Gravitational anomalies, Hawking radiation, and spherically symmetric black holes

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Gravitational anomalies, Hawking radiation, and spherically symmetric black holes

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Abstract: Motivated by the recent work of Robinson and Wilczek, we evaluate the gravitational anomaly of a chiral scalar field in a Vaidya spacetime of arbitrary mass function, and thus the outgoing flux from the time-dependent horizon in that spacetime. We show that this flux differs from that of a perfect blackbody at a fixed temperature. When this flux is taken into account, general covariance in that spacetime is restored. We also generalize their results to the most general static, and spherically symmetric spacetime.

Keywords: Black Holes in String Theory, Space-Time Symmetries, Anomalies in Field and String Theories, Black Holes.

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1. Introduction

Classically, the energy-momentum tensor of any field is expected to be covariantly conserved in a curved background. Quantum mechanically, however, this is not always the case. For example, for a chiral scalar field in $(1 + 1)$-dimensional curved spacetime, the covariant derivative of the energy-momentum tensor reads

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\nu\beta} \ , \tag{1.1}$$

the right hand side being the gravitational anomaly in that spacetime [1 – 3].

Under certain simplifying assumptions, it was shown by Christensen and Fulling [4],[1] that the above anomaly can be interpreted as a flux of radiation, which quantitatively agrees with the Hawking flux [10, 11], from a horizon in that spacetime. This means that Hawking radiation is a necessary consequence of quantization [12, 13] (just as anomaly is), and that it also helps to restore general covariance. The resultant 'total' energy-momentum tensor is covariantly conserved.

Recently, the above idea was re-visited by Robinson and Wilczek, who demonstrated that the result was valid for a wide variety of spacetimes, and without many of the previous assumptions (henceforth abbreviated as the R-W method) [14].[2] Thus Hawking radiation indeed restores general covariance for a large class of spacetimes. The spacetimes considered in the R-W method encompassed many of the known spherically symmetric black hole solutions. However, it excluded certain others, such as the Garfinkle-Horowitz-Strominger (GHS) black hole in string theory. Furthermore, black holes with non-static horizons, such as the Vaidya spacetime, were excluded as well. In this paper, we show that the R-W method can be applied to both the above scenarios. For the most general spherically

[1] For recent applications see [5 – 9].

[2] For more details see [15]

symmetric black hole (including the GHS black hole), the outgoing flux that is dictated by gravitational anomaly agrees with the flux from a perfect blackbody, radiating at the Hawking temperature of the black hole. For Vaidya spacetime, although such a flux exists, it does not agree with a perfect blackbody flux. The reason of course is that the Hawking temperature of the black hole is no longer constant in time. Turning the argument around, one can say that the flux that may be observed from an evolving horizon is the one above. This paper is organized as follows. In Section 1 we review the R-W method for the case of Schwarzschild black hole. In section 2 we extend the R-W method to the case of nonstatic Vaidya spacetime of arbitrary time-dependent mass function and derive its flux. In section 3 we generalize the R-W method to the most static, and spherically symmetric spacetimes. As an example of the generalized method, we study the stringy GHS black hole. It is shown that the gravitational anomaly in this stringy black hole background is cancelled by the total flux of a $1+1$ dimensional blackbody at the Hawking temperature of this stringy black hole. Finally, section 4 is devoted to a brief summary of our results.

2. R-W method for the Schwarzschild type black holes

Robinson and Wilczek [14] considered a d-dimensional Schwarzschild type spacetime with the metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{(d-2)} \tag{2.1}$$

where $f(r)$ is arbitrary and $d\Omega^2_{(d-2)}$ is the metric on S^{d-2}. This metric describes many interesting solutions of Einstein equations. We assume that it has a single non-degenerate horizon at $r = r_H$.

