choice of this realized volatility measure is informed by the closeness of the realized volatility to the actual volatility and its simplicity.

After the volatility proxy has been estimated the study uses four loss functions to assess the out-of-sample forecast performance of the models. The loss functions are defined below as

$$MSE = n^{-1} \sum_{t=1}^n (\widehat{\sigma}_{p,t}^2 - h_{t-1}^2(1))^2$$

$$R^2 LOG = n^{-1} \sum_{t=1}^n \left( \log \frac{\widehat{\sigma}_{p,t}^2}{h_{t-1}^2(1)} \right)^2$$

$$PSE = n^{-1} \sum_{t=1}^n \left( \widehat{\sigma}_{p,t}^2 - h_{t-1}^2(1) \right)^2 h_{t-1}^{-4}(1)$$

$$MAD = n^{-1} \sum_{t=1}^n |\widehat{\sigma}_{p,t}^2 - h_{t-1}^2(1)|$$

The Mean Square Error (MSE) is the average of the squared deviations of the forecast from the volatility proxy. Mean squared error has the disadvantage of heavily weighting outliers. This is as a result of the squaring of each term, which effectively weights large errors more heavily than small ones this means that a large deviation is assigned a much higher weight than a sum of small deviations even if the sum of the small deviations is equal to the one-time large deviation. In evaluating volatility forecasting performance, it may seem quite illogical since in general one large deviation is not more troublesome

than a sum of small deviations that sum up to the size of the large deviation since the returns accumulate over time. Moreover, single outliers will have a significant impact on the MSE criteria. The  $R^2LOG$  also give higher weight to large deviations but they are not as penalized as in the case of MSE. The Percentage Squared Errors (PSE) measures the average of the squared percentage deviation. The errors are expressed as a percentage of the forecasted volatility. The PSE corrects the challenge in measuring high variance in absolute terms and hence estimates the relative error as a percentage to account for this. The Mean Absolute Deviation (MAD) is interesting since it is very robust to outliers and this criterion actually gives equal weighting to a large deviation of size  $\varepsilon$  as to a sum of several deviations accumulating to  $\varepsilon$ .



## CHAPTER FIVE

### 5.1 Data and Results

We now proceed to tests the predictive power of volatility models using six major currencies in the world, US dollar, Canadian dollar, British pounds, the Japanese Yen, Swiss franc and the Euro. US dollar is used as the base currency against which all the other currencies are compared. The complete set of data used in this study is the weekly exchange rate data for the six currencies of which data for all of the currencies ranges from 2007w1 to 2015w52. The data used for the study is sourced from ecomagic.com.

The data are divided into 7 years 47 weeks in-sample period and 5 weeks out-of-sample period. The total number of observations used for the study therefore comes to 2340 exchange rate series.

Figure 1A plots the exchange rate series for the entire period. From the plots of the exchange rate series, it is evident is that the levels of exchange rate do not appear to be stationary and there are a clutter of activity among all the exchange rate series plotted. This indicates that, a high volatility is followed by another high volatility.

Figure 1

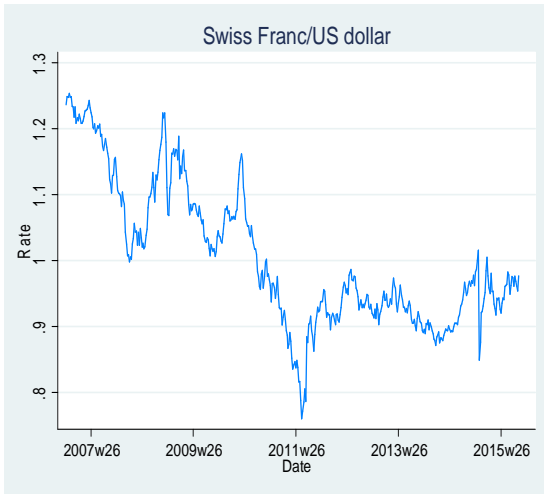


Figure 2

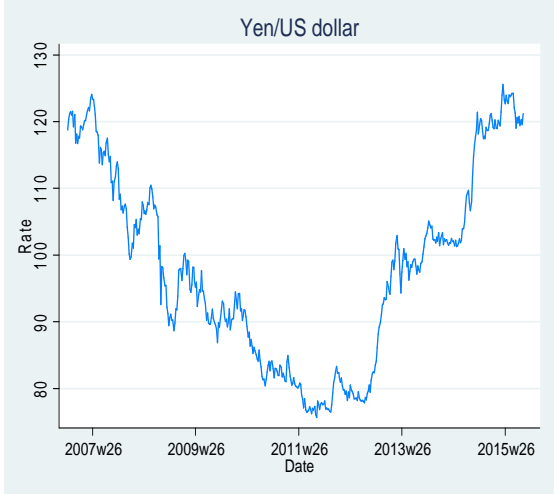


Figure 3

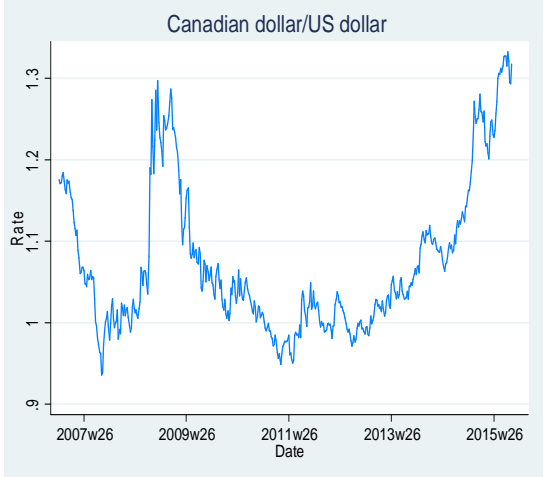


Figure 4

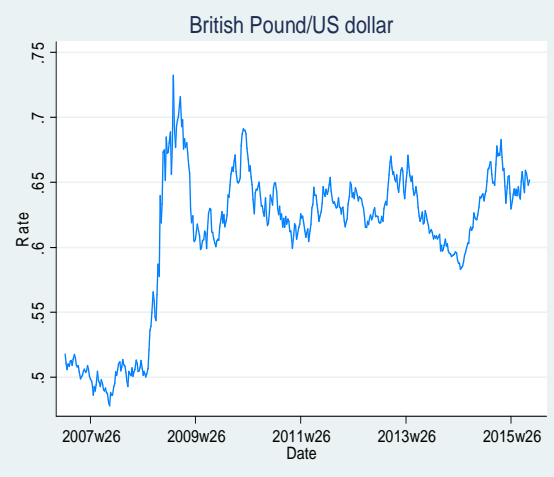


Figure 5

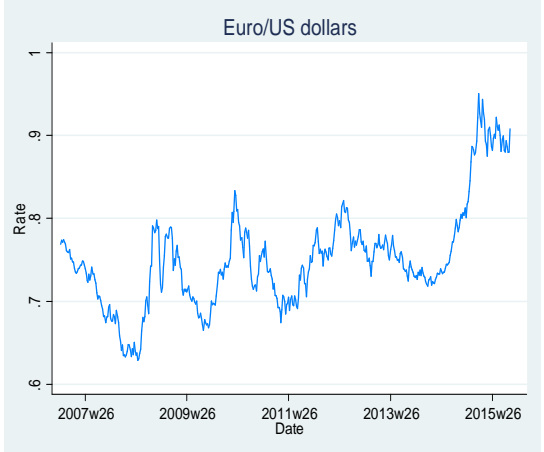


Figure 1A: Graph of various exchange rate series

As already discussed in chapter four, the main variable of interest in the study is not the price process but the weekly log return defined in equation (34). Figure 1B shows the daily return for the in-sample period. The daily return series appears to follow a stationary process with a mean close to zero. Additionally, one can observe that the assumption of constant variance is not valid for all series but with volatility exhibiting relatively calm periods followed by more turbulent periods. This is one of the key characteristics mentioned in the introduction of asset return volatility and is referred to as volatility clustering. Also the impact of the recent financial crisis though represents relatively a short period in the entire sample; however, it appears to have strong effect on the exchange rate variability in those countries. All diagrams of figure 1B except the Swiss Franc show an increase in the amplitude of its variability between the periods of 2008-2009.

Figure 6

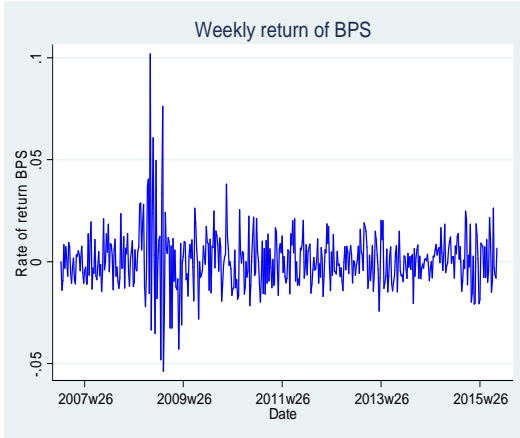


Figure 7

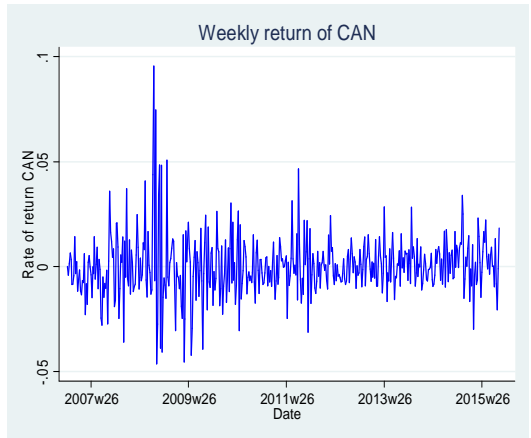


Figure 8

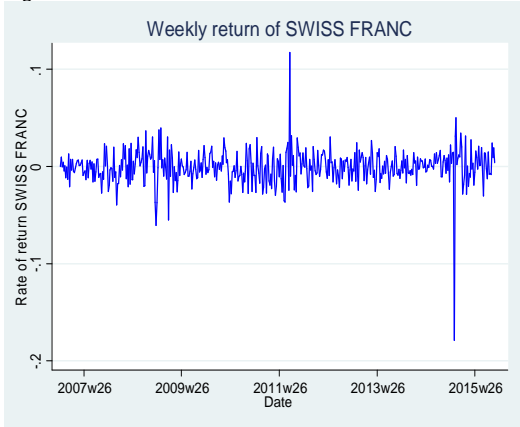


Figure 9

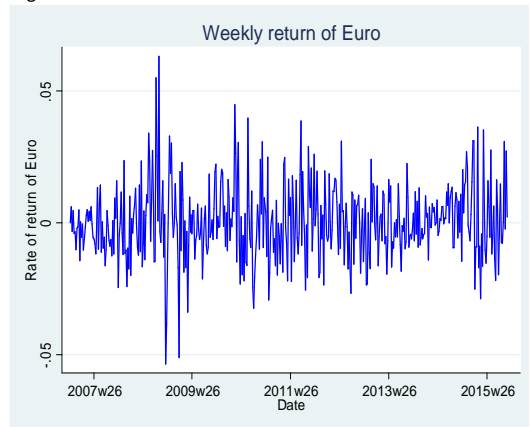
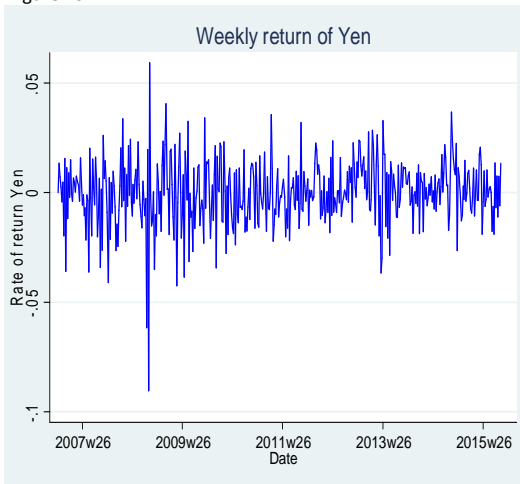


