

**DYNAMIC MOMENT ANALYSIS OF NON-STATIONARY TEMPERATURE
DATA IN ALBERTA**

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Bachelor of Management, University of Lethbridge, 2008

A Thesis

Submitted to the School of Graduate Studies
of the University of Lethbridge
in Partial Fulfillment of the
Requirements for the Degree

MASTER OF SCIENCE IN MANAGEMENT

Faculty of Management
University of Lethbridge
LETHBRIDGE, ALBERTA, CANADA

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Abstract

Strong seasonality is observed in the volatile hourly Alberta temperature and its low- and high-order statistical moments. We propose a time series model consisting of a linear combination of an annual sinusoidal model, a diurnal sinusoidal model and a fractional residual model, to study the characteristics of these spatial and time-dependent Alberta temperatures. Wavelet multi-resolution analysis is used to measure Hurst exponents of the temperature series. Our empirical results show that these Hurst exponents vary over various time scales, indicating the existence of multi-fractality in the temperatures. Such temperature models are of importance for the pricing and insurance of agricultural crops, of tourist resorts and of all forms of energy extraction and generation of importance to the resource-based economy of Alberta. Of particular interests are the observed extreme volatilities in the winters, caused by the unpredictable Chinook winds, which may be an important reason to introduce a Chinook insurance option.

Acknowledgements

My utmost gratitude goes to my thesis supervisor Dr. Cornelis A. Los for his invaluable guidance and insight over the course of this research. His expertise, patience and continual support made this thesis possible.

My thanks and appreciation goes to my thesis committee members, Dr. Ebenezer Asem and Assistant Professor John Sheriff for their valuable comments. I would also like to thank Dr. Calum G. Turvey and Dr. Helen Kelley for serving on my Examination Committee.

Last but not least, I dedicate this thesis to my mom whose love, encouragement and support is with me in whatever I pursue.

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1. Introduction

Weather has a large impact on people's living quality and a huge economic influence on businesses. The uncertainties of non-catastrophic weather events such as heat, cold, snow, rain or wind can cause major headaches for the management of many companies, just like the uncertainties of sudden catastrophic weather events. Undesirable and unwanted weather conditions can easily wipe out financial profitability and result in poor financial-economic performance, especially for weather-sensitive companies. Corporate risk managers have become increasingly aware of such weather risk. They have significantly increased their demands for effective weather hedging and for risk management products such as weather derivatives.

Since the Chicago Mercantile Exchange (CME) first launched its weather derivatives, the weather derivatives market has been the fastest-growing derivatives market in the world. As reported by Price Waterhouse, the volume and liquidity of the weather derivatives market has grown twenty-fold between 2002 and 2003 (cited in Svec & Stevenson, 2007, p.186). Today, the CME provides temperature and precipitation indices in 24 cities in the US, six in Canada, 10 in Europe, and two in the Asia-Pacific region (Morrison, 2009). Trading volumes have grown from 798,000 contracts in 2006 to around a million in 2007, with a total notional value about \$18 billion (Morrison, 2009). Thus the weather market is no longer a novelty to industries around the world.

Besides the attraction from the fast growing weather risk market, an interest in the strangely uncertain and, particularly in the winter season, unpredictable Alberta weather is another critical motivation to initiate this research. Alberta is located in western

Canada, bounded by the Provinces of British Columbia to the west and Saskatchewan to the east and the Northwest Territories to the north and the U.S. State of Montana to the south (Wikipedia, n.d.). Because Alberta extends for over 1200 kilometers from north to south, its climate varies dramatically between its various latitudinal regions.

In addition, the winter climate of the southwest of Alberta is affected by the presence of a warm, dry, adiabatic wind, known as Chinook wind (a mountainous föhn wind) blowing from the Rocky Mountains down into the prairies, which interrupts the cold winter temperatures (Wikipedia, n.d.). It is interesting to note that the Chinook wind is more prevalent over southern Alberta (Figure 1 the dark red part around Lethbridge) and is less common north of Red Deer. As reported by Natural Resource Canada, Pincher Creek, 65 miles west of Lethbridge, experienced the most extreme change in temperature in Canada in January 1962, when the temperature was driven by a Chinook wind up from minus 19°C to plus 22°C in one hour (The Atlas of Canada, 1990).

These highly volatile weather patterns have amazed meteorologists. Such dramatic temperature swings bring also major uncertainties and potential financial risk to various groups of Albertans. For example, farmers' crops may be seriously hurt by unanticipated excesses of both temperature and precipitation. Oil and gas companies as well as ski resorts may suffer from low market demand caused by a warm winter, while electricity firms lose their revenues due to abnormally cool summers. Thus, as financial-economic researchers, we are keen to study and explore the more detailed characteristics of Alberta temperature, so that these groups of people may financially benefit from an

improvement in the modeling of the financial risk characteristics of the weather in Alberta, in particular, of its temperature.

Remarkably, the strongly non-stationary Alberta temperature data behave like particular asset returns in the financial markets. Like such asset returns, temperature data exhibit continuously small fluctuations and occasionally large discrete swings over various time horizons. Indeed, the volatility of temperature data resembles strongly time-dependent stochastic processes, like the stochastic volatility examined in financial markets, but may be even more time-dependent than financial data, due to its hysteretic seasonality. We conduct a dynamic, time-dependent, moment analysis to better characterize the extraordinary behavior of non-stationary Alberta temperature. In addition, the higher-order moments of the temperature distributions, such as skewness and kurtosis, prove to matter when measuring weather risk because they are also, like the lower-order moments of mean and variance, spatially variant and time-variant.

We find that the dynamic Alberta temperature data can be parsimoniously modeled with a linear combination of an annual sinusoidal, a diurnal sinusoidal model and a fractal persistence model. It is worth noting that the temperature residuals - after removing the annual seasonality and intra-day sinusoidal temperature fluctuations - exhibit singularities, discontinuities and irregularities, which suggest that a fractality pattern exists in the non-stationary temperature data. Similar persistence characteristics have also been shown to exist in the Latin American, European and Asian financial markets (Karuppiah & Los, 2005; Kyaw, Los & Zong, 2006; Lipka & Los, 2002; Los, 2003). Mallat's (1989a, 1989b, 1989c) wavelet multi-resolution analysis (MRA) will be

conducted to identify local Hölder or Hurst exponents to measure the degrees of persistence of the temperature data. The MRA provides a complete time-frequency analysis of the statistical properties of time series data and detects patterns and singularities for the localized risk (volatility).

There has been a growing interest in recent financial literature to search for accurate and efficient pricing models for weather derivatives. Most derivatives based on heat are based on the notion of heating or cooling degree days. Current literature has debate on whether the weather derivatives pricing should estimate the dynamic path of the underlying temperature process or the process of a degree-day index itself. Turvey (2005) proposed a pricing model based upon degree-day weather index since he considered that the dynamics in temperature cannot represent those in degree-day index and should not be simply utilized to price the weather options.

Solving this dispute is beyond the scope of our study. What this study focuses on is to characterize the dynamic Alberta temperature and its risk at various locations in Alberta. The importance of understanding such characteristics of temperature is significant to the degree-day weather index pricing model because the expected weather condition (above, equal to or below normal temperature) is an important factor in the proposed model to affect the volatility of the index value, which will result in different option prices. For example, our findings about the anti-persistence for extra-day temperature patterns and significant winter temperature volatility in Alberta due to the warm Chinooks would lead to forecasts of higher than normal volatility. Then the option modeler would be prudent to increase the option values accordingly. As the measured

volatility increases, both put and call option values increase, since options are the financial instruments to directly measure the value of increased volatility. Therefore, regardless of the pricing model, an in-depth understanding of basic temperature characteristics, like changing volatility and skewness, as well as kurtosis (for the option smile) is significant to price all weather-related optional hedging and other risk-related financial management products.

This paper is organized as follows: in section 2 we present a brief, precise and concrete literature review on dynamic moment analysis, the theory of long term dependence and fractality or similarity scaling and of temperature modeling and weather derivatives pricing. The data sets are described in section 3. In Section 4, a simple temperature model is proposed consisting of an annual sinusoidal, a diurnal sinusoidal and an intra-day persistent fractal model. In particular, we emphasize introducing wavelet Multi-resolution Analysis (MRA) to measure the degrees of non-sinusoidal (non-Fourier) persistence of temperature residual series. A synthesis of fractional Brownian motion using wavelets is conducted in section 5 to verify the quality of our temperature model. Detailed empirical findings are discussed in section 6. Section 7 discusses the importance of Alberta temperature for the pricing and trading of natural gas in the Great Lakes Midwestern region. Section 8 discusses the contribution of our study, e.g., the proposal for an optional Chinook insurance, and concludes the paper.

2. Literature Review

This section summarily reviews the existing literatures on dynamic moment analysis, theory of long term dependence, fractality and temperature modeling and weather derivative pricing. Compared with the plentiful literatures on the other two topics, the literature on the new dynamic, time-varying moment analysis is still scarce.