Now, the classical action functional for gravity coupled to matter, $S[matter, g_{\mu\nu}]$, under general coordinate transformations, changes as

$$\delta_\lambda S = -\int d^d x \sqrt{-g} \lambda^\nu \nabla_\mu T^\mu_\nu \tag{2.2}$$

(where λ is the variational parameter), and that the symmetry of the classical action requires that

$$\delta_\lambda S = 0 \Rightarrow \nabla_\mu T^\mu_\nu = 0 \ . \tag{2.3}$$

R-W propose however, that to avoid problems of divergence associated with the Boulware vacuum, propagating modes along one lightlike direction are absent. The price to pay is that the resultant theory is chiral, for which the above condition is violated quantum mechanically due to chiral anomaly. Now instead, the general covariance of the full quantum theory requires the variation of the effective action $W[g_{\mu\nu}]$ to be zero

$$\delta_\lambda W = 0 \ . \tag{2.4}$$

Explicit variation yields

$$-\delta_\lambda W = \int d^2 x \sqrt{-g} \lambda^\nu \nabla_\mu T^\mu_\nu \, , \tag{2.5}$$

where

$$T^{\mu}_{\nu} = T^{\mu}_{i\,\nu}\Theta_{-} + T^{\mu}_{o\,\nu}\Theta_{+} + T^{\mu}_{\chi\,\nu}H \ . \tag{2.6}$$

$\Theta_{\pm} = \Theta\left(\pm(r-r_H)-\epsilon\right)$ are step functions and $H \equiv 1-\Theta_{+}-\Theta_{-}$, which is equal to unity between $r_H \pm \epsilon$ and zero elsewhere. $T^{\mu}_{i\,\nu}$ and $T^{\mu}_{o\,\nu}$ are covariantly conserved inside and outside the horizon respectively. However, $T^{\mu}_{\chi\,\nu}$ in (2.6) is not conserved due to the chiral anomaly at the horizon, which is timelike and given by [2]

$$\nabla_{\mu}T^{\mu}_{\chi\,\nu} \equiv A_{\nu} \equiv \frac{1}{\sqrt{-g}}\partial_{\mu}N^{\mu}_{\nu} \tag{2.7}$$

where

$$N^{\mu}_{\nu} = \frac{1}{96\pi}\epsilon^{\beta\mu}\partial_{\alpha}\Gamma^{\alpha}_{\nu\beta} \tag{2.8}$$

and $\epsilon^{\beta\mu}$ is the two dimensional Levi-Civita tensor. eq. (2.5) can be simplified as

$$\begin{aligned} -\delta_{\lambda}W = &\int d^2x\sqrt{-g}\lambda^{t}\left\{\partial_r\left(N^{r}_{t}H\right) + \left(T^{r}_{o\,t} - T^{r}_{\chi\,t} + N^{r}_{t}\right)\partial\Theta_{+} + \left(T^{r}_{i\,t} - T^{r}_{\chi\,t} + N^{r}_{t}\right)\partial\Theta_{-}\right\} \\ &+ \int d^2x\sqrt{-g}\lambda^{r}\left\{\left(T^{r}_{o\,r} - T^{r}_{\chi\,r}\right)\partial\Theta_{+} + \left(T^{r}_{i\,r} - T^{r}_{\chi\,r}\right)\partial\Theta_{-}\right\} \ , \end{aligned} \tag{2.9}$$

which when combined with eq. (2.4) yields the following solution

$$\begin{aligned} T^{t}_{t} &= -\frac{(K+Q)}{f} - \frac{B(r)}{f} - \frac{I(r)}{f} + T^{\alpha}_{\alpha}(r), \\ T^{r}_{r} &= \frac{(K+Q)}{f} + \frac{B(r)}{f} + \frac{I(r)}{f}, \\ T^{r}_{t} &= -K + C(r) = -f^2 T^{t}_{r}, \end{aligned} \tag{2.10}$$

where

$$B(r) = \int_{r_H}^{r} f(x)A_r(x)dx, \tag{2.11}$$

$$C(r) = \int_{r_H}^{r} A_t(x)dx, \tag{2.12}$$

$$I(r) = \frac{1}{2}\int_{r_H}^{r} T^{\alpha}_{\alpha}(x)f'(x)dx \ , \tag{2.13}$$

and K, Q are constants of integration. Here, it is assumed that $\frac{I}{f}\Big|_{r_H} = \frac{1}{2}T^{\alpha}_{\alpha}\Big|_{r_H}$ is finite, and

$$\lim_{(r-r_H)\to 0_-}\left(\frac{1}{f}\right) = -\lim_{(r-r_H)\to 0_+}\left(\frac{1}{f}\right) \ . \tag{2.14}$$