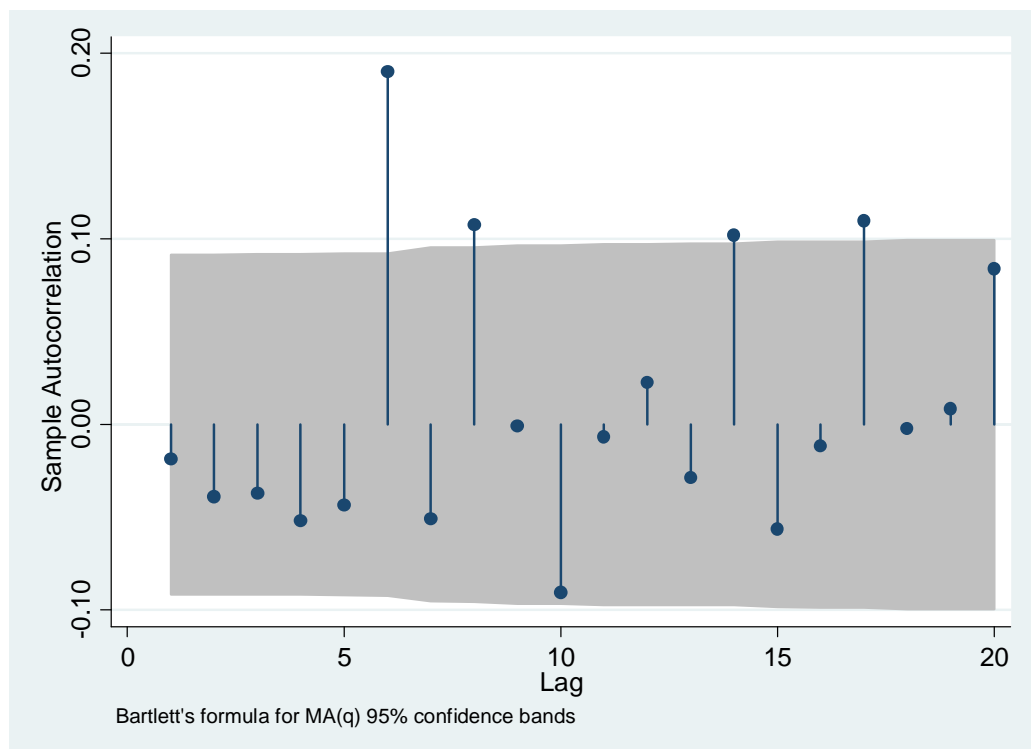
Figure 10



**Figure 1B: Weekly return of the various exchange rate series.**

### 5.1.1 Sample Autocorrelation function for Canadian dollar

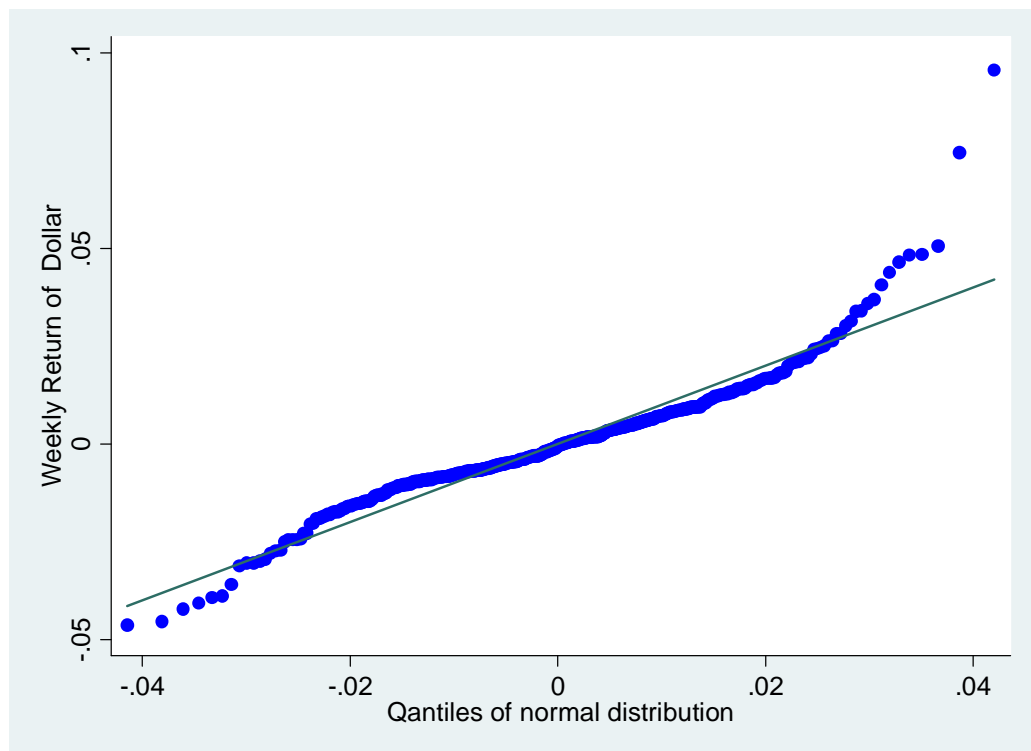
The ARCH/GARCH model can successfully be implemented when the data exhibit some minor serial correlation. To check for serial correlation in the data we use Sample Autocorrelation function and the graph are presented for analysis. The plot in figure 2 shows the Sample Autocorrelation Function for the daily returns of lags 0 to 20 for the Canadian dollar. The Sample Autocorrelation function is a very useful tool and can be used for checking for serial correlation in the return data. Based on ocular inspection of the Sample Autocorrelation Function plot, it is not completely clear whether the data is serially correlated even though it has minor significant serial correlation at lag 6 and 8. Similar conclusions are reached on all the other exchange rate series which are presented in appendix B.



**Figure 2: Sample Autocorrelation function for Canadian dollar**

### 5.1.2 QQ plot of sample data versus distribution

After determining that the data exhibit some minor serial correlation, the next step is to choose the appropriate error distribution for the sample forecast. Hence to choose the appropriate error distribution we use the q-q plot in figure 3. In Figure 3, q-q plot, plots the empirical distribution of the weekly returns (y-axis) against the best fitted normal distribution (x-axis).



**Figure 3:** Quantile plot of Canadian dollar

From figure 3 it is clearly seen that the best fitted normal distribution does not provide a good fit as a reference distribution. This is largely due to the fact that the empirical distribution of the weekly returns exhibit significantly heavier tails than the reference distribution. This requires the consideration of another distribution (t-distribution) which

has the capacity to provide a good fit for data with fat tails. This behavior of the return also reaffirms the summary statistics in table 2 where the weekly return of Canadian dollars has kurtosis of (9.004) which far exceeds the threshold of 3 for the normal distribution. Similar conclusion is reached for the other exchange rate series.

**Table 1: Descriptive statistics of exchange rate returns**

Variable	Mean	Median	Max	Min	SD	Skewness	Kurtosis
Franc	-0.00048	-0.00035	2.6262	-2.6065	0.1719	0.1766	229.545
Yen	0.000027	0.00250	0.05910	-0.09035	0.0148	-0.5844	6.5832
CAN	0.0004	-0.000187	0.0956	-0.0464	0.0146	0.9935	9.0087
BPS	0.00054	-0.00030	0.1019	-0.0538	0.0140	1.1794	11.3154
Euro	0.00037	-0.00045	3.1783	-3.1693	0.2080	0.06011	232.3214

Table 1 reports standard summary statistics for returns of the five exchange rate series. The return distribution is positively for all the currencies except the Japanese Yen. The mean return for Japanese Yen is close to zero and it is 0.0027% per week while the mean return for Swiss Franc, Euro, Canadian dollar and British Pound rate stand at -0.00048, 0.00037, 0.0004 and 0.00054. This shows that within the given periods, the British pound has the highest return than the other currencies. The kurtosis is large for both Euro/US (232.3214) and the Swiss Franc (229.5454) but fairly large for Canadian dollar/US dollar rate (9.009), Yen/US dollar rate (6.583) and Pounds/US dollar rate (11.3154). The weekly standard deviation (SD) shows that the Euro return is the most volatile with a

value of (0.2080), followed by the Swiss Franc (0.1719) while the rest of the series exhibit less volatility around 1.5% in the given period. All the currencies show evidence of fat tails since their kurtosis exceed 3 while negative skewness for the Yen indicates that left tails are particularly extreme. This means significant exchange rate movements in either direction (positive or negative) occur in the forex market with greater frequency than will be predicted under, for example, a normal distribution setting.

### 5.1.3 Jarque-Bera Test

To test for normality of the skewness and kurtosis, Jarque-Bera test is used and the results of the Jarque-Bera test is presented in table 1. The Jarque-Bera test is a goodness-of-fit test which determines if the kurtosis and skewness of the sample data are similar to a normal distribution and it is given by:

$$JB = \frac{n}{6} \left( S^2 + \frac{1}{4} (K - 3)^2 \right) \text{ and}$$

$$S = \frac{n\sqrt{n-1}}{n-2} \frac{\sum_{i=1}^n (X_i - \bar{X})^3}{\left( \sum_{i=1}^n (X_i - \bar{X})^2 \right)^{3/2}}, K = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^4}{\left( \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \right)^2} - 3$$

where  $n$  is the sample size and  $x$  is an exchange rate serie.  $S$  is the sample skewness (sample skewness is the measure of the asymmetry of the probability distribution of a real-valued random variable about its mean) and  $K$  is the sample kurtosis. If the sample data follows the normal distribution, then the  $JB$  should, asymptotically, have a chi-squared distribution with two degrees of freedom. The null hypothesis is that the sample data have a skewness of zero and an excess kurtosis of 3 which is the characteristics of a



normally distributed data. The Jarque-Bera test for the descriptive statistics is determined with Stata in built function sktest.