I. Dynamic Moment Analysis

Although many studies have applied various statistical approaches to study the underlying data, very few (e.g., an early example is Los, 1984) have discussed and explored the properties of time-varying moment distribution analysis. The conventional statistical analysis often assumes a normal distribution for the examined variable, i.e., the variable distribution has a zero mean and a constant volatility which, normalized, equals to one. Many financial-economic and other theories have been seriously criticized for making such a simple assumption, since in the past two decades it has been repeatedly demonstrated that empirical distributions are rarely normal and that their statistical moments are not time-invariant. For instance, the prevalent Black and Scholes option pricing model (Black & Scholes, 1973) erroneously presumes the normality of the distribution of assets returns and their constant volatility or risk. In recent years, many researchers have noticed the importance of non-normal distribution and varying moments (Los, 2008, for a recent survey).

Now more attention has been attracted to use also higher-order moments (like skewness and kurtosis) to study non-stationary data and non-linear systems. Fung and Hsieh (1999) correctly claim that the mean and variance are not sufficient to represent the

distribution of hedge funds returns, if those returns are not normally distributed. Lee, Phoon, and Wong (2006) demonstrate that their first- and cross-moment analysis is a useful and efficient tool to evaluate the risk and performance of hedge funds, even when the fund returns have a distribution with skewed fat tails and sharp peaks. Currently, higher-order moments are getting more emphasis in exploring characteristics for non-stationary data; unfortunately, very few financial-economic researchers have noticed the time-varying dynamical moments or initiated robust and in-depth research into this interesting topic, although it is a topic known to signal processing and filtering engineering since the introduction of the Kalman filter in 1959 (Los, 1984 for a literature survey, mathematical theory, programming, and simulations). In the engineering fields of signal processing and filtering the emphasis is now again on higher-order dynamic (time-varying) moment filtering (e.g., the recent December 2009, Vol 29 (6) issue of *IEEE Control Systems Magazine* for several articles on new “Applications of the Kalman Filter.”), after a 25-year hiatus.

II. Long-term Dependence and Fractality

Conventional statistical analysis begins by assuming that the measured events of a system must be “independent and identically distributed” and the distribution of such random system events must be Gaussian. Such restrictive assumptions have long been superimposed onto the modeling process of most large, complex systems especially in the financial literature. Empirical findings from our real life observations, however, clearly show that these complex systems are not “normal” and that the Gaussian distribution is not a sufficient tool to describe and study the complicated characteristics of dynamic

systems. A substantial number of researchers are now looking at the long-term dependence (or Long Memory) of nonlinear systems.

The Long Memory process was first introduced by the British hydrologist, H.E. Hurst, who worked on the Nile River Dam Project in the early 20th century. He found that the Nile River's overflows follow a persistent pattern in which larger-than-average overflows are more likely to be followed by more large overflows while a lower-than-average overflow is followed by other lower-than-average overflows. Generally speaking, a Long Memory effect is that what happens today will impact the future forever in a nonlinear fashion; that is, an event in such dynamical system is not necessarily serially correlated but nonlinearly related to many of its precedents.

Since Mandelbrot (1972) first introduced the concept of such long-term persistence based on fractality in economic series and in financial markets, extensive studies have been conducted using various renewed fractal models to test the long-term dependence and measure the degrees of persistence of financial markets (Los, 2003). In the past decade or so, substantial evidence has been emerging that the fractal model residual, compared with its Gaussian counterpart, is a better model to identify the statistics of nonlinear, dynamical and complex processes.

Peters (1994) asserts that the Fractal Market Hypothesis, which is an application of long-term dependence concept to modern financial markets, should replace the Efficient Market Hypothesis (Fama, 1970) to be the dominant paradigm for how the markets work, because the former releases the latter's oversimplified statistical assumptions and fits the real life observed facts better (p. 40). In his book, Peters applies

R/S analysis to analyze the five-day, 20-day, and 60-day returns for Dow Jones Industrial Average from 1888-1990. The results show that all three returns exhibit persistence patterns with Hurst exponents larger than the neutral 0.5 (p. 113). He further proves that the volatility or standard deviation of 22-day returns for the S&P 500 from 1945 to 1990 is anti-persistence and has a Hurst exponent of 0.31 below the neutral 0.5 (p. 148).

Kyaw, Los and Zong (2006) examine the persistence of the financial rates of return from six Latin American stock markets and five currency markets. Their results show all stock markets and currency markets exhibit Long Memory, but the degrees of persistence vary from markets to markets. Specifically, Chile and Venezuela have persistent stock markets while Argentina, Colombia and Mexico markets are all anti-persistent. The Brazilian stock market is the only one which shows the independence of the innovations of the Geometric Brownian Motion with a neutral Hurst exponent 0.5. Similar research was conducted in eight supposedly illiquid Asian currency markets compared to the deeply liquid Yen/US dollar and German Mark/US dollar market by Karuppiah and Los (2005). Their discovery of the anti-persistence of the Japanese Yen/US dollar rates has had a profound effect on the current understanding of the dynamic behavior of currency markets (Elliott & Van der Hoek, 2003).

Mulligan (2004) applies five self-affine fractal analysis techniques to measure the Hurst exponents for highly volatile technology equities. His results suggest that most technology securities have Hurst exponents in the anti-persistence range, meaning that they are hyper-efficiently valued by the market with very fast mean-reversion.

There is also growing academic interest in the examining the Long Memory properties of commodity futures using fractional models. Both the paper of Barkoulas, Labys, and Onochie (1999), and the paper of Crato and Ray (2000) examine 17 agricultural commodity futures return series for evidence of Long Memory. The former finds strong evidence of Long Memory in futures return, while the latter finds no evidence. These disparate results were reexamined by Elder and Jin (2009). These authors employ the advanced wavelet multi-resolution analysis to decompose 15 commodity futures price series, including six grains, three soft commodities, three meats and three metals. They report that half of examined commodities display strong evidence of Long Memory in the form of anti-persistence, while the three metal futures do not exhibit such long term dependence property. In other words, the occurrence or non-occurrence of Long Memory is market-dependent and characterizes the empirical market. That is an important finding for both traders and portfolio managers: the degree of Long Memory or persistence, so important for the price discovery process and for financial risk management, has to be empirically measured and cannot be assumed to be market-neutral.

III. Temperature Modeling and Weather Derivatives Pricing

The revenues of utilities, agricultural sectors, energy industry, travel industry and many other economic sectors are affected by weather, driving the accelerating growth of demand of weather derivatives products around the world. Compared with the standard insurance products, weather derivatives are able to cover the damage caused by relatively high-frequency and limited loss events rather than the infrequent, abnormal weather

catastrophes, providing those weather sensitive producers, such as farmers and energy producers, with an alternative handy tool in weather risk management (Richards, Manfredo & Sanders, 2004; Svec & Stevenson, 2007; Zapranis & Alexandridis, 2008). However, the unique but distinctive weather risk, differing from traditional commodity price risk and other source of risk, poses a great challenge on the weather volatility modeling and the pricing of weather derivatives. The main stream of temperature modeling and weather derivatives pricing have been focusing on various types of stochastic approaches.

For example, the Ornstein-Ühlenbeck stochastic process was first introduced by Dornier and Queruel (2000) to study the temperature variations with standard Brownian motion white noise. This model was further extended by Brody, Syroka, and Zervos (2002) and then Benth and Šaltytė-Benth (2005). Brody et al. substitute the white noise part with fractional Brownian noise and discover Long Memory phenomena in the temperature data of London, UK. Benth and Šaltytė-Benth (2005) propose a Lévy-based mean-reverting stochastic model in which the Norwegian temperature data is proved to fit well to a seasonally-varying variance combined with a heavy-tailed distribution. In their 2007 study, the same authors adopt a standard Brownian motion to model the temperature variations in Stockholm, Sweden.

In their 2007 study, Benth and Šaltytė-Benth acknowledge that the only reason they assume the normality of temperature distribution, and choose to use standard Brownian motion for modeling the noise term, is that the Lévy process dynamics complicate the theoretical pricing of derivatives. In both 2005 and 2007, the authors have

derived explicit solutions to temperature option pricing based upon the assumption that the processes followed by the temperature dynamics are a Lévy process and geometric Brownian motion, respectively. Svec and Stevenson (2007) have done similar work in modeling temperature for Sydney, Australia, but their remarkable contribution to the literature is that they used an advanced wavelet transformation model to capture both cyclical and jumping behaviour in high frequency non-stationary temperature data (half hourly).

Recent progress in temperature modelling is made by Zapranis and Alexandridis (2008) who proved that the classic mean-reverting temperature model, which assumes a constant mean-reversion parameter, would largely improve its accuracy by varying the speed of the mean reversion parameter with time. By making the speed time-dependent, the authors modified Benth and Šaltytė-Benth's (2007) temperature option pricing model; however, the study fails to provide a complete mathematical solution due to the resulting complexity of the model and they have to simulate the data with their model. Admittedly, the study does tremendous work on temperature modeling and brings the theoretical modelers closer to the real, but complex empirical world.

3. Data Description and Initial Analysis

I. Alberta Temperature Data Description

The temperature data used in this paper was downloaded from Environment Canada, a leading government organization offering reliable and ample weather information for all cities across Canada for weather researchers and other interest groups. Temperature data are collected for Edmonton, Calgary and Lethbridge, the three largest cities in Alberta.

Edmonton, as the capital of Alberta, is located at the geographical center of the province. More importantly, Edmonton, once called “Oil Capital of Canada”, is a major center for oil and gas industry and has a significant economic impact on the whole province (Wikipedia, n.d.).