Next, in the limit $\epsilon \to 0$, using eq. (2.14) and

$$\partial_{\mu}\Theta_{\pm} = \delta^{r}_{\mu}\left(\pm 1 - \epsilon\partial_r \pm \frac{1}{2}\epsilon^2\partial_r^2 - \ldots\right)\delta\left(r-r_H\right), \tag{2.15}$$

the variation of the effective action (2.9) takes the form

$$\begin{aligned}
-\delta_\lambda W = & \int d^2x\lambda^t \left\{ \left[K_o - K_i\right] \delta\left(r - r_H\right) \right. \\
& - \epsilon \left[K_o + K_i - 2K_\chi - 2N_t^r\right] \partial\delta\left(r - r_H\right) + \ldots \} \\
& - \int d^2x\lambda^r \left\{ \left[\frac{K_o + Q_o + K_i + Q_i - 2K_\chi - 2Q_\chi}{f} \right] \delta\left(r - r_H\right) \right. \\
& \left. \left. - \epsilon \left[\frac{K_o + Q_o - K_i - Q_i}{f} \right] \partial\delta\left(r - r_H\right) + \ldots \right\} \right. . \qquad (2.16)
\end{aligned}$$

It is easily seen in equation (2.16) that the values of the energy-momentum tensor on the horizon contribute to the gravitational anomaly. Also, the parameters λ^t and λ^r being independent, the necessary and sufficient conditions for eq. (2.16) to hold are

$$K_o = K_i = K_\chi + \Phi \qquad (2.17)$$

$$Q_o = Q_i = Q_\chi - \Phi\,, \qquad (2.18)$$

where

$$\Phi = N_t^r\Big|_{r_H} \ . \qquad (2.19)$$

The energy-momentum tensor now assumes the form

$$T^\mu_\nu = T^\mu_{c\nu} + T^\mu_{\Phi\,\nu}\,, \qquad (2.20)$$

where $T^\mu_{c\nu}$ represents the conserved energy-momentum tensor without any quantum effects, and $T^\mu_{\Phi\,\nu}$ is a conserved tensor with $K = -Q = \Phi$, representing the flux Φ.
For the specific Schwarzschild type black hole spacetime described by (2.1), one can show that

$$\begin{aligned}
N_t^t &= N_r^r = 0 \\
N_t^r &= \frac{1}{192\pi}\left(f'^2 + f''f\right) \qquad (2.21) \\
N_r^t &= -\frac{1}{192\pi f^2}\left(f'^2 - f''f\right) \ ,
\end{aligned}$$

implying

$$\Phi = N_t^r\Big|_{r_H} \qquad (2.22)$$

$$= \frac{1}{192\pi} f'^2(r_H) \ . \qquad (2.23)$$

Now, it is well known that the surface gravity κ in this case is given by

$$\kappa = \frac{1}{2}\frac{\partial f}{\partial r}\Big|_{r=r_H} \qquad (2.24)$$

$$= \frac{1}{2} f'(r_H) \ , \qquad (2.25)$$

which implies the following Hawking temperature

$$T_H = \frac{\kappa}{2\pi} \tag{2.26}$$

$$= \frac{f'(r_H)}{4\pi} \ . \tag{2.27}$$

On the other hand, a beam of massless black body radiation moving outwards in the radial direction at a temperature T_H has a flux of the form

$$\Phi = \frac{\pi}{12} T_H^2 \ . \tag{2.28}$$

Therefore it is evident that the flux (2.23) is nothing but the Hawking flux, which exactly cancels the gravitational anomaly! Recently, Iso, Umetsu, and Wilczek [16] showed that in the case of a charged black hole apart from the gravitational anomalies, gauge anomalies show up. These are cancelled by the Hawking radiation of charged particles from the charged black hole. Furthermore, extended versions of [14] were presented in [17] which included 4-dimensional rotating black holes as well as in [18] which included the $(2+1)$-dimensional rotating BTZ black hole.