**Table 2: Jacque-Bera test for Canadian dollar**

Variable	Observation	pr(Skewness)	pr(Kurtosis)	adjchi2(2)	prob>chi2
Return of CAN	463	0.000	0.000	.	0.000

The Jarque-Bara test uses the p-value for the sample Skewness and p-value for the sample Kurtosis to test for the normality of the data set in what is commonly called the portmanteau test. From the table 2 the main focus is on the p-value of the chi square which is 0.000. Since the p-value for the chi square is less than 0.05, it indicates that there is enough evidence to reject the normality of the return series. Similar analysis has been conducted on all the other return series and the same conclusion has been reached however the detail of the analysis has been presented in the appendix in order to facilitate the smooth reading of the paper.

Prior to modeling the exchange rate return series, Augmented Dickey–Fuller (ADF) test is employed to determine the stationarity of the return of exchange rate series based on the following regression.

$$\Delta r_t = \varphi + \beta t + \alpha r_{t-1} + \sum_i^p d_i \Delta r_{t-i} + \mu_t \quad (37)$$

where  $\mu_t$  is a white noise error term,  $dr_{t-i} = r_t - r_{t-1}$ ,  $dr_{t-1} = r_{t-1} - r_{t-2}$ . Equation (37) tests the null hypothesis of a unit root against a stationary alternative and the results are presented in table 3.

**Table 3: ADF test of exchange rate returns**

Variable	Coefficient	Std. Error	t-Statistic	ADF Prob.
Euro	-0.019877	0.01405	-4.36568	0.0000
Franc	-1.035831	0.17280	-5.994199	0.0000
Pounds	-1.176629	0.263190	-4.470649	0.0000
Dollar	-0.970818	0.247912	-3.915979	0.0001
Yen	-0.744175	0.219412	-3.391672	0.0008

-2.573460\*\*\*, -1.94199\*\*, -1.615923\* are 1%, 5% and 10% ADF critical values respectively

Table 3 shows the results of Augmented Dickey–Fuller (ADF) unit root tests for the exchange rate return series. Since the values of the t-statistics are more negative than the ADF critical values at 1%, 5% and 10% levels, we reject the null hypothesis of unit roots (random walk) in the exchange rate returns series. The ADF probability confirms that the test statistics are statistically significant. This means that the exchange rate return series are stationary and therefore rejects the notion of first differencing of the return series.

#### **5.1.4 Model Selection for Forecasting**

Forecasting exchange rate volatility requires the selection of the model with the appropriate lag length for each of the exchange rate series being investigated. As already discussed in chapter three, a more general way to select the model with the optimal lag

length for financial time series data is to use information criterion. This study however employs AIC to select the models with the appropriate lag length for forecasting. For each exchange rate series, the AIC is used to select one model from each of the family of volatility models (ARCH, GARCH, EGARCH) being investigated. As already discussed in chapter 3, the underlining assumption that is used for the analysis in this chapter is the student-t distribution. The models selected based on the AIC criterion is presented in table 4.

In table 4 below, the models with the appropriate lag length which is used for forecasting has been presented. Each row presents a particular exchange rate series with the selected models and their corresponding minimum value of AIC.

**Table 4: Optimal Lag Length for the Estimated Volatility Models**

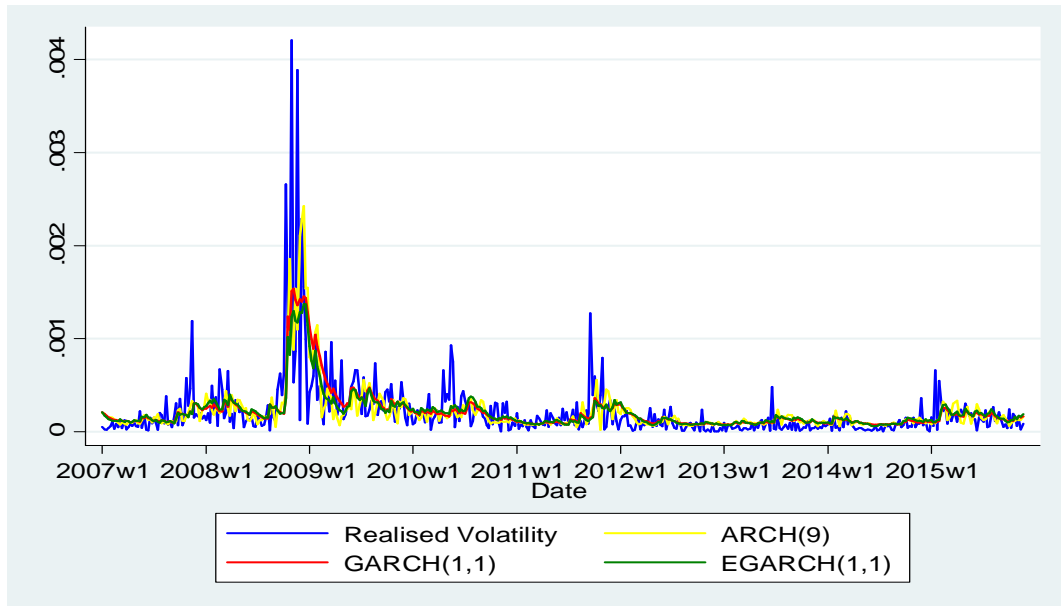
Exchange Rate Returns	Optimal Lag Length					
	ARCH(8)	ARCH(9)	GARCH(1,1)	EGARCH(1,1)	EARCH(2,1)	EGARCH(2,2)
Dollar		-2736.7	-2736.6	-2732.6		
Franc		-2568.3	-2573.5	-2577.3		
Pounds	-2778.9		-2792.3		-2794.1	
Euro	-2675.7		-2673.5	-2676.9		
Yen	-2630.6		-2632.2			-2620.5

From table 4, it is evident that the volatility models with the appropriate lag length for forecasting the Canadian dollar and the Swiss franc are ARCH(9), GARCH(1,1) and EGARCH(1,1). A very coherent results from the table is that, among the GARCH family models the GARCH(1,1) is ranked as the model with the optimal lag length for modeling all the exchange rate series. Contrary to our expectation, the ARCH(8) is selected as the model with optimal lag length for forecasting BPS, Euro and Yen over the ARCH(9) which is a higher order is. The EGARCH(1,1) is selected by the AIC among the

EGARCH family models as having the appropriate lag length for modeling CAN, Franc and Euro while EGARCH(2,1) and EGARCH(2,2) are selected for forecasting BPS and Yen respectively. Having selected the models with the appropriate lag length for forecasting does not give information on the forecasting performance of the models hence the next section examines the in-sample forecasting performance of the selected models. The selection of the GARCH(1,1) as the appropriate model among the GARCH family models for all the exchange rate series conforms to the claim by Javed and Mantalos (2013), that out of the numerous studies that have investigated into model selection for forecasting volatility, the “performance of the GARCH (1,1) model is satisfactory”. Javed and Mantalos (2013) further assert that the first lag of the GARCH(1,1) is sufficient to capture the movements of the volatility.

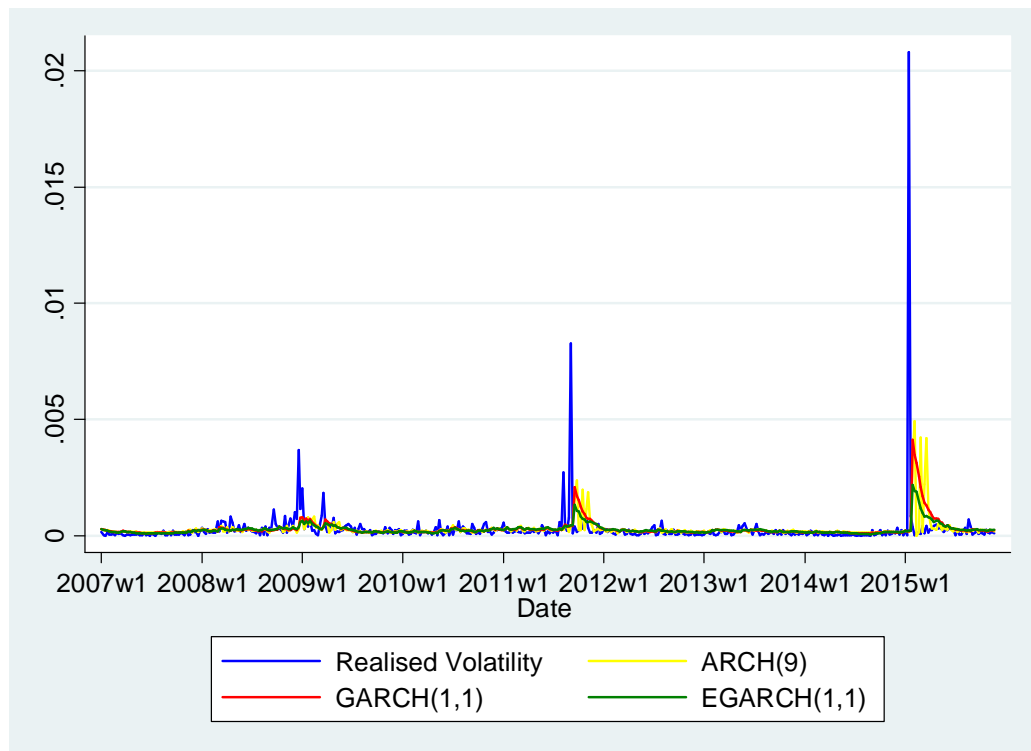
## **5.2 Evaluation of in-sample fit of the models.**

This section assesses the performance of the selected models in section 5.1 to determine the best model for the in-sample forecast of the various exchange rate series. We use both graphs and loss functions to assess the performance of the volatility models. Figures 3-7 below graphically display the internal forecasts of the volatility of CAN, Swiss franc, BPS, Euro and Yen. The figures plot the realized volatility and the models used for forecasting that particular exchange rate series. Also yellow, red and green lines are used for plotting volatility models for ARCH, GARCH and EGARCH family respectively. For each figure the vertical axis measures the conditional variance whilst the horizontal axis measures the duration of the time for the internal forecast.