Calgary is located at the transition zone between the Canadian Rockies foothills and the Canadian Prairies and has the largest population in Alberta. The factor that makes Calgary critical to our research is not merely the large population but also its contributions to the economy. Like Edmonton, the energy industry dominates Calgary’s economy. Calgary is a crucial transportation and distribution hub for agricultural and ranching industries. Moreover, Calgary is very close to Canada’s first national park – Banff National Park, which makes Calgary an ideal staging point for tourists (Wikipedia, n.d.).

Lethbridge is the largest city in southern Alberta. Most of the city’s economy is derived from or related to agricultural business. Compared with Edmonton and Calgary, Lethbridge is closest to the Rocky Mountains, which means it is affected the most

significantly by Chinook winds among the three cities (Wikipedia, n.d.). In Figure 1 the map shows where the Chinook winds occur most frequently.



Figure 1 Alberta map shows where Chinook winds occur most frequently. The dark red represents the area has the most frequent winds.

Clearly, Lethbridge, which is within the dark red area, experiences the most Chinook winds while Calgary has less and Edmonton has the least. Due to the economic importance of these three cities and strong ties between their businesses and temperature, we consider them as the best samples to efficiently represent the picture of Southern Alberta temperatures, which affect the Great Lakes Midwestern area. This area is of crucial importance to the natural gas futures market in the USA. In addition, because of different latitudes, we expect to see the similarities, but also the differences among the latitudinal temperature distributions of the three cities. The collected data are fairly high frequency, hourly, data covering the period from January 1, 1998 to December 31, 2009.

The observation time contains three leap years, which are 2000, 2004 and 2008 and nine ordinary years, resulting in three data series of 105,192 observations each. Any missing observations in the data are replaced with fitted values constructed by interpolating the average of the temperature preceding and following the missing observations. There are 19 interpolated data points in Edmonton, 34 in Calgary, and 71 in Lethbridge.

II. Dynamic Moment Analysis of Temperature Data

First of all, the raw hourly temperature data exhibits two distinctive patterns—chaos and sinusoidal-like patterns plus randomness, as shown in Figure 2. The stochastic data barely has trends or patterns, while the annual sinusoidal-like pattern fluctuates around the mean temperature. There appears to be larger-amplitude randomness and extreme jumps and singularities in winter.

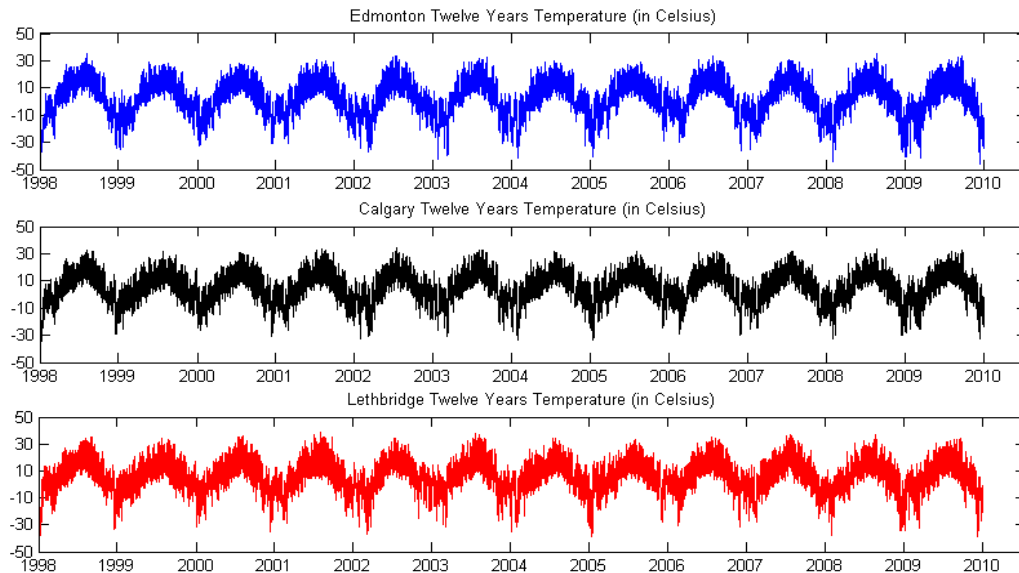


Figure 2 Twelve Years Hourly Temperature Series for Three Locations

Second, we observe that the Alberta temperature exhibits a strong seasonality, which appears to be hysteretic. The annual seasonality is caused by the obliquity of the earth's rotational axis relative to the sun in combination with the earth's annual rotation around the sun. Given a constant sun radiation, the outputs, temperature and its distribution, are varied corresponding to the specified seasons and spatial differences. In other words, the characteristics, such as the shape of temperature distributions, in the summer should differ from those in the winter. The seasonal distributional attributes differ among three cities.

Figure 3 and Figure 4 clearly demonstrate that the statistical temperature distributions are time- and spatially-dependent, which strongly contradicts the constant normal or Gaussian distributions assumed in classical temperature modeling and weather risk valuation. Figure 3 shows very different characteristics of temperature distributions for the four seasons in Lethbridge (The same binning or aggregation process has been used to generate distributions for Calgary and Edmonton also and the results are similar). Spring and autumn distributions are fairly normal with mean around zero and symmetric bell shape. Both summer and winter distributions are absolutely non-normal and non-symmetric. The summer distribution has a long tail to the right (higher extremes) and winter has one to the left (lower extremes). Moreover, the kurtosis or "peakedness" of the summer and winter distributions is lower than the one of the spring and autumn distributions, indicating that summer and winter have lower probability of having values near the mean. Figure 4 displays the summer and winter distributions for all three locations.

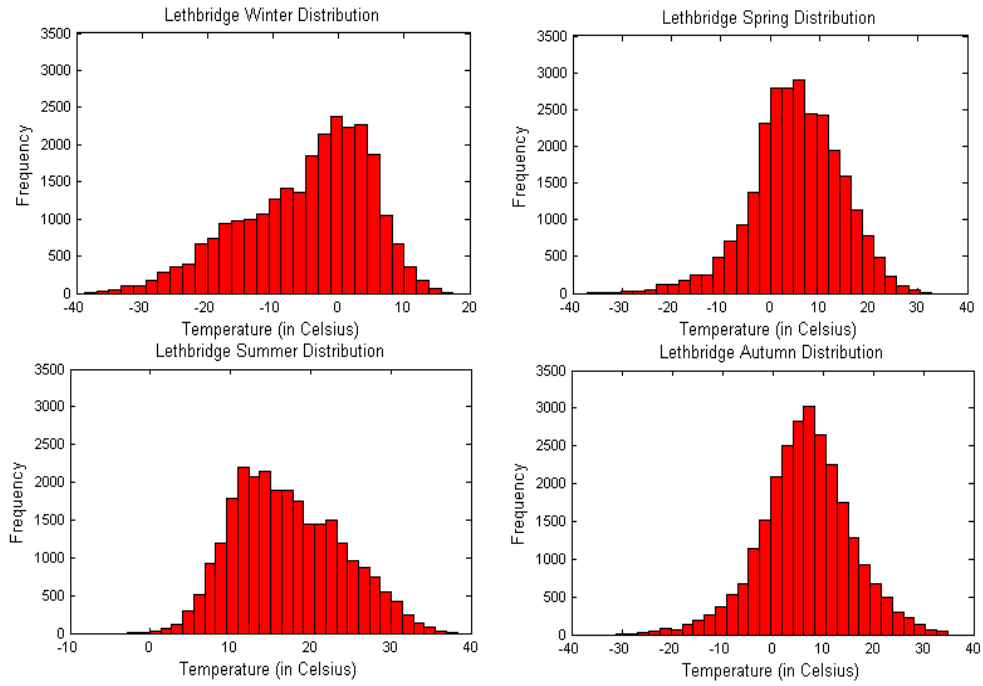


Figure 3 Temperature Distributions for Four Seasons in Lethbridge

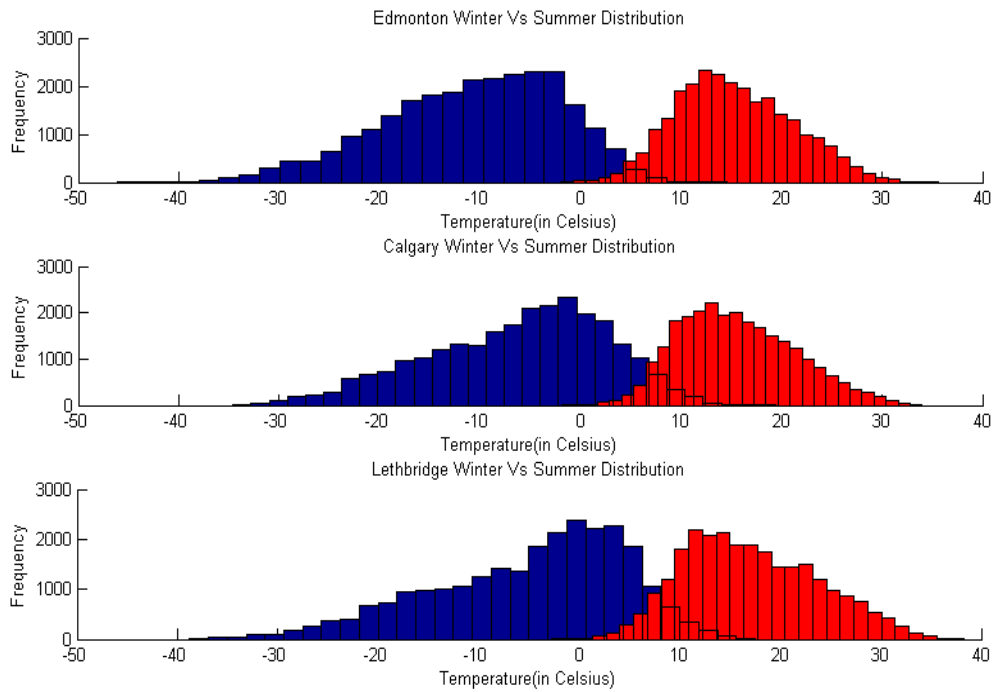


Figure 4 Summer and Winter Distributions in Three Locations

Finally, having derived the four monthly statistical moments (mean, variance, skewness and kurtosis) from the time- and spatially-dependent distributions, we notice that the four moments are not independent from each other, but exhibit some systems of fairly stable relationships among them. For example, the first three monthly statistical moments of Alberta temperature series possess clear seasonality, like the raw hourly data distributions. In particular, the summers have high positive average temperature, while the winters have low negative averages.

