Cosmic coincidence or graviton mass?

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Using the quantum corrected Friedmann equation, obtained from the quantum Raychudhuri equation, and assuming a small mass of the graviton (but consistent with observations and theory), we propose a resolution of the smallness prroblem (why is observed vacuum energy so small?) and the coincidence problem (why does it constitute most of the universe, about 70%, in the current epoch?).

As a result of some remarkable recent advances in astrophysical observations of high red-shift supernovae [1, 2] and the cosmic microwave background radiation [3, 4], we now have a better than ever understanding of the large scale structure of our universe and its evolution. It is now more or less accepted that it was created at the big-bang singularity about 14 billion years ago, that it underwent a short but rapid inflationary phase, then an expanding phase in which it transited from radiation to a matter dominated era, and is currently homogeneous and isotropic and in an accelerating phase and made up of about 70% *Dark Matter* characterized by a pressure to density ratio $w \equiv p/\rho = -1$, and the remaining non-relativistic matter (mostly dark), with $w = 0$. One also assumes that that universe obeys the laws of general relativity and quantum mechanics, the latter being important at very early times. Beneath this apparently simplicity problems remain however, among them perhaps the most notorious being the extremely small value of the cosmological constant Λ for it to be a candidate for dark energy, about 10^{-124} in Planck units, known as the *smallness problem* (e.g. vacuum energy of quantum fields predict 50 orders of magnitude or more, greater than the observed value), and also its almost equality with H_0^2/c^2, where $H_0 =$ the current value of the Hubble parameter, also known as the *coincidence problem*. In this article, we show that both these problems can be resolved in one stroke, provided one assumes that the origin of Λ lies in the quantum wavefunction of gravitons (of photons) which pervade our universe, albeit having a small mass, but consistent with all observations.

Since the Friedmann equation, the guiding equation of cosmology, can be derived from the Raychaudhuri equation, we start with the recently obtained quantum corrected Raychaudhuri equation (QRE), obtained by replacing geodesics with quantal (Bohmian) trajectories [5], associated with a wavefunction $\psi = \mathcal{R}e^{iS}$ of the fluid or condensate filling our universe ($\mathcal{R}(x^\alpha), S(x^a) =$ real), and giving rise to the four velocity field $u_a = (\hbar/m)\partial_a S$, and expansion $\theta = Tr(u_{a;b}) = h^{ab}u_{a;b},\ h_{ab} = g_{ab} - u_a u_b$ [6] [1].

$$\frac{d\theta}{d\lambda} = -\frac{1}{3}\,\theta^2 - R_{cd}u^c u^d + \frac{\hbar^2}{m^2}h^{ab}\left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} \quad (1)$$

The second order Friedmann equation satisfied by the scale factor $a(t)$ can be derived from the above, by replacing $\theta = 3\dot{a}/a$ and $R_{cd}u^c u^d \to \frac{4\pi G}{3}(\rho + 3p)$ [7]

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\,(\rho + 3p) + \frac{\hbar^2}{3m^2}h^{ab}\left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} . \quad (2)$$

The quantum correction ($\mathcal{O}(\hbar^2)$) term in Eqs.(1) and (2), also known as *'quantum potential'* (a term coined by Bohm [5]), vanish in the $\hbar \to 0$ limit giving back the classical Raychaudhuri and the Friedmann equations. Note that since Bohmian trajectories do not cross [8, 9], it follows that even when θ (or $\dot{a}$) $\to -\infty$, the actual trajectories (as opposed to geodesics) do not converge and there is no counterpart of geodesic incompleteness or the classical singularity theorems, and singularities such as big bang or big crunch can in fact avoided. Next, we interpret the correction term as the cosmological constant

$$\Lambda_Q = \frac{\hbar^2}{m^2c^2}h^{ab}\left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} . \quad (3)$$

Although Λ_Q depends on the form of the amplitude of the wavefunction $\mathcal{R}$, for any reasonable form such as a Gaussian wave packet $\psi \sim \exp(-r^2/L^2)$, or for one which results when an interaction is included in the scalar field equation $[\Box + g|\psi|^2 - k]\psi = 0$, namely $\psi = \psi_0 \tanh(r/L\sqrt{2})$ $(g > 0)$ and $\psi = \sqrt{2}\ \psi_0\ \mathrm{sech}(r/L)$ $(g <$

[1] We use the metric signature $(-,+,+,+)$ here, as opposed to $(+,-,-,-)$ in [6], resulting in opposite sign of the $\hbar^2$ terms. Here we concentrate on the more important of the two correction terms.

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0) [10] it can be easily shown that $(\Box\mathcal{R}/\mathcal{R})_{;a;b} \approx 1/L^4$, where L is the characteristic length scale in the problem, typically the Compton wavelength $L = \hbar/mc$ [11] over which the wavefunction is non-vanishing. This gives

$$\Lambda_Q = \frac{1}{L^2} = \left(\frac{mc}{\hbar}\right)^2 \tag{4}$$

which has the correct sign as the observed cosmological constant. Next to estimate its magnitude, we note that if L is identified with the linear dimension of our observable universe, then m can be regarded as the small mass of gravitons (or photons), with gravity (or Coulomb field) following a Yukawa type of force law $F = -\frac{Gm_1m_2}{r^2}\exp(-r/L)$. Since gravity (and light) has not been tested beyond the above length scale, this interpretation is natural and may in fact be unavoidable. If one invokes periodic boundary conditions, this is also the mass of the lowest Kaluza-Klein modes. Substituting $L = 1.4 \times 10^{26}$ $metre$, one obtains $m \approx 10^{-68}$ kg or 10^{-32} eV, quite consistent with the estimated bounds on graviton masses from various experiments [12], and also from theoretical considerations [13–16]. Finally, plugging in the above value of L in Eq.(4), we get

$$\Lambda_Q = 10^{-52}\ (\text{metre})^{-2} \tag{5}$$

$$= 10^{-123}\ \ (\text{in Planck units})\ , \tag{6}$$

which is indeed the observed value. Also since the size of the observable universe is about c/H_0, where H_0 is the current value of the Hubble parameter, one sees why the above value of Λ_Q numerically equals H_0^2/c^2 (which is $8\pi G/3c^4 \times \rho_{crit}$, the critical density), offering a viable explanation of the coincidence problem.

In summary, gravitons and photons which pervade our universe and collectively described by a wavefunction, necessarily give rise to a quantum potential which manifests as cosmological constant in the quantum corrected Friedmann equation. Furthermore for all reasonable choices of the wavefunction, its magnitude turns out to be $(observable\ universe\ size)^{-2}$, which remarkably matches with the accepted minute value of the cosmological constant. Note that the argument goes beyond just a dimensional one, and this identification (again which appears unavoidable) readily provides a natural explanation of the smallness and coincidence problem in cosmology, which were sought for a long time. Further extensions of these results can be found in [17].

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