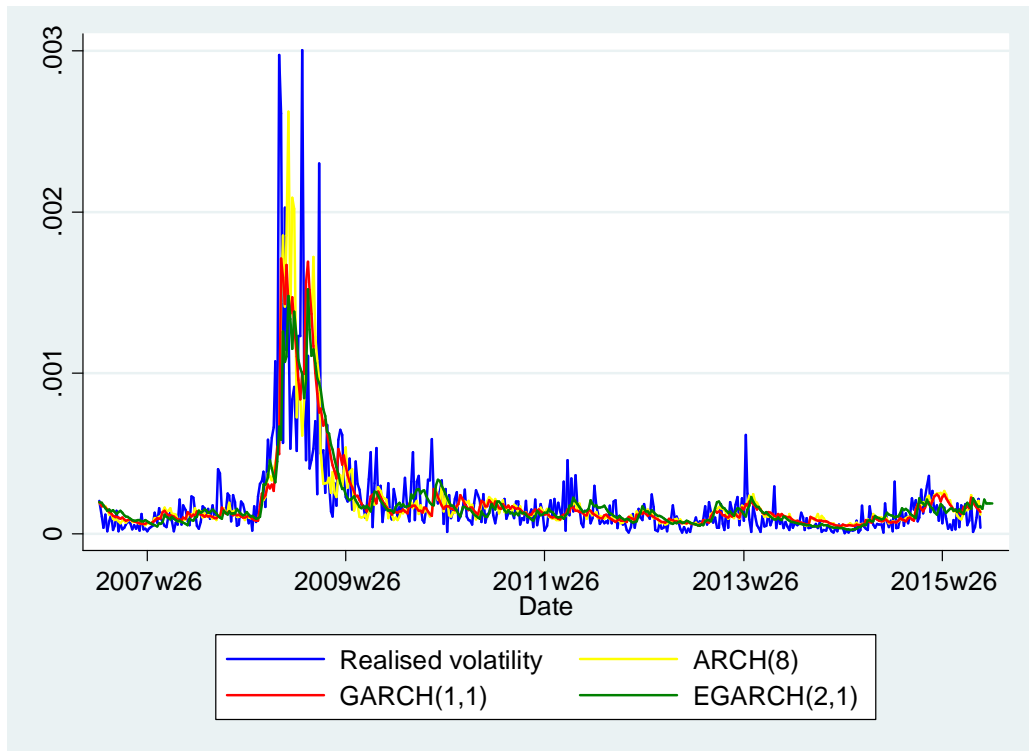
**Figure 3: In-sample forecast of Canadian dollar**

From figure 3, all the volatility models are able to trace the realized volatility effectively. However during the spike period the ARCH(9) performs slightly better than the other models. This is quite an interesting result which debunks the fact that the complex models always perform better in more turbulent periods like the 2008/2009 season than the parsimonious models.



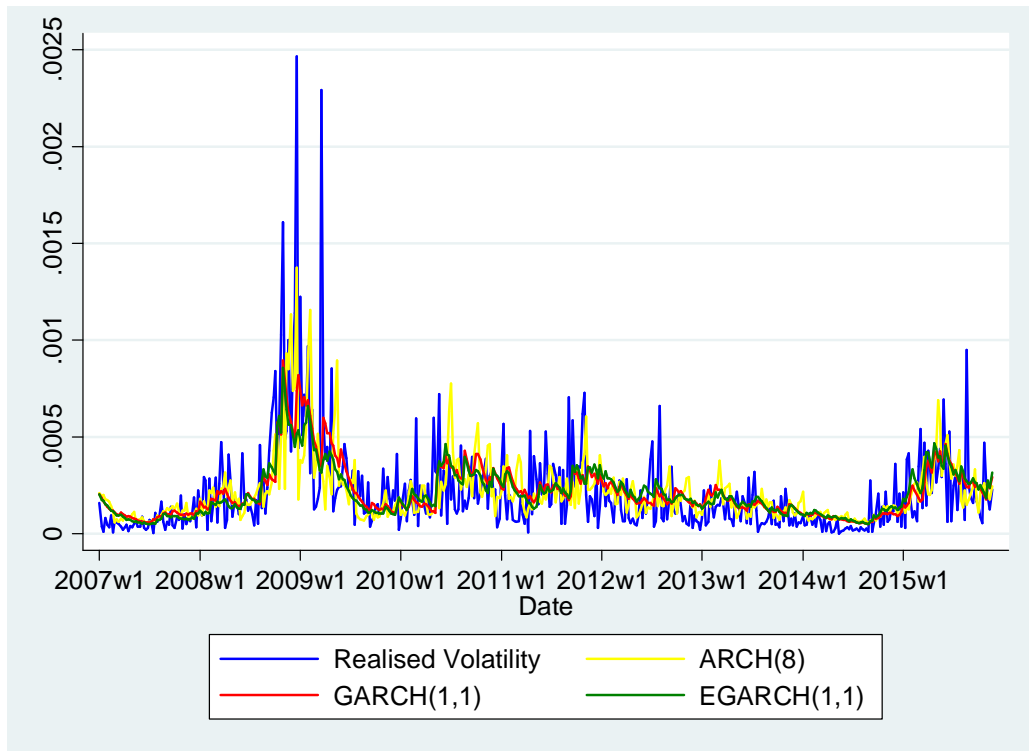
**Figure 4: In-sample forecast of Swiss Franc**

From Figure 4 it is observed that the Swiss Franc has been stable over the years until the overvaluation in 2011 where the Swiss Franc is set 1.2 to 1 Euro. This is seen in the graph as the depreciation of the Swiss franc in 2011w26. Prior to this period all the volatility models are able to trace the realized volatility well however during the spike period it is seen that the GARCH(1,1) model perform better than the both the ARCH and the EGARCH model. Also in December 2015 the Swiss Bank announced it negative interest rate on financial assets and that led to a huge depreciation of the Swiss Franc. Here also the GARCH(1,1) traces better than all the other models. It can therefore be concluded that the GARCH(1,1) performs better during turbulent times which is also in line with available literature.



**Figure 5: In-sample forecast of the Pounds**

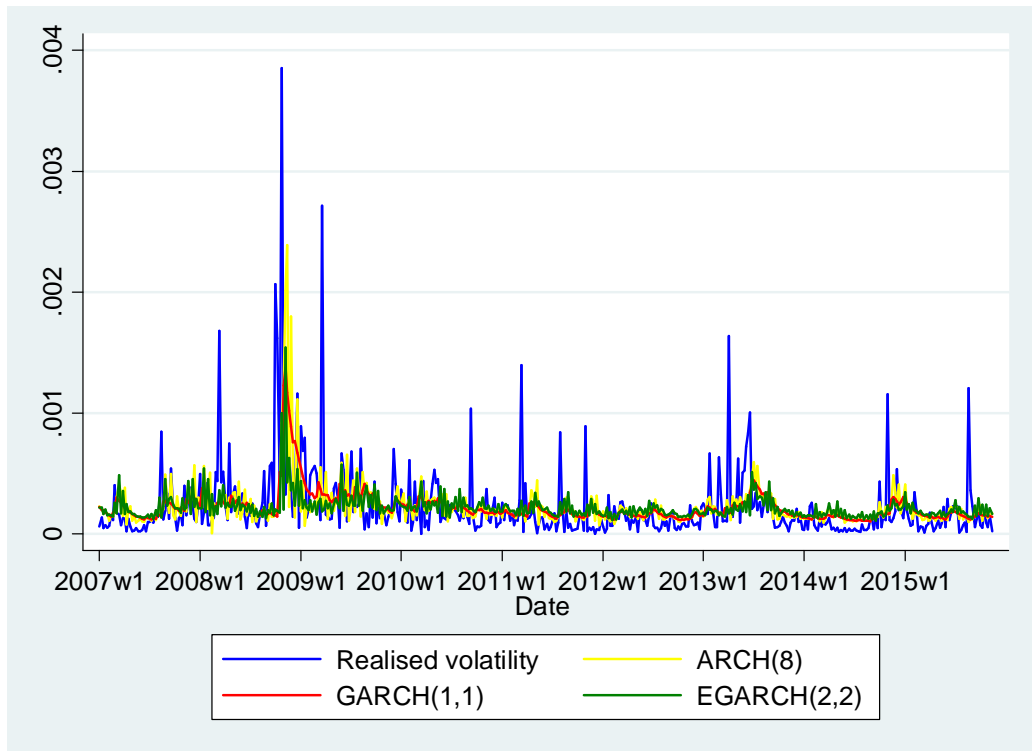
A critical observation of figure 5 indicates that the GARCH(1,1) performs better than the other volatility models. While the ARCH(8) model fails to capture most volatile period as and when they occur, the GARCH(1,1) is able to capture the volatile periods immediately. This also proves the superiority of the GARCH over the ARCH in terms of internal forecast. Though the EGARCH models traces the volatility during the financial crisis in 2008/2009 well, it is still outperformed by the GARCH(1,1). It can be concluded that we cannot necessarily rule out the performance of the more complex models like the EGARCH to effectively model volatility in turbulent time.



**Figure 6: In-sample forecast of the Euro.**

From figure 6 it is evident that the ARCH is not able to traces most of the spike periods especially from 2011-2015 but the GARCH does an excellent work by effectively tracing the volatility during those times. This also confirms the out-performance of the GARCH over the ARCH in the in-sample forecast. From the graph it is difficult to tell the difference between the performance of the GARCH and the EGARCH models. This explains the need for using the loss functions to support the ranking of the model with the best predictive power in the out-of-sample. In relation to the Euro, it can be concluded the complex models performs better in the in-sample forecast than the simple models.





**Figure 7: In-sample forecast of Yen**

Looking at the internal forecast from figure 7, it is difficult to tell which volatility model is able to trace volatility of Yen better. However one could slightly be moved to rank ARCH(8) as the best model for forecasting in-sample because it performs relatively well in the crisis time.(2008/2009).

In assessing the performance of the volatility models we use the loss function to complement the ranking of the models by the graph. Table 5 presents the summary of MSE, R2LOG, MAD and the PSE for the compared volatility models. For all the loss functions presented in table 5, the smallest value is preferred.

**Table 5: MSE, R2LOG, MAD, and PSE values of In-Sample Forecasts**

Models	MSE	R2LOG	MAD	PSE
Canadian dollar				
ARCH(9)	9.63E-08	1.053723	0.0001416	1.698711
GARCH(1,1)	7.87E-08	0.9943268	0.0001310	0.8635479
EGARCH(1,1)	8.07E-08	1.009881	0.0001312	0.8915312
Swiss Franc				
ARCH(9)	1.30E-06	1.254530	0.0002690	36.24738
GARCH(1,1)	1.30E-07	1.254532	0.0002693	37.24738
EGARCH(1,1)	1.16E-06	1.078927	0.000238	21.38693
BPS				
ARCH(8)	6.97E-08	0.6725403	0.0001164	0.6053682
GARCH(1,1)	5.70E-08	0.6448407	0.0001080	0.5599768
EGARCH(2,1)	5.73E-08	0.6630972	0.0001083	0.5837312
EURO				
ARCH(8)	4.37E-08	0.8830094	0.0001314	0.9074312
GARCH(1,1)	3.88E-08	0.7951659	0.000119	0.6377019
EGARCH(1,1)	4.10E-08	0.793628	0.000119	0.646408
Japanese YEN				
ARCH(8)	1.11E-08	1.127133	0.0001706	2.617992
GARCH(1,1)	9.83E-08	1.061603	0.0001587	1.195548
EGARCH(2,2)	9.30E-08	1.164466	0.0001629	1.817026

With reference to the return on Canadian dollars in table 5, a very coherent finding is reached. All the loss functions unequivocally rank the GARCH(1,1) model as the best models for the in-sample forecast of the return of Canadian dollar whilst ARCH(9) suggested by all the loss functions as the worst model in terms of the in-sample forecast. A possible explanation for such finding is that a shock to the forex market in Canada last for a very long time hence the failure of the ARCH(9) to perform better than the GARCH(1,1) since the ARCH is nested in the GARCH model. It is also evident that the

GARCH(1,1) marginally performs better than the EGARCH(1,1). This results supports the findings of Zivot (2009) and Hansen and Lunde (2005) who assess the performance of volatility models and reached a conclusion that the GARCH(1,1) model perform better than all the other volatility models in terms of in-sample forecast.