Also, we notice that in most winters there exists two periods of large temperature declines. Winter, in general, has a higher volatility (variance) than summer, meaning that the winters exhibit much larger temperature swings than do the summers. Furthermore, summers have a higher probability of having high and positive extreme values; hence, the summer distribution is positive or right skewed, while the winters have a longer tail on the left side, hence the winter distribution is negative or left skewed.

The seasonality in kurtosis is less obvious than in other three moments. The positive excess kurtosis (in excess of the "normal" kurtosis level of 3) usually occurs in the spring and autumn while the negative excess kurtosis usually occurs in the summer and winter at least measured as monthly averages. Figure 5 (a, b, c, d) contains plots of monthly mean, variance, skewness and kurtosis in three cities. The results of comparison of these data among cities again confirm the conclusion that the statistical moments of distributions are time- and spatially- varying. For example, because of the latitudinal variation, Lethbridge (225km south of Calgary) has the hottest average summer temperatures, while Edmonton (500 km north of Calgary) has the coldest average winter

temperatures. Edmonton is the most northerly city in North America with a metropolitan population of over one million.

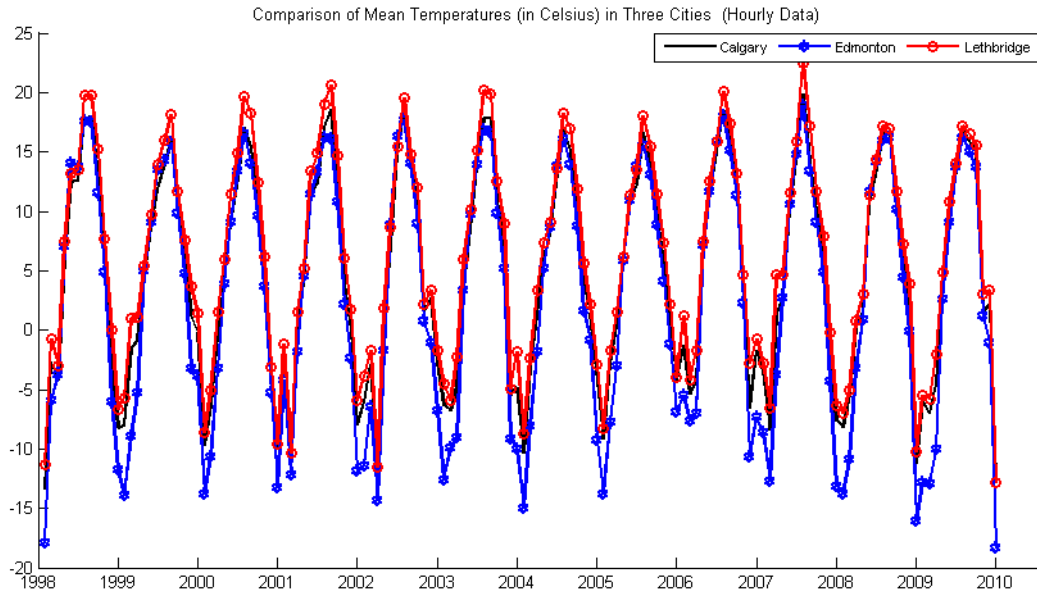


Figure 5a Monthly Average Temperature Plots in Three Cities

Figure 5b shows that the summer and winter temperature variances are largest in Lethbridge, while the temperature variance in Edmonton is overall the least of the three cities.

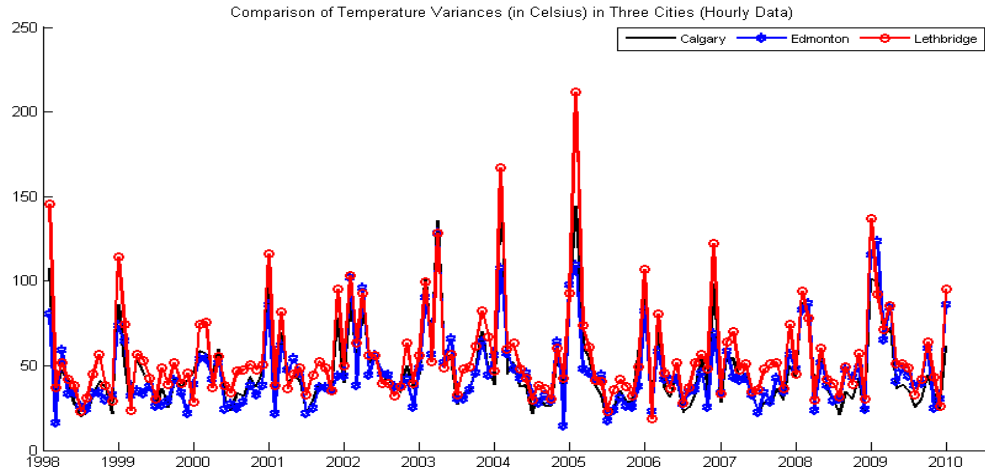


Figure 5b Monthly Temperature Variance Plots in Three Cities

Interestingly, Lethbridge exhibits the most extreme negative temperature skewness of the three cities, while Calgary exhibits the most extreme positive temperature skewness during the summer in the past twelve years.

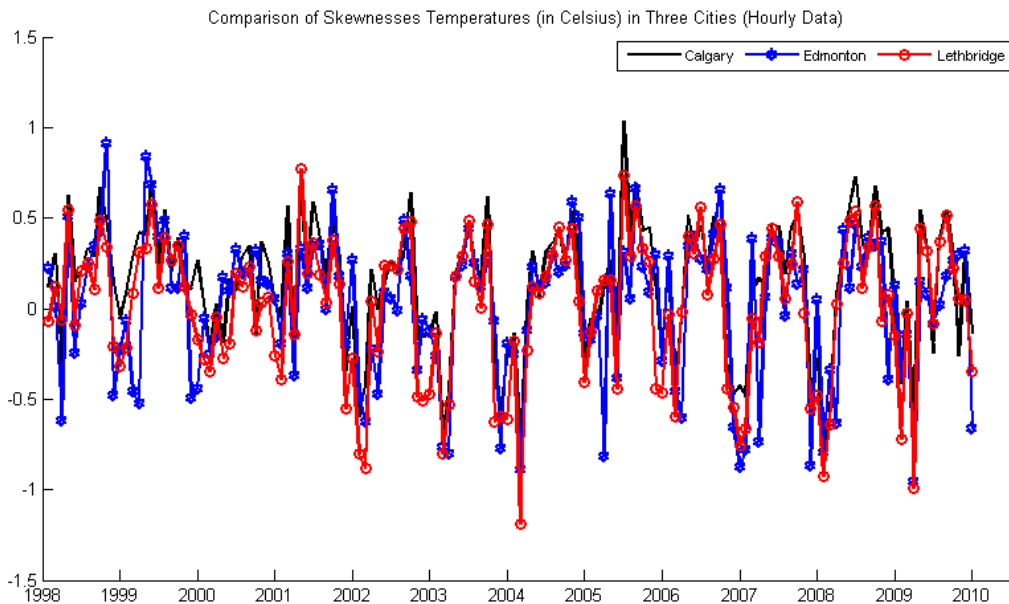


Figure 5c Monthly Temperature Skewness Plots in Three Cities

The most extreme values of kurtosis are observable in both Lethbridge and Edmonton. For example, 2004 showed both winter (platy-) and spring (lepto-) extremes in kurtosis for Lethbridge, while 2006 winter showed a positive extreme in (lepto-) kurtosis for Edmonton. Overall, the excess kurtosis per se looks rather random over time, but appears to have a slight negative bias towards platy-kurtosis around -0.5, with some dramatic exceptions in the early spring of 2004 and of 2007, for example. High leptokurtosis indicates that the majority of temperature variation is found in the immediate neighbourhood of the annual sinusoidal average temperature, with regular reversions towards the annual sinusoidal mean, while low platykurtosis indicates that the temperature variation is widely dispersed around the same annual sinusoidal temperature with much less reversion towards the mean.