3. The Vaidya Metric

In this section we examine the R-W method for nonstatic spacetimes, of which one of the simplest is given by the Vaidya metric

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2 \tag{3.1}$$

where $v = t + r^\star$ is the advanced time coordinate ($r^\star$ is the tortoise coordinate) and the mass M is a function of the advanced time v. This spacetime accommodates two kinds of surfaces of particular interest. The apparent horizon is at $r_{\rm AH} = 2M$, whereas the event horizon is denoted by $r_{\rm EH} = r_h$ [19]. To determine the null-surface $r_h = r_h(v)$, one first defines

$$\tilde{v} = v \ \text{and} \ \tilde{r} = r - r_h \ , \tag{3.2}$$

in terms of which the line element (3.1) can be written as [20]

$$ds^2 = -\left(1 - \frac{2M(v)}{r} - 2\dot{r}_h\right) d\tilde{v}^2 + 2d\tilde{v}dr + \tilde{r}^2 d\Omega^2 \ . \tag{3.3}$$

Then the event horizon r_h satisfies the null-surface condition

$$1 - \frac{2M(v)}{r_h} - 2\dot{r}_h = 0 \, , \tag{3.4}$$

which yields

$$r_h = \frac{2M(v)}{1 - 2\dot{r}_h} \, , \tag{3.5}$$

where $\dot{r}_h = dr_h/dv$. The surface gravity is given by

$$\kappa = \frac{M(v)}{(1-2\dot{r}_h)r_h^2} \tag{3.6}$$

and the corresponding radiation temperature by [21]

$$T = \frac{1-2\dot{r}_h}{8\pi M(v)}\,, \tag{3.7}$$

which, using (3.5) becomes

$$T = \frac{1}{4\pi r_h}\ . \tag{3.8}$$

It should be noted that since r_h depends on v, the location of the event horizon as well as the shape of the black hole change with time.

Since the Vaidya metric can be written in the form of eq. (2.1), in order to evaluate the corresponding flux one can evaluate the quantity N_v^r on the event horizon. Using eq. (2.8), this is given by

$$\begin{aligned} N_v^r &= \frac{1}{96\pi}\epsilon^{\beta r}\partial_\alpha\Gamma^\alpha_{v\beta} \\ &= \frac{1}{96\pi}\epsilon^{vr}\partial_\alpha\Gamma^\alpha_{vv} \\ &= \frac{1}{96\pi}\left(\frac{6M^2(v)}{r^4} - \frac{2M(v)}{r^3}\right)\ . \end{aligned} \tag{3.9}$$

The corresponding gravitational anomaly evaluated on the event horizon is

$$\Phi = N_v^r\Big|_{r_h} \tag{3.10}$$

$$= \frac{1}{96\pi}\left(\frac{6M^2(v)}{r^4} - \frac{2M(v)}{r^3}\right)\Big|_{r_h} \tag{3.11}$$

$$= \frac{1}{96\pi r_h^2}\left(\frac{6M^2(v)}{r_h^2} - \frac{2M(v)}{r_h}\right)\ . \tag{3.12}$$

If one now considers the Vaidya metric to be radiating at the radiation temperature T, then using eqs. (3.5) and (3.8), the flux is given by

$$\Phi = \frac{\pi}{12}T^2\left(1 - 8\dot{r}_h + 12\dot{r}_h^2\right) = \frac{\pi}{12}\xi\ T^2 \tag{3.13}$$

where

$$\xi \equiv 1 - 8\dot{r}_h + 12\dot{r}_h^2\ . \tag{3.14}$$

Thus, it is seen that the flux from the horizon of Vaidya spacetime is not the blackbody (thermal) flux given by (2.28). The underlying reason for this difference is the non-constant temperature in this case, owing to its time-dependent mass [22]. The factor of ξ expresses this dependence, and as expected, for the special case $M \to$ a constant, $\xi \to 1$ and flux (3.13) yields the previously obtained flux (2.28) for a Schwarzschild black hole.