In relation to the performance of the volatility models in table 5, all the loss functions suggested the EGARCH(1,1) as the best model for the in-sample forecast of the return of the Swiss franc. Similar to the findings for the Canadian dollar, the ARCH(9) is outperformed by all the volatility models under consideration. An interesting conclusion from table 5 is that the more complex models are able to perform better than the more parsimonious model in the in-sample forecast.

Similar to the findings on the return on Canadian dollar from table 5, all the loss functions rank the GARCH(1,1) as the best model for in-sample forecast of BPS. ARCH(8) model is ranked the worst in terms of the in-sample fit for BPS. Additionally there is no evidence of the EGARCH model performing better than the GARCH(1,1) model.

With respect to the internal forecast of the Euro, the GARCH(1,1) model emerges as the most preferred in terms of the MSE, R2LOG and MAD. The only loss function suggesting another model is the R2LOG. Apart from that the conclusion is that the GARCH(1,1) outperforms all the other volatility models in the in-sample whilst all the loss functions suggest the ARCH(8) as the worst in terms of the in-sample fit.

Lastly, in assessing the in-sample performance of the volatility models for Yen, all the loss function except MSE suggest the GARCH(1,1) as the most preferred model.

However, no coherent conclusion is reached on the worst model for the in-sample forecast. Another interesting finding worth noting is the fact that, one model is suggested as the best while the same model is ranked the worst by a different loss function.

### **5.3 Summary of the In-Sample performance of the Volatility Models**

From the analysis of the in-sample forecast of the models, it is seen that the GARCH (1,1) model outperforms all the other (ARCH and EGARCH) models. The ranking of the GARCH(1,1) over the ARCH models is expected because the ARCH model is nested in the GARCH model. So if the GARCH(1,1) model does not provide a better in-sample fit then there is no point in setting the extra parameter of the GARCH term in the GARCH model to anything other than zero which will then reduce the GARCH model to the simple ARCH model. Another interesting detail in Table 5 is that the higher order GARCH does not necessarily provide a better in-sample fit than the GARCH(1,1) which is in line with the finding of Zivot (2009) who finds that the GARCH(1,1) model is just enough to model financial time series data. This conclusion is also in line with the findings of Hansen and Lunde (2005) who examine the performance of ARCH models and conclude that the GARCH (1,1) model outperforms all the complex and parsimonious models.

As emphasized throughout this paper, it is vital to assess the out-of-sample forecasting performance of the volatility models in addition to the in-sample performance. The reasoning for this comes from the objective of the analysis which is also to forecast volatility and not only to model its behavior in the past. From the assessment of the performance of the volatility models in the in-sample forecast, we conclude that the

GARCH(1,1) outperforms all the other models hence the natural question is that, is the GARCH(1,1) going to perform better than the other models in terms of the Out-of-sample forecast ? This question is totally discussed in the next section.

#### 5.4 Evaluation of Out-of-sample forecast

This section discusses one of the objectives of the thesis which is to compare different volatility models in terms of their out-of-sample forecasting performance. The variable under study here is the different models for the conditional variance for each exchange rate series. Each of the volatility models is presented and explained in chapter 3. The main theme is whether the more complex models which are able to exhibit more of the stylized facts and characteristics of asset price volatility provide a better out-of-sample-fit than the more parsimonious models. Also based on previous discussion in terms of in-sample fit, the GARCH(1,1) outperformed all the other volatility models, so the obvious question is that, will the same conclusion be drawn in terms of the out-of-sample forecast? This question is answered using loss functions and graphs to assess the performance of the various volatility models. The ranking of the volatility models by the loss functions is presented in table 6.

Table 6 presents the summary of MSE, R2LOG, MAD and the PSE for the compared volatility models in terms of their out-of-sample performance. For all the loss functions presented in table 6, the smallest value is preferred.

**Table 6: MSPE,R2LOG, MAD and PSE values of Out-of-Sample Forecasts**

Models	MSE	R2LOG	MAD	PSE
Canadian dollar				
ARCH(9)	9.04E-09	.9920989	.0000853	.3055288
GARCH(1,1)	1.09E-08	1.022823	.0000929	.3241589

EGARCH(1,1)	1.11E-08	1.033794	0.0000932	.3230322
	Swiss Franc			
ARCH(9)	9.13E-08	0.5562008	0.001749	1.59861
GARCH(1,1)	9.31E-08	0.5874234	0.0001877	1.630322
EGARCH(1,1)	9.11E-08	0.6088972	0.0001897	1.445158
	BPS			
ARCH(8)	7.93E-09	0.335138	0.0000841	0.3811536
GARCH(1,1)	5.96E-09	0.288671	0.00007355	0.2650327
EGARCH(2,1)	7.04E-09	.3286808	0.0000804	0.2403774
	EURO			
ARCH(8)	1.76E-07	0.8830094	0.0001314	0.9074312
GARCH(1,1)	1.45E-07	0.9701986	0.000262	2.58507
EGARCH(1,1)	1.42E-07	1.006068	0.000266	2.20899
	YEN			
ARCH(8)	9.20E-09	0.9236588	0.0000851	0.2849881
GARCH(1,1)	8.82E-09	0.9214961	0.0000826	0.3114989
EGARCH(2,2)	1.15E-08	1.063917	0.0000949	0.2645036

Table 6 presents the most coherent ranking across the different loss functions in assessing the out-of-sample performance of the various volatility models for the return on Canadian dollar. All the loss functions suggest the ARCH(9) as the most preferred model while the complex parametric Exponential GARCH is ranked the worst. This finding differs from the conclusion in Table 4, where all the loss functions unequivocally rank the GARCH(1,1) as the most preferred model for the in-sample forecasting of the return of Canadian dollar. A lot of valuable lessons can be learnt from this finding. First, it is not necessarily the model with the best in-sample fit that provides the best out-of-sample forecast of exchange rate volatility. An interesting finding contrary to our a priori expectation is that the simple models can provide a better out of sample forecast than the more complex models. This result supports the research work by (Dimson and Marsh, 1990) which concludes that those who are interested in forecasts with reasonable predictive accuracy, the best forecasting models are the simple models.

In relation to the Return on Swiss Franc, Table 5 presents a consistent ranking across the different loss functions. All the four loss functions unequivocally rank EGARCH(1,1) as the most preferred model. Similarly all the four loss functions suggest the GARCH(1,1) model as the worst among the volatility models being compared. Though the ARCH model is preferred to the GARCH model, the EGARCH is preferred to the ARCH as well. This suggests that the more complex models do provide a better out-of-sample fit than the more parsimonious models.

With respect to the return on British Pound Sterling, all the three loss functions (MSE, R2LOG and MAD) rank GARCH(1,1) model as the most preferred. A different conclusion is reached by PSE which rank EGARCH(2,1) as the preferred model. So apart from the ranking of the PSE, the GARCH(1,1) is the best model for out-of-sample forecast of the British Pound Sterling. However all the four different loss functions unequivocally rank the ARCH(8) as the worst in terms of its out-of-sample forecast. This result also suggests that at times the model with the best in-sample fit is able to forecast better than the other models.

Similar to the findings on the return on British Pound Sterling, three loss functions (R2LOG, MAD and PSE) suggest ARCH(8) as the best model among the volatility models compared in out-of-sample forecast for the Euro. However this same volatility model is ranked as the worst model by MSE. This finding illuminates the need for selecting adequate loss function for the intended purpose of the forecast. It can be concluded that the ARCH(8) is suggested as the overall best model for forecasting the out-of-sample volatility of the Euro. With respect to the worst model for the out-of-sample forecast, the results from table 5 is inconclusive as different loss functions suggest

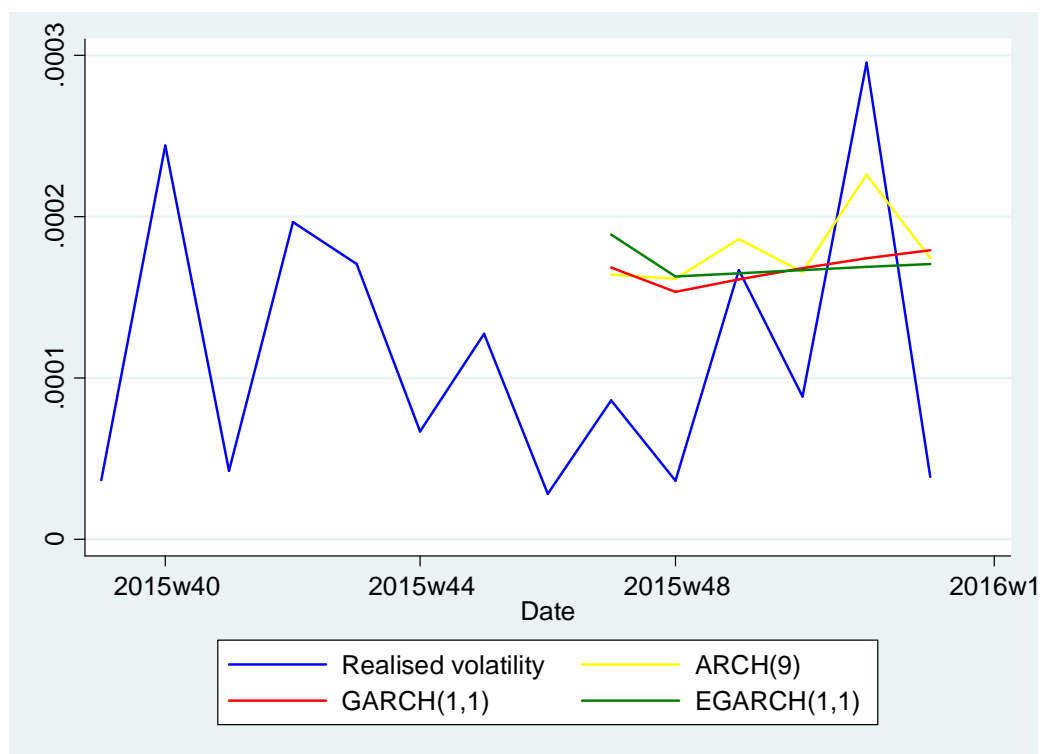
different models as the worst for the out-of-sample forecast. It is quite a contrast that one loss function suggests that a particular model is the worst and another loss function suggests that same model to be the best. This illuminates the importance of choosing an adequate loss function for the intended purpose of the forecast. The conclusion is that it is not the model with the best in-sample fit that provides the best out-of-sample fit. Also the simple models are able to forecast better than the complex models in terms of the out-of-sample.