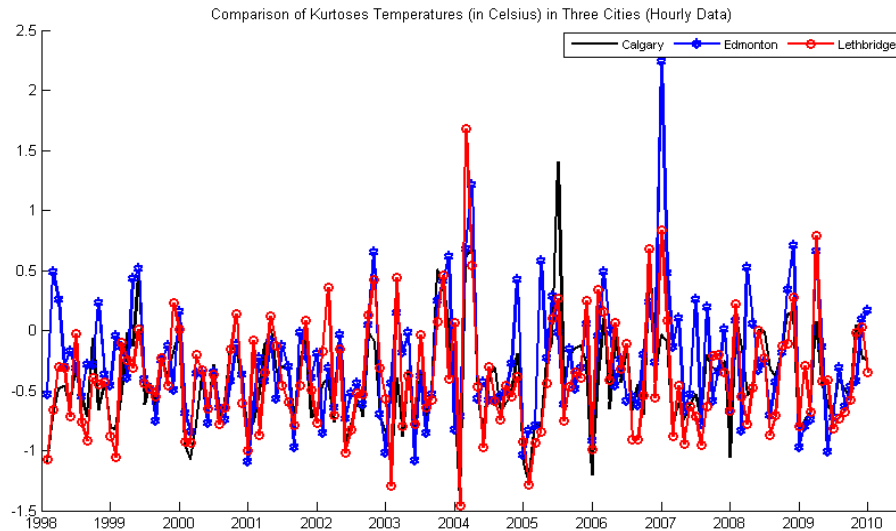


Figure 5d Monthly Temperature Excess Kurtosis Plots in Three Cities

Through examination of 2D and 3D graphics of the uncertain temperature system relationships between and among the four moments, we observe in Figure 6 that the relation between temperature mean (average) and variance displays a hyperbolic shape,

meaning that a high temperature mean will bring a low volatility while a low temperature mean will have a high volatility. In contrast, the scatter plots of the monthly averages and time-varying skewness cluster around a linear relationship. This linear relation implies that a high mean is correlated with a high and positive skewness and vice versa. Even though there are a few outliers, the relation between mean and kurtosis tends to exhibit a parabolic shape, from which we conclude that the extremely positive and negative temperatures tend to be accompanied by negative excess kurtosis (= platy-kurtosis), while the moderate temperatures tend to be accompanied by standard normal kurtosis (equalling 3; i.e., the excess kurtosis equals zero).

However, occasionally we observe extremely positive kurtosis (= leptokurtosis) close to zero degrees Celsius. This may be related to the “snap” or “freak” snow storms or blizzards (especially in November-December and in March –April) for which Southern Alberta is well-known.

Since the seasonality within all moments appears to be significantly driven by the value of the first moment or average, the study focuses on relationships between the time-dependent average (mean) and the other three moments, but not on the relations among the other three moments (although those are implied). Figure 6 displays 2D plots to show the relationships among four moments in Lethbridge. These relationships exhibit the same patterns in Calgary and Edmonton, as in Lethbridge.



Figure 6 Relationships between Mean and Other Three Moments in 2D (Lethbridge)

In all, we conclude that the first four moments of the Alberta temperature distributions are definitely non-stationary, time-varying and spatially (qua latitude) different. In order to accurately model the temperature variations (or volatility) and the financial risks for weather-related derivatives, a more advanced temperature distribution model is clearly called for, instead of the usual affine model of a static normal distribution.

4. Methodology and Analysis

I. Additive Sinusoidal Temperature Model

Our proposed model of the hourly temperature can be written as the following additive hourly time series

$$T_t = AS_t + IS_t + \varepsilon_t, \quad t=0, 1, 2, \dots \text{hours}$$

Here, T_t is our modeled temperature at hour t , AS_t and IS_t refer to an annual sinusoidal model and a diurnal (or intra-day) sinusoidal model, respectively, and ε_t is the residual, possibly long-term dependent, noise.

As discussed in the dynamic moment analysis section, we always experience an obvious annual seasonal pattern: the summer have higher temperatures, while winters have lower temperature. From a closer observation on the temperature data, we also find out that temperature follows another sinusoidal pattern within the 24 hours time range. We model both the annual cycle and the diurnal (intra-day) cycle of temperature with simple sine functions for each of the three locations, Edmonton, Calgary and Lethbridge, respectively:

$$AS_t = a_0 * \sin [(2\pi/8760)t + a_1] + a_2$$

$$IS_t = b_0 * \sin [(2\pi/24)t + b_1] + b_2$$

Where a_0 and b_0 are describing the amplitudes of the temperature, a_1 and b_1 are the phase shifts, and a_2 and b_2 define the constant average levels of the temperatures over a whole year, respectively a whole day.

We first employ the Nonlinear Least-squares regression fitting (nlinfit) function in MATLAB to fit the annual sinusoidal function AS_t to the original hourly temperature data. In Table 1, we report the different parameters for the annual sinusoidal model AS_t at each location. Next, the modeled annual seasonality is subtracted from the original temperature data and the function IS_t is fitted to the now annually deseasonalized temperature data. Table 2 displays the parameters for the intra-day sinusoidal model. Although we do not assume that the distribution of the residual noise ε_t is Gaussian in the following sections, for completeness, the t-values, which are based on precisely that assumption, are reported in parentheses under each estimated coefficient. Indeed, it would be very remarkable if that residual noise would be Gaussian: we find non-Gaussian persistence patterns and multi-fractality in the residual noise ε_t .

Table 1 The Fitted Parameters for the Annual Sinusoidal Model AS_t

	Lethbridge	Calgary	Edmonton
a_0	12.1916 (358.16)	12.0928 (387.07)	14.5861 (471.09)
a_1	4.3807 (777.95)	4.3989 (844.67)	4.4813 (1049.48)
a_2	6.0903 (252.53)	4.6586 (210.49)	2.7382 (124.88)
R^2	0.5495	0.5876	0.6785

From Table 1, we notice that Edmonton has the largest annual temperature amplitude $a_0 = 14.6^\circ\text{C}$ among these three Albertan cities, while Lethbridge has the highest annual average temperature $a_2 = 6.1^\circ\text{C}$, due to its lower geographical latitude (closer to the equator). On average, during the whole year, Lethbridge is 3.3 degrees warmer than Edmonton and 1.4 degrees warmer than Calgary, which is why it is

preferred as a location for retirement in Alberta. Notice that Edmonton has a slightly larger phase shift a_1 than either Lethbridge or Calgary. The explanatory power of these simple annual sinusoidal average temperature models varies between 55% (lowest in Lethbridge) to 68% (highest in Edmonton). This indicates that the average temperature in Lethbridge is 13% less predictable than in Edmonton.

Table 2 The Fitted Parameters for the Intra-day Sinusoidal Model IS_t

	Lethbridge	Calgary	Edmonton
b_0	4.7970 (155.73)	4.0950 (142.66)	4.2775 (152.07)
b_1	3.7009 (289.75)	3.6576 (262.44)	3.6431 (278.65)
b_2	0	0	0
R^2	0.1888	0.1635	0.1815

As seen in Table 2, the intra-day diurnal (“day-and-night”) sinusoidal temperature model – based on the earth’s rotation around its axis – adds another 16% variation explanation (lowest in Calgary) to 19% (highest in Lethbridge). The 3% difference in explanatory power of the diurnal temperature model is, maybe, attributable to the generally overcast skies in Calgary (with a slight radiation reversal) in contrast to the clear skies in Lethbridge (which has substantial radiation reversals between day and night). Indeed, Lethbridge has the largest average intra-day temperature amplitude $b_0 = 4.8^\circ\text{C}$. The average daily temperature difference between day and night temperatures in Lethbridge is substantial and, on (annual) average, half a degree Celsius more than in Edmonton and 0.7 degrees Celsius more than in Calgary. As expected, because of more

cloud formation, Calgary has the mildest temperature amplitudes b_0 for both the annual and the diurnal intra-day sinusoidal models.

II. Wavelet Multi-resolution Analysis of Temperature Residuals ε_t

The temperature residual ε_t is obtained by subtraction of both modeled annual and intra-day sinusoidal models from the original Alberta hourly temperature series. Figure 7 shows the temperature residuals series for Lethbridge and its plot of monthly variances.

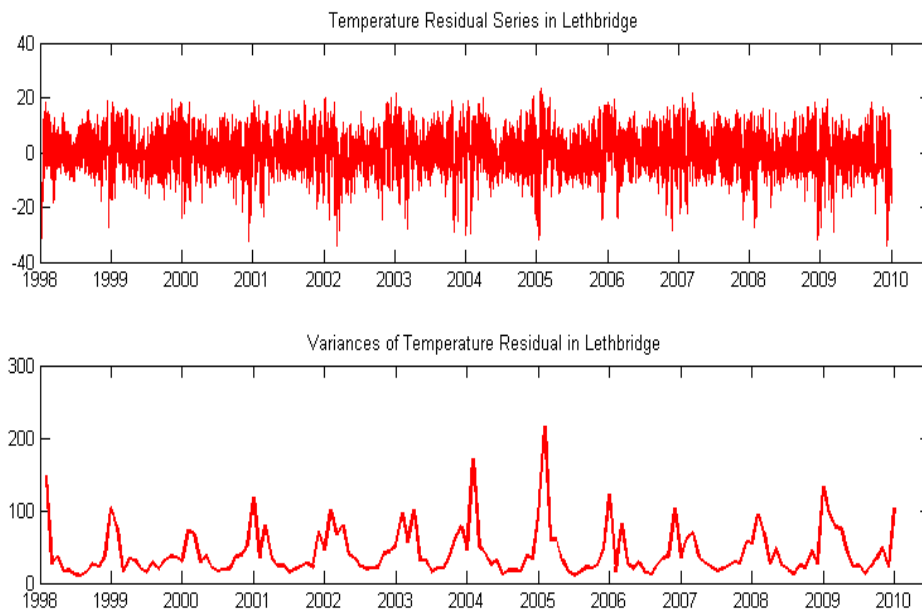


Figure 7 Temperature Residuals in Lethbridge and Its Monthly Variance Plot

We can clearly observe that the annual and diurnal seasonal patterns are no longer visible in the residuals. However, there is clearly still another temporal dependency in the volatility of residuals. The variations of the residuals are much higher in the winter periods than in the summer periods. The residuals show the cyclical recurrence of singularities and other irregularities in the winters. More specifically, the temperature residuals and their measured volatility or risk follow an aperiodically cyclical pattern: the

volatility of temperature reaches its peak every year in the winter period, but the exact timing of such peaks is uncertain. The recurrence of such aperiodically cyclical peaks in statistical variance indicates an underlying heretic or possibly chaotic system with two co-existing equilibrium points (Los, 2003, pp. 327 – 328).