4. Generalizing to non-Schwarzschild type black holes

In this section, we generalize the R-W method to the case of non-Schwarzschild type black holes, i.e the most general static, spherically symmetric (non-Schwarzschild type black holes) metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + +r^2 d^2\Omega_{(d-2)} \tag{4.1}$$

where

$$f(r) \cdot g(r) \neq 1 \ . \tag{4.2}$$

If it has a horizon at $r = r_H$ then close it, one can write

$$f(r) \approx f'(r_H) \cdot (r - r_H) \tag{4.3}$$

$$g(r) \approx g'(r_H) \cdot (r - r_H) \ . \tag{4.4}$$

The corresponding surface gravity and the Hawking temperature are given respectively by

$$\kappa = \frac{1}{2}\sqrt{f'(r_H)g'(r_H)} \tag{4.5}$$

$$T_H = \frac{\sqrt{f'(r_H)g'(r_H)}}{4\pi} \ . \tag{4.6}$$

Thus, a beam of massless blackbody radiation moving in the positive radial direction at a temperature T_H will have a flux of the form

$$\Phi = \frac{1}{192\pi} f'(r_H)g'(r_H) \ . \tag{4.7}$$

As for R-W, we assume that the physics near the horizon is described by a 1+1 dimensional field theory, in the the 'r-t' section of the spacetime (4.1), and as before, the form of the energy-momentum tensor after variation of the effective action (2.9) is given by (up to constants K, Q and the trace T^α_α)

$$T^t_t = -\frac{(K+Q)}{f} - \frac{B(r)}{f} - \frac{I(r)}{f} + T^\alpha_\alpha(r), \tag{4.8}$$

$$T^r_r = \frac{(K+Q)}{f} + \frac{B(r)}{f} + \frac{I(r)}{f}, \tag{4.9}$$

$$T^r_t = -K + \bar{C}(r) = -f(r)g(r)T^t_r. \tag{4.10}$$

Now the quantities $B(r)$, $C(r)$, and $I(r)$ are defined as follows

$$B(r) = \int_{r_H}^{r} f(x)A_r(x)dx, \tag{4.11}$$

$$\bar{C}(r) = \sqrt{\frac{g(r)}{f(r)}} \int_{r_H}^{r} \sqrt{\frac{f(x)}{g(x)}} A_t(x)dx, \tag{4.12}$$

$$I(r) = \frac{1}{2}\int_{r_H}^{r} T^\alpha_\alpha(x)f'(x)dx \ . \tag{4.13}$$

Continuing the R-W method, we end up with an identical expression for the energy-momentum tensor due to the gravitational anomaly, which is expressed through the pure flux (2.19).

However the explicit expression associated with the spacetime under consideration is quite different from the one given for the Schwarzschild type black holes, i.e. expression (2.23). This difference stems from the fact that the components of N^{μ}_{ν} for the non-Schwarzschild type black holes are now given by

$$\begin{aligned} N^t_t &= N^r_r = 0 \\ N^r_t &= \frac{1}{192\pi}\left(f'g' + f''g\right) \\ N^t_r &= -\frac{1}{192\pi g^2}\left(g'^2 - g''g\right) \ . \end{aligned} \tag{4.14}$$

Therefore the quantity Φ that describes the pure flux for the non-Schwarzschild type black holes reads

$$\Phi = \frac{1}{192\pi} f'(r_H) g'(r_H) \ . \tag{4.15}$$

We see that this is identical to the expression (4.7), derived using black hole thermodynamics. Thus, once again, the gravitational anomaly is cancelled by the Hawking flux.
As an application of the above result, we examine the GHS black hole [23] which is member of a family of solutions to low-energy string theory, described by the action (in the string frame)

$$S = \int d^4x\sqrt{-g}\, e^{-2\phi}\left[-R - 4\left(\nabla\phi\right)^2 + F^2\right] \tag{4.16}$$

where ϕ is the dilaton field and $F_{\mu\nu}$ is the Maxwell field associated with a U(1) subgroup of $E_8 \times E_8$ or $Spin(32)/Z_2$. Its charged black hole solution is given as

$$ds^2_{\rm string} = -\frac{\left(1 - \frac{2Me^{\phi_0}}{r}\right)}{\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right)}dt^2 + \frac{dr^2}{\left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right)} + r^2 d\Omega \tag{4.17}$$