Similar to the findings on the return on Euro, three loss functions (MSE, R2LOG, and MAD) rule that the GARCH(1,1) is the best of all the volatility models compared in terms of their out-of-sample forecast for the Yen. However this same volatility model is ranked as the worst model by PSE. This finding also reaffirms the assertion of the need for selecting adequate loss function for the intended purpose of the forecast. It can be concluded that the GARCH(1,1) is suggested as the overall best model for forecasting the out-of-sample volatility of the YEN. It also evident that the GARCH(1,1) marginally performed better than the ARCH(8) which also indicates the strength of the of the simple models in forecasting out-of-sample volatility. Again with respect to the worst model for the out-of-sample forecast, all the four different loss functions suggest EGARCH(2,2) as the worst. Similar to previous findings there is a discrepancy in the ranking of the volatility models one loss function suggests that a particular model is the worst while another loss function suggest that same model to be the best. This also alludes to the importance of choosing an adequate loss function for the intended purpose of the forecast. The conclusion is that, the model with the best in-sample fit also provides the best out-of-sample fit. Furthermore the simple models are able to forecast better than the



complex models in terms of the out-of-sample. Having assess the out-of-sample performance of the volatility models with the loss function we now proceed to examine graphically how the volatility models trace the pattern of the various exchange rate series. Figures 8-12 presents the graph of the performance of the volatility models in the out-of-sample

Figure 8 displays the forecast of the CAN by ARCH(9), GARCH(1,1) and EGARCH(1,1). Realized volatility is in blue line while the red yellow and green lines represents GARCH(1,1), ARCH(9) and EGARCH(1,1) respectively.

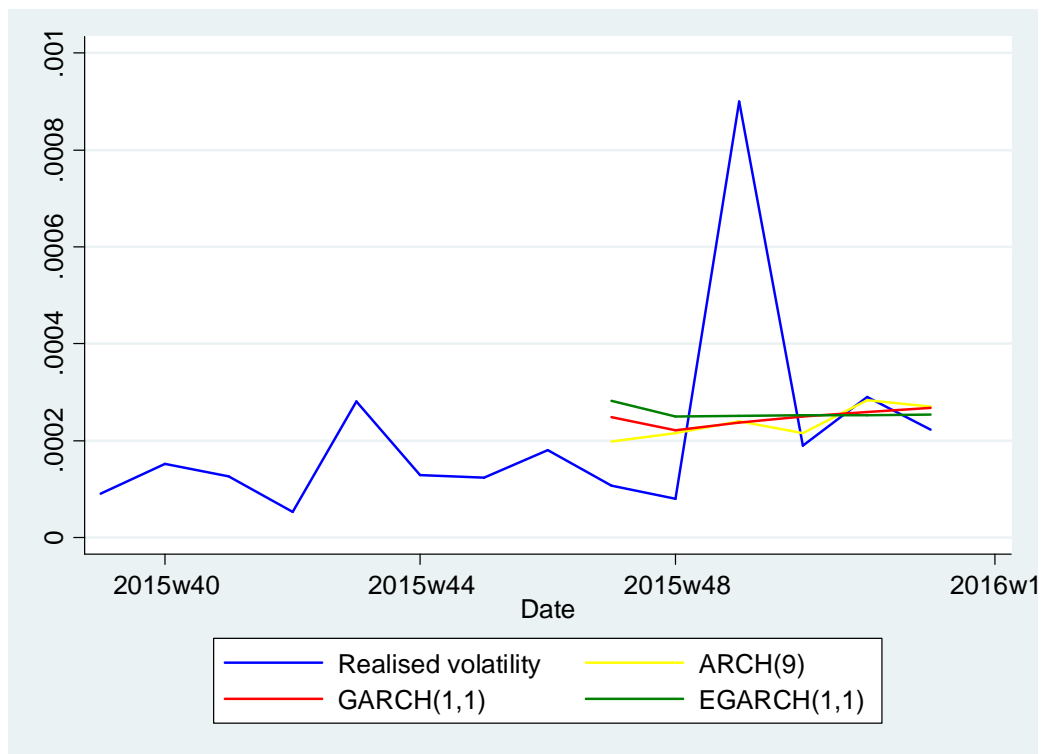


**Figure 8: Out-of-sample forecast of Canadian dollar**

A close examination of figure 8 indicates that the ARCH(9) forecasts the out-of-sample volatility of CAN better than all the other models. This result confirms the findings in table 5. Even though the GARCH(1,1) and EGARCH(1,1) are more closer to the realized

volatility however they fail to trace the actual pattern of volatility of the Canadian dollar. The reason for the relative success of the ARCH(9) over the other complex models could emanate from the nature of monetary policy of the Bank of Canada. The bank of Canada implements monetary policy by influencing the short term interest-rates. It does this by raising and lowering the target for the overnight rate (also known as the key policy rate). The overnight rate is basically the interest rate at which major financial institutions borrow and lend one-day (or overnight) funds among themselves. This affects the volatility of exchange rate in Canada through a transmission mechanism. That is following the announcement of the Bank's policy action to increase its target for the overnight rate, the actual overnight interest rate adjusts almost instantly. As the overnight interest rate rises, two responses are observed. First, the rise in the overnight rate results in an increase in longer-term interest rates in Canada. This increase occurs because there is an entire spectrum of financial assets, ranging from overnight loans to 30-year bonds, and their rates move together. Secondly, as Canadian interest rates rise, financial capital from around the world flows into Canada in pursuit of higher yields. This capital inflow leads to an appreciation of the Canadian dollar and vice versa. The monetary policy by the bank of Canada has proved successful in maintaining a very stable currency even during the crisis in 2007-2008. As a result the Canadian currency do not exhibit much of volatility which therefore enables simpler models like the ARCH(9) to forecast it better than the more complex models.

Figure 9 shows the out of sample forecast of the Swiss franc for ARCH(9), GARCH(1,1) and EGARCH(1,1). Realized volatility is in blue line while the red yellow and green lines represents GARCH(1,1), ARCH(9) and EGARCH(1,1) respectively. .

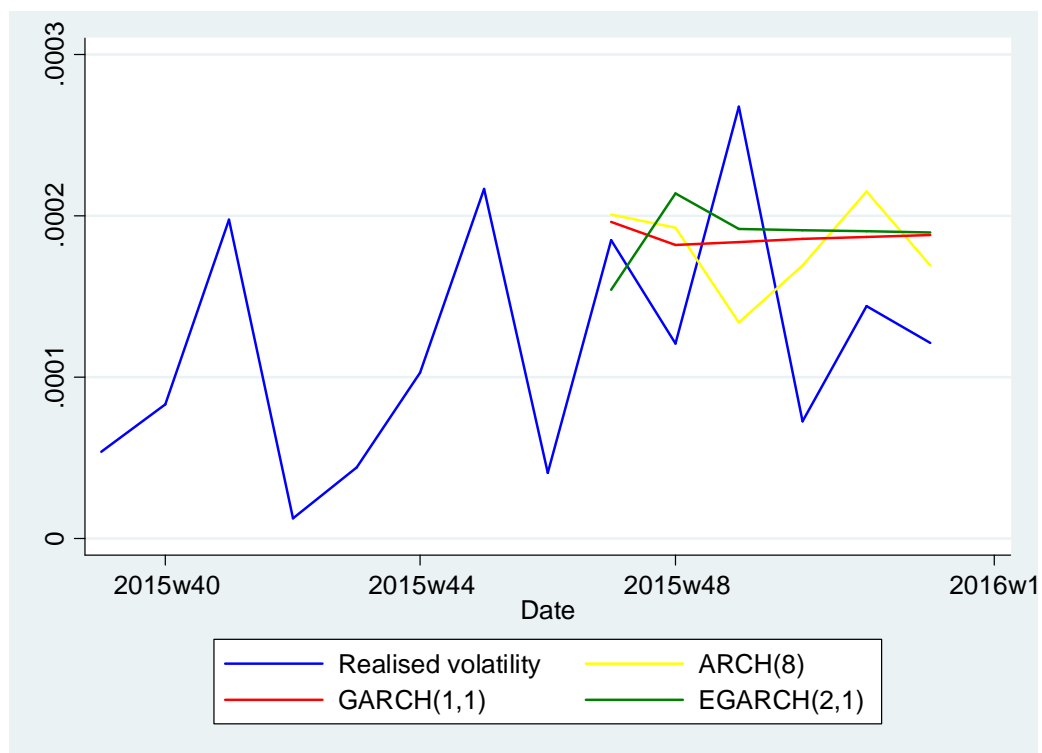


**Figure 9: Out-of-sample forecast of Swiss Franc**

Looking at figure 9, it is quite difficult to determine the models with the best out-of-sample forecast. However assessing the first step forecast illuminates the model that has the best out-of-sample forecast. While the ARCH(9) predicts opposite to the realized volatility in 2015w7, the EGARCH(1,1) model is able to trace the realized volatility of the Swiss. This is shown by the downward sloping portion of the EGARCH(1,1) model from the figure 9. Though the GARCH(1,1) exhibits the same trend as the EGARCH(1,1), but the gap between the EGARCH(1,1) and the realized volatility is smaller than that of

GARCH(1,1). This shows that the forecast from the EGARCH(1,1) is closer to the true volatility than the GARCH(1,1)

Figure 10 shows the out of sample forecast of the BPS for ARCH(9), GARCH(1,1) and EGARCH(1,1). Realized volatility is in blue line while the red yellow and green lines represents GARCH(1,1), ARCH(9) and EGARCH(1,1) respectively.