The sharp discontinuities and singularities observed in both temperature residuals and its variance indicate that the residual series are fractional, non-stationary and exhibit seasonal long-memory behaviour. To test the persistence characteristics of the data series, we estimate their mono-fractal Hurst exponents identified from the wavelet multi-resolution analysis (MRA) of Mallat (1989a, 1989b, 1989c). The wavelet analysis, compared with the ordinary Fourier analysis which only provides information about the types of periodic frequencies, can describe localized risk in both time and frequency domains, simultaneously. It enables researchers to detect non-stationary spikes, discontinuities and singularities in a time series data. Scalograms (= colorized “coefficients of determination”), which are colorized visualizations of the wavelet resonance coefficients, identify the degree of correlation of the time series with a particular mother wavelet (from the dyadically tiled orthogonal wavelet base) at various time scales (= inverses of frequencies) and measure the power of the variations, discontinuities and singularities. The Hurst Exponent is derived from the slope of the scalegram, which is the time-average of a scalogram. A detailed scalogram and scalegram computation procedure has been precisely described by Los (2003). The scalogram is defined as the squared wavelet resonance coefficients $\{I_{j,n}^2\}$ while the scalegram is the variances of wavelet detail coefficients based on (usually) dyadic scaling

$$\begin{aligned}\text{Var} \{d_{j,n}\} &= E \{I_{d_{j,n}}^2\} \\ &= (\sigma_\varepsilon^2 / 2) V_\psi(H)(2^j)^{-(2H+1)}\end{aligned}$$

By taking the dyadic logarithm, the (negative) slope of the scalegram $b = 2H+1$ can be obtained, from which we can compute H . All scalograms and scalegrams in this paper are computed using MATLAB software (version R12.0). The mother wavelet we utilized in our MRA analysis is Daubechies wavelet at level 6 (db6)¹. Scales used in the analysis are dyadic of 14 levels, i.e., based on the power of 2, meaning that each scale $a = 2^j$, where $j=1$ to 14.

The Hurst exponent H helps to analyze the global dependence characteristics of a time series and to measure the degrees of such dependence. A normal geometric Brownian motion process has a neutral Hurst exponent $H = 0.5$. The GBM process has independent increments without any particular trends or tendency. When the Hurst Exponent is $0.5 < H < 1$, the time series is called persistent and is characterized by a Long Memory effect. If a persistent temperature change has been up (down) in the last period, then the chances are that it will continue to increase (decrease) in the next period. For example, trends and tendencies are clear in persistent financial markets. The closer the Hurst exponent H is to 1, the smoother the trend of a process will appear, but it can exhibit sharp, singular interruptions, like fault lines. A system with a Hurst Exponent $0 < H < 0.5$ is called anti-persistent. An anti-persistent series is also said to be fast mean-reverting, meaning that the system needs to reverse itself more frequently to the initial point than a strictly random series, by covering less temperature change in the same time.

¹ To ensure that our results are not sensitive to the particular wavelet chosen, we recalculated the wavelet coefficients and the slope of global wavelet for Morlet, another frequently used mother wavelet. The results are robust using these different wavelets.

Both persistent and anti-persistent processes seem to abound in nature, as demonstrated by Mandelbrot (1982). Anti-persistence is more likely to occur in relaxation processes, like fluid turbulence or deeply liquid foreign exchange markets (like the Yen/US dollar, or Euro/US dollar rates, which are based on reversing swaps) or fast-moving futures markets; persistence appears in long-run cyclical processes, like river levels, tides levels, stock market and real estate price changes (Peters, 1994).

5. Wavelet-Based Synthesis for Fractional Brownian Motion

Fractional Brownian motion (fBm, in short) is a zero mean, continuous-time random process. Because it presents characteristics of self-similarity, fractal, long-range dependence, fBm has quickly become a major research tool for various fields where such properties are relevant. Compared with ordinary Brownian Motion, the fractional Brownian motion process is much more accurate for modeling non-stationary stochastic processes and therefore, much more helpful in studying our real world phenomena. In addition, even though the fBm process itself is non-stationary, its increments are stationary, which makes simulation of such a process executable. Many efforts have been devoted to the possibility of performing numerical simulation for such a process. Sellan (1995) proposed a powerful additive wavelet-based synthesis of fBm and a practical implementation of such simulation was first launched by Abry and Sellan (1996). Sellan proposed that the wavelet representation for fBm $B_H(t)$ is:

$$B_H(t) - b_0 = \sum_k b_H(k) \Phi_{0,k}^{(s)}(t) + \sum_{j \leq 0, k} \gamma_j(k) 4^{-s} 2^{-js} \varphi_{j,k}^{(s)}(t)$$

$$\text{Where } \varphi_{j,k}(t) = 2^{-j/2} \varphi_0(2^{-j}(t-k)), \text{ with } (j, k) \in (\mathbb{Z}^+, \mathbb{Z})$$

Here $s = H+0.5$, b_0 is an arbitrary constant, γ_j are independent identically distributed Gaussian random variables, $b_H(k)$ is a fractionally ARIMA(0, s, 0) process, and $\Phi^{(s)}$ and $\varphi^{(s)}$ are suitably defined fractional scaling and wavelet functions, respectively. From the equation, the fBm can be interpreted as a $\sum_k b_H(k) \Phi_{0,k}^{(s)}(t)$ over which are superimposed a succession of details $\sum_{j \leq 0, k} \gamma_j(k) 4^{-s} 2^{-js} \varphi_{j,k}^{(s)}(t)$. The white Gaussian process γ_j carries the Gaussian nature of the fBm while the orthonormal wavelet basis $\varphi_{j,k}^{(s)}(t)$ captures the exact short-term correlation structure of the fBm. The long-term dependence

is mainly caught by the FARIMA process $b_H(k)$ as discussed in Abry and Sellan (1996). Flandrin (1992) interpret the detail resonance coefficients as “the difference in information between two successive approximations”.

6. Empirical Findings and Simulations

I. Measurement of Global Dependence: Hurst Exponent Results

To observe the effectiveness and efficiency of identification of our simple temperature model, scalegrams are produced of both the original temperature series and the twice (annually and diurnally) deseasonalized temperature residual series. Figure 8 displays the scalegrams using both series in the three cities in Alberta. All scalegrams in Figure 8 are not straight lines, indicating the existence of multi-fractality: there exist different degrees of persistence corresponding with various time scales.

We conclude that these scalegrams can be divided into three scale ranges. The first scale range includes short term scales from 2-hour (2^1) to 16-hour (2^4) (= intra-day, less than 24 hours), the temperature data in that time range is moderately persistent with values ranging from $H = 0.715$ to 0.770 in all three cities, with the greatest and identical intra-day persistence of $H = 0.77$ observed both in Lethbridge and Edmonton and the least in Calgary (again, we suspect, due to Calgary's regular cloud cover). The second scale range consists of medium term scales from 32-hour (2^5) to 1024-hour (2^{10}) (= extra-day, more than 24 hours to ca. 1.5 months). The slope of the scalogram flattens out in the medium term scales and indicates that the extra-monthly residual series exhibits anti-persistence (= mean-reversing and in the highest medium scale ranges possibly chaos).

As reported in Table 3, the values of Hurst exponents in the medium term scales are fairly small, i.e., $H \approx 0.10$, less than 0.2, so those series indicate turbulence or chaos, which is the co-existence of multiple temperature equilibria. Beyond the 2048-hour (2^{11}) (= one quarter year or seasonal scale), we observe that the slopes of the scalegrams are

negative. At these scales the residual series is truly completely random, without any form of persistence.

Compared with the deseasonalized temperature residuals scalegram, the scalegrams using the original temperature data exhibit peak power at the 24-hour diurnal temperature cycle and the annual temperature cycle.

While the scalegram refers to the solar power measured by the variance or resonance coefficients, we can conclude that the variances of temperature residual series like Hurst exponents are time-dependent. In other words, the “volatilities” of temperature noise are not fixed or unique but alter over time. Our temperature residual series indeed is thus modeled by a non-stationary stochastic process, the fractional Brownian motion (fBm), indexed by the Hurst exponent H .