where ϕ_0 is the asymptotic constant value of the dilaton. This metric describes a black hole with an event horizon at

$$r_+ = 2Me^{\phi_0} \tag{4.18}$$

when $Q^2 < 2e^{-2\phi_0}M^2$. For the aforementioned black hole we have

$$f(r) = \frac{\left(1 - \frac{2Me^{\phi_0}}{r}\right)}{\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right)} \tag{4.19}$$

$$g(r) = \left(1 - \frac{2Me^{\phi_0}}{r}\right)\left(1 - \frac{Q^2e^{3\phi_0}}{Mr}\right) \ . \tag{4.20}$$

The corresponding Hawking temperature follows from eq. (4.6)

$$T_H = \frac{1}{8\pi Me^{\phi_0}} \tag{4.21}$$

when the metric elements (4.19) and (4.20) of GHS black hole are used. One can see that the Hawking temperature of the GHS black hole is independent of the charge Q, for $Q < \sqrt{2}e^{-\phi_0}M$.
At extremality, i.e. when $Q^2 = 2e^{-2\phi_0}M^2$, the GHS black hole solution (4.17) becomes

$$ds^2_{\rm string} = -dt^2 + \left(1 - \frac{2Me^{\phi_0}}{r}\right)^{-2} dr^2 + r^2 d\Omega \ . \tag{4.22}$$

and its Hawking temperature vanishes, since the corresponding Euclidean section is smooth without any identifications. The quantity N^r_t given by (4.14), reads

$$N^r_t = \frac{1}{192\pi}\left(g'f' + gf''\right) \ . \tag{4.23}$$

When we evaluate this quantity at r_H the second term is zero since $g(r_H) = 0$. Thus,

$$N^r_t\Big|_{r_H} = \frac{1}{192\pi} g'(r_H) f'(r_H) \tag{4.24}$$

$$= \frac{1}{192\pi}\frac{e^{-2\phi}}{4M^2} \tag{4.25}$$

$$= \frac{\pi}{12}\left(\frac{e^{-2\phi}}{64\pi^2 M^2}\right) \tag{4.26}$$

$$= \frac{\pi}{12}\left(\frac{1}{64\pi^2 M^2 e^{2\phi}}\right) \tag{4.27}$$

$$= \frac{\pi}{12}\left(\frac{1}{8\pi M e^{\phi}}\right)^2 \ . \tag{4.28}$$

Therefore, comparing with (4.21), we get

$$N^r_t\Big|_{r_H} = \frac{\pi}{12} T^2_H \ . \tag{4.29}$$

As for the extremal case (4.22), it is obvious that the generalized R-W method gives the correct null result (due to the vanishing Hawking temperature, i.e. $T^{\rm ext}_H = 0$) since in this case $f(r) = 1$ and thus $f'(r) = 0$.

5. Conclusions

In this work, we have computed the gravitational anomaly for chiral scalar fields for the nonstatic Vaidya spacetime of arbitrary mass function. According to R-W, this is the flux of radiation from a horizon in the above spacetime, such that general covariance at the quantum level is restored. To our knowledge, this is the first time that Hawking flux from such a dynamical spacetime has been computed. There have been some computations in the past but only for specific mass functions. In addition in these cases the flux was in a rather complicated form, contrary to our result derived here. In the limiting case where the mass function is equal to the ADM mass of the Schwarzschild black hole, we recover the R-W results. Furthermore, we have generalized their method to the most general

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static, and spherically symmetric spacetimes. We then applied the generalized method to the Garfinkle-Horowitz-Strominger stringy black holes. The gravitational anomaly of this stringy black hole is cancelled by the flux of a beam of massless $1+1$ dimensional particles at the Hawking temperature of this black hole. Moreover, at extremality we get the known zero temperature and correspondingly a null flux indicating that there is no gravitational anomaly.

It would be interesting to study the physical implications of our result for other dynamical spacetimes, on which we hope to report elsewhere.

Note: A related work by Keiju Murata and Jiro Soda [24] appeared on the archive on the same day we submitted our paper.

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