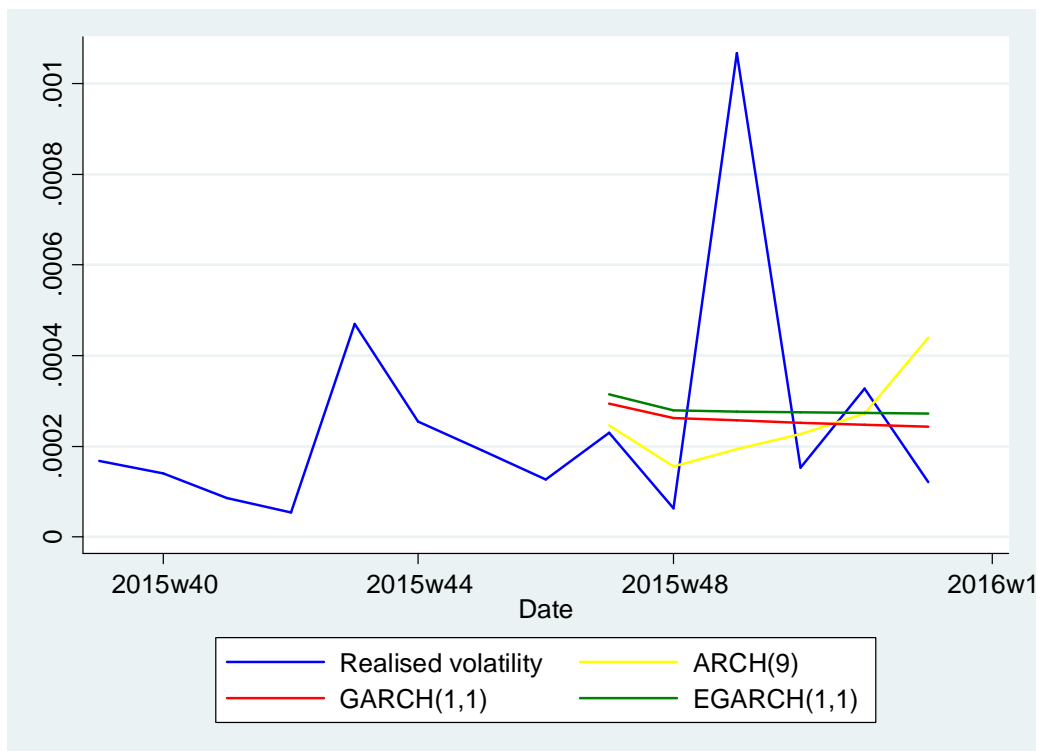


**Figure 10: Out-of-sample forecast of BPS**

From figure 10, there seems to be contradiction on what the loss functions suggest and the observation from the graph. While the loss function suggests GARCH(1,1) as the best model, the graph appears to support ARCH(8) as the best. Though the GARCH(1,1) is able to model effectively the first step out-of-sample forecast, the rest of the forecast is almost a straight line. However, the ARCH(8) is able to trace effectively the realized

volatility in both the first and the last two forecast in the out-of-sample. This results confirms the strength of the basic models in the out-of-sample forecast of financial time series data.

Figure 11 shows the out of sample forecast of the CAN for ARCH(9), GARCH(1,1) and EGARCH(1,1). Realized volatility is in blue line while the red yellow and green lines represents GARCH(1,1), ARCH(9) and EGARCH(1,1) respectively.

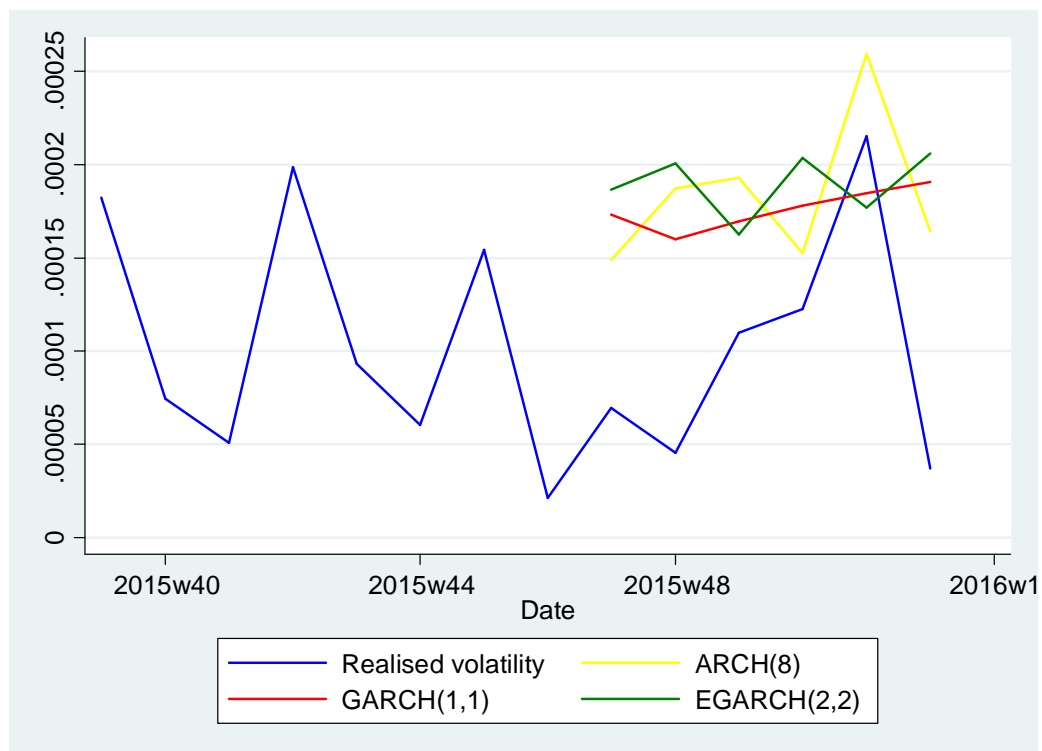


**Figure 11: Out-of-sample forecast of Swiss Franc**

From figure 11, none of the models seems to have the best predictive power the volatility of the euro since both the GARCH and the EGARCH produce almost a straight line whiles the ARCH model is able to trace only the first step and deviates as the forecast sample increases. The vertical distance between the ARCH(8) and the realized volatility

is very small indicating the closeness of the forecast to the true volatility. The behaviour of the ARCH(8) in the out-of-sample forecast is consistent with theory which says that, as the out of sample increase then the forecast turn to deviate from the true volatility since the margin of error increases. Based on the first forecast, it can be concluded that the simple ARCH(8) model performs better than the complex models.

Figure 12 shows the out of sample forecast of the Yen by ARCH(9), GARCH(1,1) and EGARCH(1,1). Realized volatility is in blue line while the red yellow and green lines represents GARCH(1,1), ARCH(9) and EGARCH(1,1) respectively.



**Figure 12: Out-of-sample forecast of Yen**

A critical examination of figure 12 indicates that, the ARCH(8) provides a better out-of-sample forecast than the rest of the models since it is able to trace the pattern of the exchange rate volatility better than GARCH(1,1) and EGARCH(2,2). With reference to the realized volatility graph in appendix B, it is clearly seen that the Yen exhibit periods of high and low volatility hence the a priori expectation is that the EGARCH(1,1) should outperform all the other volatility models however contrary to our expectation the ARCH(8) is able to trace the out-of-sample volatility better than the complex models. This also confirms that the simple models should be given be given the due consideration when forecasting volatility.

## CHAPTER SIX

### 6.1 SUMMARY CONCLUSION AND RECOMMENDATIONS

This section presents the summary conclusion and recommendation of the thesis. The study focuses on three main themes, first the basic structure of the modeling framework is investigated together with the error distribution and the conditional variance to ascertain the best error distribution for forecasting both in the in-sample and out-of-sample. The study also examines whether the more complex models which are able to exhibit more of the stylized facts and characteristics of asset price volatility provide a better in-sample fit and/or out-of-sample-fit than the more parsimonious models. Finally, the study finds out whether the model with the best in-sample fit also produces the best out-of-sample volatility forecast.

The study adopted AIC to determine the best error distribution for the various exchange rate series and concludes that assuming a student-t distribution provides a better in-sample and out-of-sample fit than the normal distribution. The result is unambiguous and expected considering the q-q plots in addition to the Jarque-bera test in chapter 5 which find that the empirical distribution of the return series of the various exchange rate series display significantly heavier tails than the normal distribution.

In terms of the performance of the volatility model in the in-sample, the results are not very conclusive as to which volatility model has the worst in-sample fit even though the GARCH(1,1) is ranked to have the best in-sample forecast for most of exchange rate series under study. The main conclusion here is that yes; the more complex models do provide a better in-sample fit than the more parsimonious models. This finding is also



reemphasized by the EGARCH(1,1) clearly outperforming ARCH type model and the GARCH type models for most of the currencies found in the Euro zone area. In line with our expectation is the superior performance of the GARCH(1,1) model over the ARCH type models in the in-sample forecast. Since the ARCH model is nested in the GARCH model. Hence one could easily reason that if the GARCH(1,1) model did not provide a better in-sample fit there would be no point in setting the extra parameter to anything other than zero which would then reduce the GARCH model to the simple ARCH model. So the conclusion that can be drawn from here is that probably the shock to the financial market does persist forever hence reducing the power of the ARCH(9) to perform better than the GARCH models. Another interesting result is that the higher order GARCH do not necessarily provide a better in-sample fit than the GARCH (1,1) which is in line with previous studies.

However, when analyzing the out-of-sample performance of the conditional variance models the result has been strikingly different. While the GARCH(1,1) outperformed all the volatility models in the in-sample, the out-of-sample volatility models produced an inconclusive results as no particular model either supersedes all the other models the other models in the out-of-sample forecast of the exchange rate volatility across the data set. A more coherent results are found in favour of the GARCH(1,1) and the ARCH(9) which are ranked the best out-of-sample forecast models by all the loss functions for consistently forecasting the volatility of Canadian dollar and Swiss franc. In relation to other exchange rate series the results are not conclusive as to which specific volatility model performs better than the rest of the models across the data set. This is because a particular volatility model is ranked differently by the different loss functions. This

finding emphasizes the importance of choosing the appropriate loss function for the forecast. Another interesting conclusion from the study is that while the GARCH(1,1) generally performed better than all the other volatility models in the in-sample, the out-of-sample performance of the models produced mixed results. This means that it is not necessarily the model with the best in-sample fit that provides the best out-of-sample forecast.

The discrepancy in the performance of the volatility models in both in-sample and out-of-sample forecast can result from the fact that the dynamics of the volatility may have changed during the long time horizon of the data and the volatility of exchange rate may have shifted over time. The dynamics of volatility is non-stationary and is expected to be more rampant especially over a long time horizon from 2007 until 2014, (over 8 years). In addition, during that time period the world witnessed one of the greatest financial crises of all time which quite likely might have changed the dynamics of the market. Another reason for the divergence between the in-sample and the out-of-sample performance may be due to the nature of the model fitting. That is a model that is backtested to perfection and has a good in-sample performance can become sluggish and unresponsive to changes in the volatility and sudden shocks while a model which performs poorly in the in-sample fit might be more flexible and hence be able to accommodate changes in volatility dynamics and shocks. There might also be a trade-off between fitting the model to the in-sample data and the model's alertness to new inputs. An important finding is not only limited to the different ranking of the models when using different loss functions, but also how dramatically it can differ. It is quite a contrast that one loss function suggests that a particular model is the worst and another loss

function ranks that same model to be the best. This highlights the importance of choosing an adequate loss function for the intended purpose of the forecast.