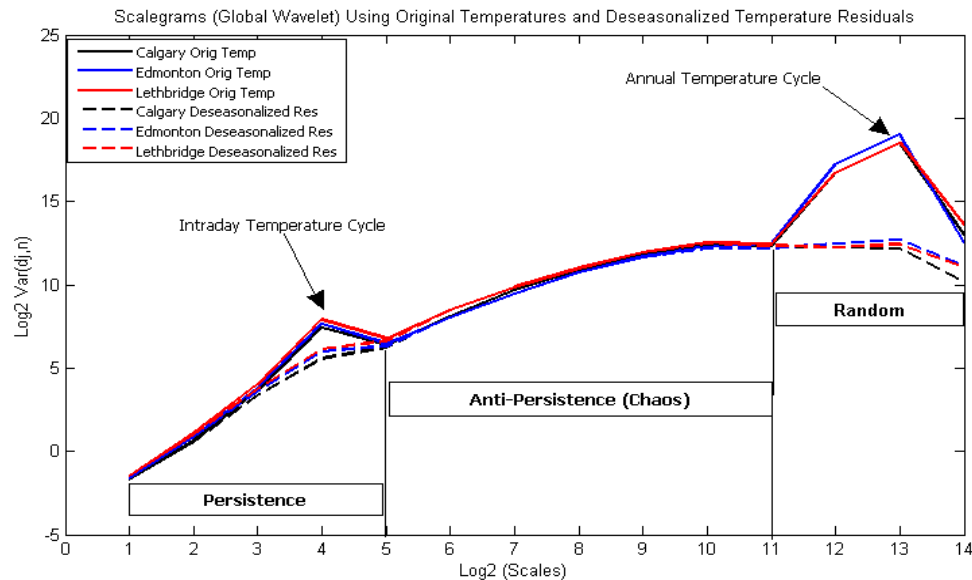


Figure 8 Scalegrams using Original Temperatures and Deseasonalized Temperature Residuals for Three Cities in Alberta

Table 3 Annual Hurst Exponents by Scales (Deseasonalized Temperature Residuals)

	Scale $2^1 - 2^4$	Scale $2^5 - 2^{10}$	Scale $2^{11} - 2^{14}$
Edmonton	0.770	0.091	≤ 0
Calgary	0.715	0.111	≤ 0
Lethbridge	0.769	0.089	≤ 0

Next, in order to see if winter temperatures are statistically different from summer temperatures, monthly Hurst exponents for the three cities are computed. In Table 4, all Hurst exponents for the intra-day scales are above 0.5, meaning that all seasons have persistent intra-day, diurnal temperatures. The winter temperature residuals over the medium term scales show H close to 0.10, indicating chaos in the period November through February/March for Lethbridge, November through March for Calgary and November through March/April for Edmonton.

Table 4 Monthly Hurst Exponents by Scales (Deseasonalized Temperature Residuals)

Monthly Hurst Exponents by Scales									
	Edmonton			Calgary			Lethbridge		
	Scale $2^1 - 2^4$	Scale $2^5 - 2^{10}$	Scale $2^{11} - 2^{14}$	Scale $2^1 - 2^4$	Scale $2^5 - 2^{10}$	Scale $2^{11} - 2^{14}$	Scale $2^1 - 2^4$	Scale $2^5 - 2^{10}$	Scale $2^{11} - 2^{14}$
Jan	0.763	0.089	≤ 0	0.684	0.140	≤ 0	0.779	0.115	≤ 0
Feb	0.670	0.063	≤ 0	0.633	0.108	≤ 0	0.741	0.090	≤ 0
March	0.722	0.087	≤ 0	0.710	0.076	≤ 0	0.737	0.039	≤ 0
April	0.787	0.037	≤ 0	0.754	≤ 0	≤ 0	0.775	≤ 0	≤ 0
May	0.778	≤ 0	≤ 0	0.728	≤ 0	≤ 0	0.747	≤ 0	≤ 0
June	0.745	≤ 0	≤ 0	0.646	≤ 0	≤ 0	0.739	≤ 0	≤ 0
July	0.742	≤ 0	≤ 0	0.698	≤ 0	≤ 0	0.790	≤ 0	≤ 0
Aug	0.837	0.030	≤ 0	0.816	0.028	≤ 0	0.852	≤ 0	≤ 0
Sept	0.829	≤ 0	≤ 0	0.793	≤ 0	≤ 0	0.794	≤ 0	≤ 0
Oct	0.742	≤ 0	≤ 0	0.684	≤ 0	≤ 0	0.653	≤ 0	≤ 0
Nov	0.735	0.081	≤ 0	0.654	0.069	≤ 0	0.684	0.086	≤ 0
Dec	0.758	0.065	≤ 0	0.704	0.110	≤ 0	0.766	0.106	≤ 0

Thus, Edmonton has the longest truly chaotic or turbulent winter season and Lethbridge the shortest. There also appears to be a hypo-anti-persistent or turbulent period (with $H = 0.03$) in August in both Edmonton and Calgary, which is, perhaps, tornado related.

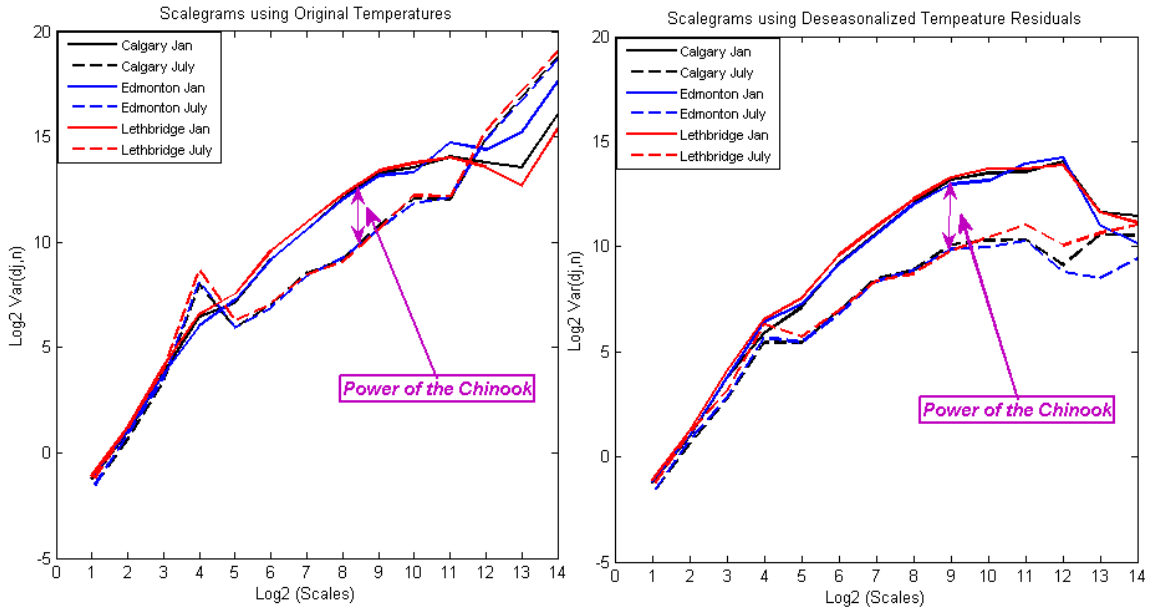


Figure 9 January and July Scalegrams using Original Temperatures and Deseasonalized Temperature Residuals for Three Cities in Alberta

From Figure 9, we notice that there is a power gap between the winter (solid line for January) and summer (dotted line for July) over the medium term scales, but not over short term: the medium scale January resonance coefficients are much larger than those in July. This significant energy gap between winter and summer is caused by the abnormally warm Chinook föhn wind in the middle of the winter (mainly January). Again, the variances (= measurable energy) of the temperature are reduced after both the diurnal intraday and annual temperature cycle peaks are taken out by the deseasonalization of our diurnal and annual sinusoidal models. One interesting finding

from original temperature scalegram is that the summer has a clear intraday temperature cycle while that cycle is less obvious in the winter.

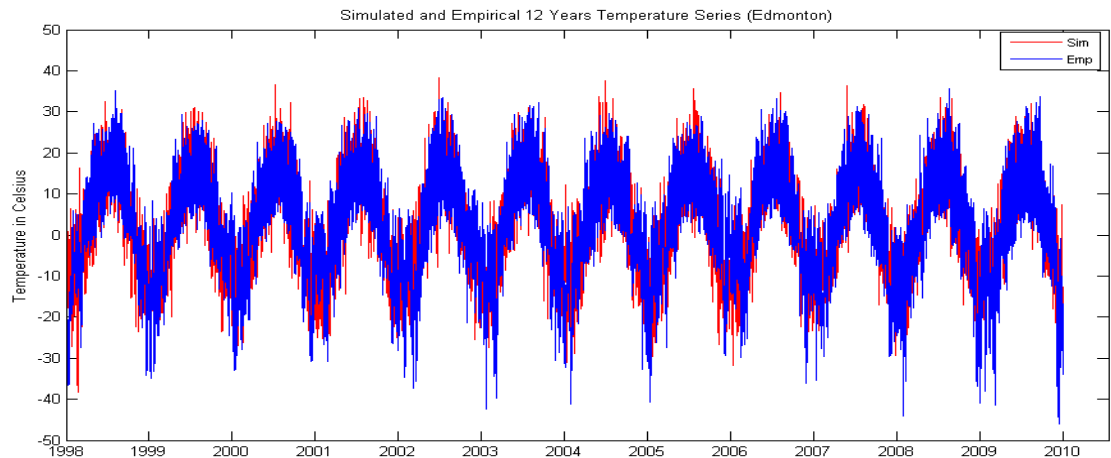
In summary, the volatilities of temperature residuals cannot simply be assumed to be constant (stationary) or stable as the conventional models assume. The classic Geometric Brownian Motion (GBM) is able neither to accurately reflect the persistence characteristics of the time-dependent temperature series, nor to model the pervasive non-stationarity and non-linearity of return series for temperature related futures and options.