Another coherent finding is that all the loss function unanimously rank the EGARCH(1,1), GARCH(1,1) and the ARCH(8) models as the worst in terms of the out-of-sample forecast for Canadian dollar, Swiss Franc and BPS respectively. A vital conclusion in the evaluation of the out-of-sample performance of the volatility model is that, the simple models are able to perform better than the more complex models in the out-of-sample. This is evidenced by the higher performance of the ARCH models over the EGARCH models in most of the exchange rate series. This leads to the conclusion that; in forecasting out-of-sample volatility the emphasis should not only be on the complex models but also the simple models.

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## APPENDIX A

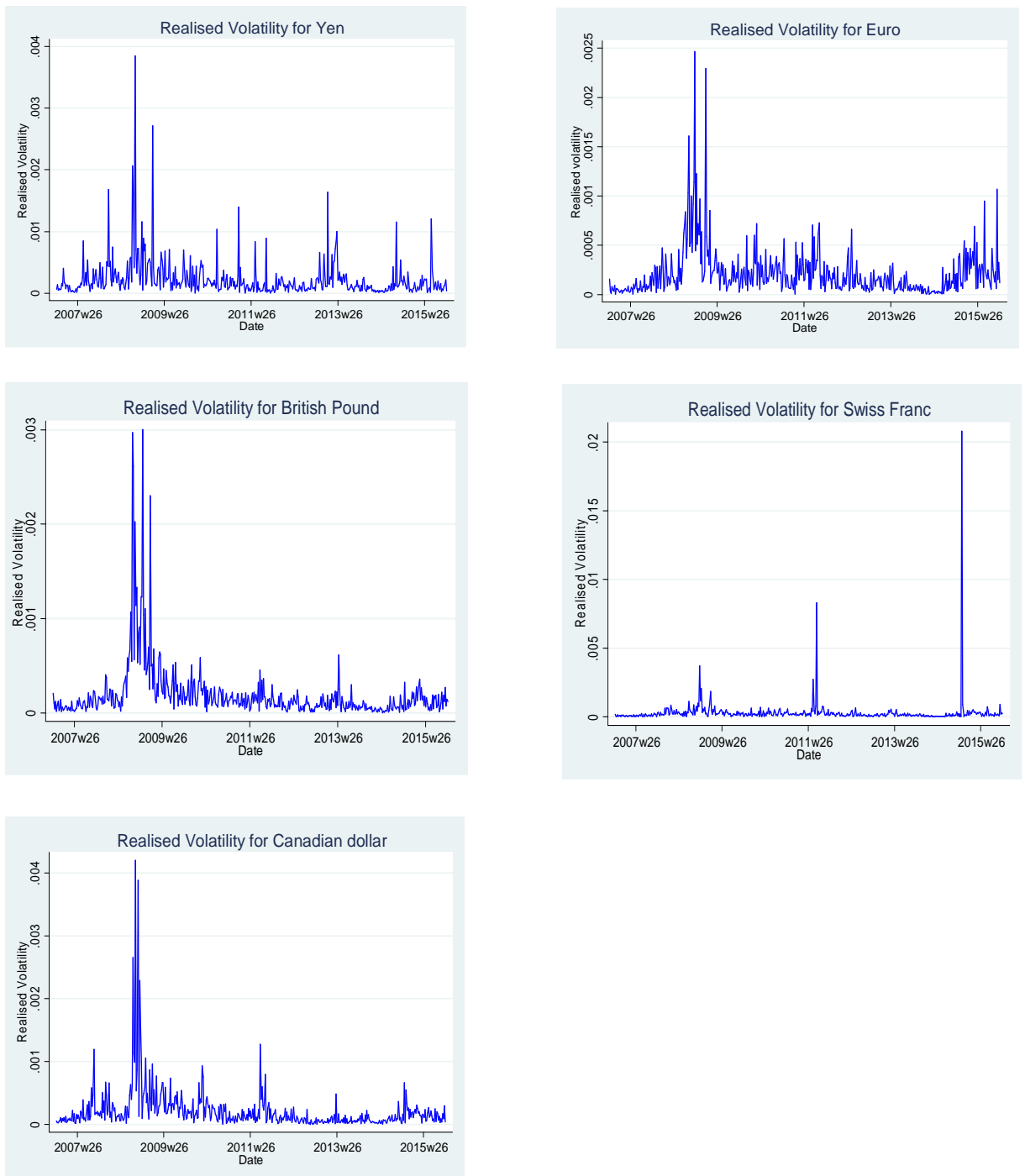
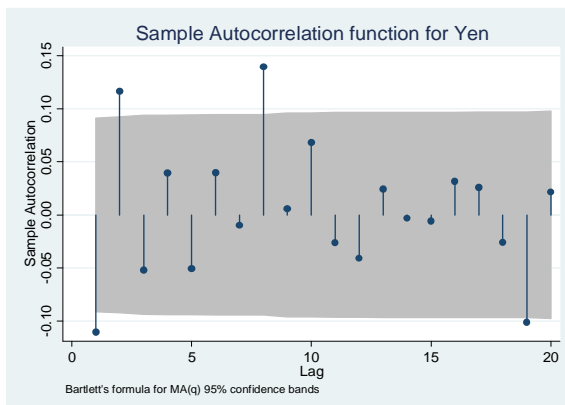
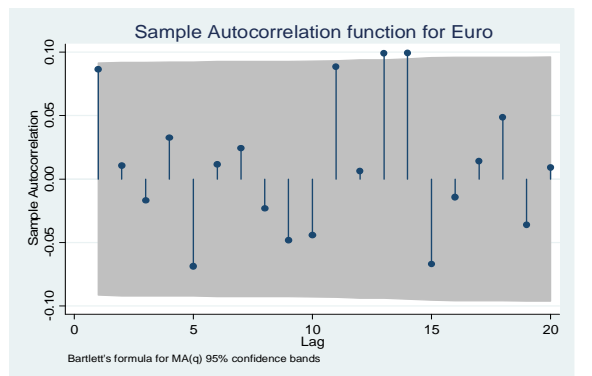
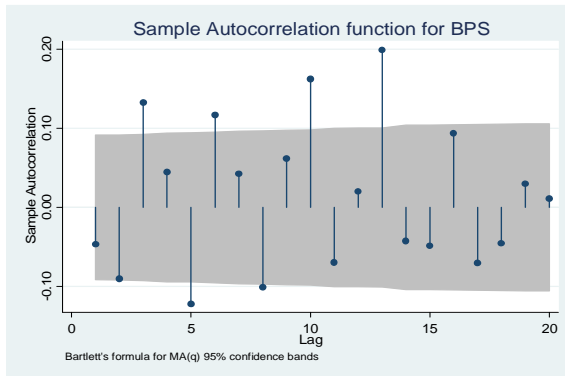
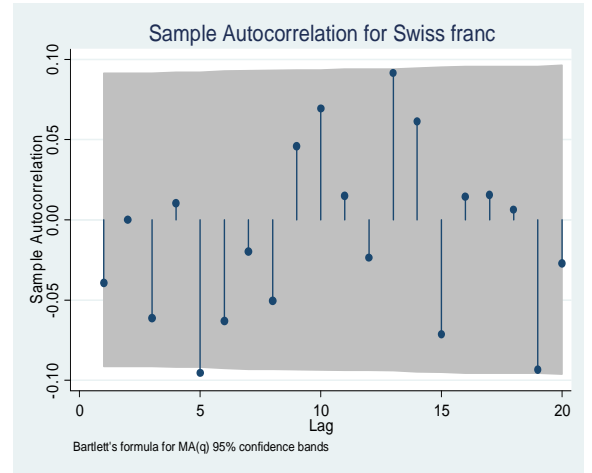
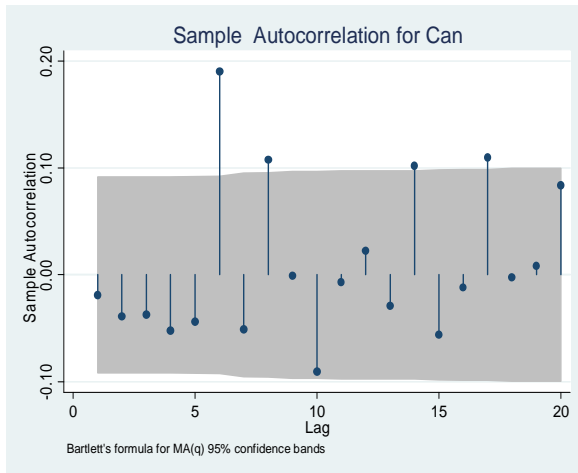


Figure 14: Realised volatility for the various exchange rate series



## APPENDIX B



**Figure 15: Sample Autocorrelation for the various exchange rate series**

**APPENDIX C****Table: AIC for the various volatility models and the respective Exchange rate series**

Models	CAN	SFRANC	BPS	EURO	YEN
ARCH(9) AIC	-2736.70	-2568.269	-2777.137	-2674.579	-2628.586
ARCH(8) AIC	-2733.89	-2565.618	-2778.991	-2675.684	-2630.586
ARCH(7) AIC	-2732.244	-2562.771	-2778.764	-2660.169	-2628.956
ARCH(6) AIC	-2727.785	-2561.871	-2775.665	-2648.408	-2630.947
ARCH(5) AIC	-2722.412	-2563.877	-2775.95	-2644.3	-2632.492
ARCH(4) AIC	-2718.11	-2567.775	-2767.373	-2645.399	-2624.77
ARCH(3) AIC	-2719.835	-2567.318	-2766.54	-2645.729	-2626.769
ARCH(2) AIC	-2717.316	-2564.376	-2761.33	-2641.732	-2625.519
ARCH(1) AIC	-2705.182	-2552.678	-2745.603	-2642.609	-2617.198
GARCH(1,2) AIC	-2727.853	-2562.589	-2784.748	-2666.818	-2628.003
GARCH(2,1) AIC	-2735.098	-2563.803	-2788.471	-	-2629.384
GARCH(2,2) AIC	-2725.691	-2563.321	-2778.258	-2658.165	-2621.888
EGARCH(1,1) AIC	-2732.562	-2577.281	-2736.037	-2676.925	-
EGARCH(1,2) AIC	-2680.909	-2549.841	-2720.546	-2634.751	-2610.013
EGARCH(2,1) AIC	-	-	-2794.081	-	-

EGARCH(2,2) AIC	-2722.013	-2570.443	-2776.741	-2633.759	-2620.457

## APPENDIX D

**Table : Descriptive statistics of exchange rate returns and Jacque-Bera test**

Variable	Mean	SD	Skewness	Kurtosis	Jacque-Bera	Jacque-Bera test
Franc	-0.00048	0.1719	0.1766	229.545	1002936	0.0000***
Yen	0.000027	0.0148	-0.5844	6.5832	277.5960	0.0000***
CAN	0.0004	0.0146	0.9935	9.0087	782.7032	0.0000***
BPS	0.00054	0.0140	1.1794	11.3154	1459.955	0.0000***
Euro	0.00037	0.2080	0.06011	232.3214	1027664	0.0000***

Note : \*\*\*, \*\*, \* represents 1%, 5% and 10% significantly.