Futures and forwards price curves reflecting temperatures, such as contracts related to energy or agricultural products, are closely tied to different investment horizons. Based upon our research, different investment horizons must lead to variously time-scaled volatilities. Therefore, financial-economic researchers must be extra cautious of such time-dependent volatilities, which do not scale like the square-root rule of the GBM, when pricing temperature related financial products such as oil, natural gas futures and weather options.

II. Simulation Results

How well does our model statistically identify the empirical hourly temperature data? To answer that question, in Figure 10, the twelve-year empirical temperature series in three cities of Alberta (in blue) are compared with their corresponding simulated counterparts (in red). For the simulation we used our additive model of the diurnal and annual sinusoidal models, combined with a model generating multi-scale fractal Brownian motion, i.e., properly persistence-parameterized and time-scaled fractal Brownian motion. In all three cities, the simulated series statistically represent the

empirical data quite well in both magnitude and fluctuations. Also the summer peak temperatures are properly captured by the simulated data; however, the winter extremes appear much harder for our simulated data to represent properly. This is especially true in Edmonton where extreme winter temperatures happen much more frequently than in the other two cities, our simulated data can only catch up to -35°C but not the singular extreme lower temperatures below that level. Moreover, the low extreme temperatures in the statistically simulated data do not take place as frequently as in the empirical series. We suspect that the low frequency of extremes leads to incorrectly measured skewness and kurtosis, which will be discussed later on.



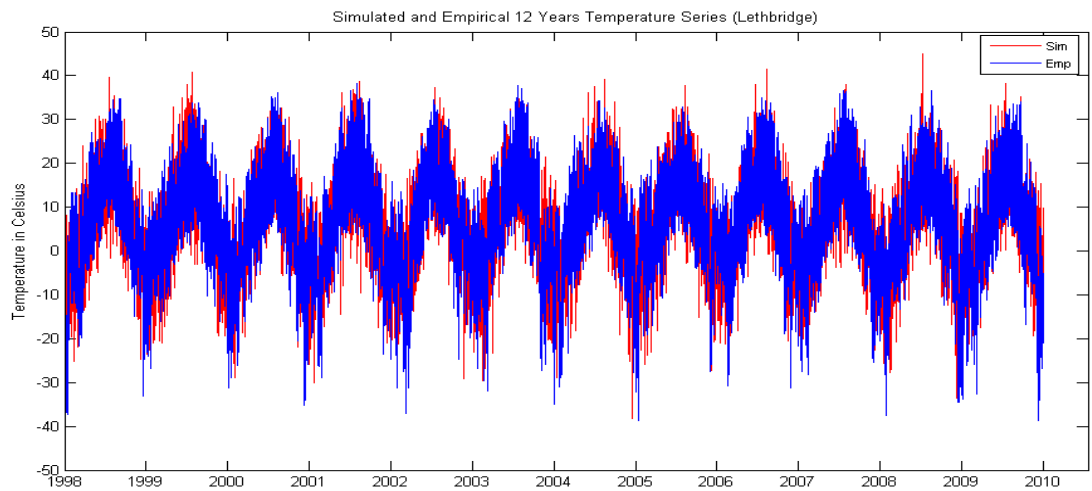
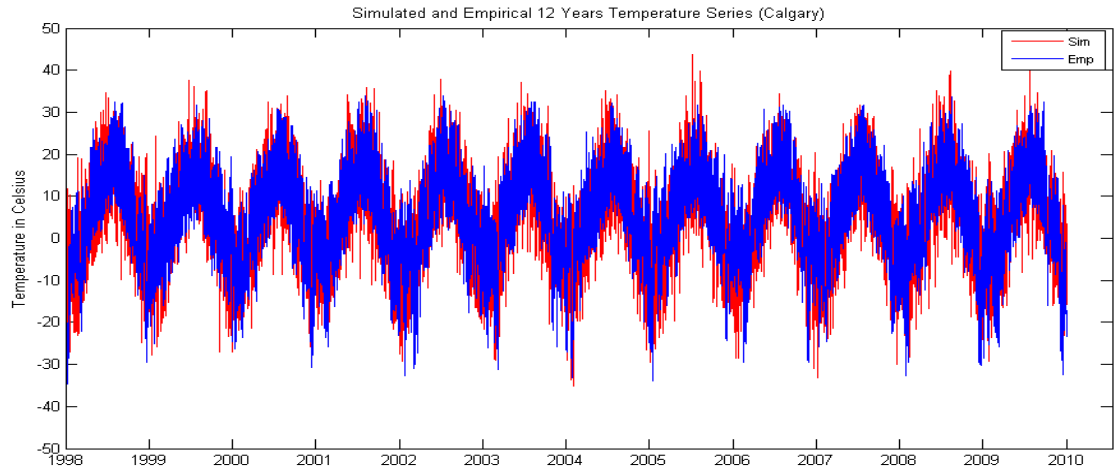


Figure 10 Twelve Years Empirical and Simulated Temperature Series for Three Cities in Alberta

The comparison of empirical and simulated scalograms is reported in the Figure 11. Both diurnal and annual sinusoidal phenomena are clearly exhibited in the empirical data as well as in the simulated data by the Fourier spectral “bands” at the dyadic diurnal ($j = 16$ hours) and annual ($j = 4096$ hours) scales. But our simulated data report less power or energy at scales 2, 4, 1024 and 2048 hours than the empirical data. Because of

lack of extreme low temperature in the simulated data, singularities and sharp jumps are become less obvious and more difficult to detect than in the empirical data.

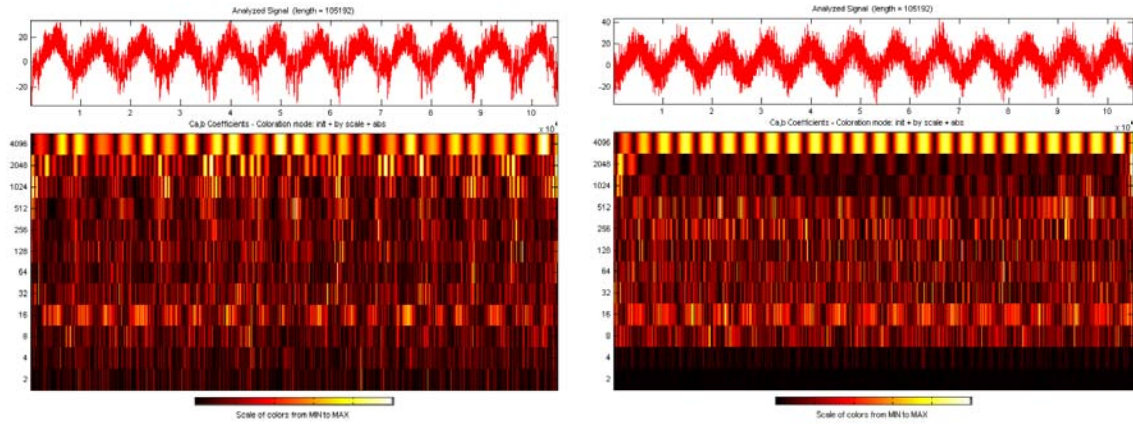


Figure 11 Scalograms of 12 Years Calgary Empirical and Simulated Temperature Data
Note. Left: empirical temperature series; Right: simulated temperature series.

For comparison, we also computed monthly statistical moments for the simulated data. The results for Lethbridge are shown in Figure 12. The same results were obtained for Calgary and Edmonton. Those results are similar and are available upon request. As displayed in the graphs, the seasonally varying averages are well identified. Our simulated data is able to catch the general fluctuations in the variance, but unfortunately, does not succeed in representing the very extreme values of the temperature volatilities in the winter periods, probably caused by the intervening Chinook winds.

Both the time-varying skewness and kurtosis are also not well identified by our simulated data during our examined years. The simulated skewness and kurtosis are able to follow the general patterns in the empirical ones sometimes, but not all the time. Regarding the skewness, this may be because the additive model is not completely correct, but should be replaced by a linear combination model with differentiated weights for the annual and daily sinusoidal patterns. Regarding the kurtosis, this may be because

the modeled kurtosis is partly distorted by the empirical fractality statistically measured by the Hurst exponent. The Hurst exponent, which, in its range (0, 1) is the inverse of a distribution's stability exponent, actually determines the kurtosis of a distribution.

This demonstrates how difficult it is to identify the true time-varying skewness and kurtosis of the non-stationary temperature data, even after incorporating the seasonality in the time-varying averages (means). The system of relationships between these seasonally fluctuating means and skewness and kurtosis is thus a still more complex system of relationships than our simple addition of seasonal sinusoidal models suggests.

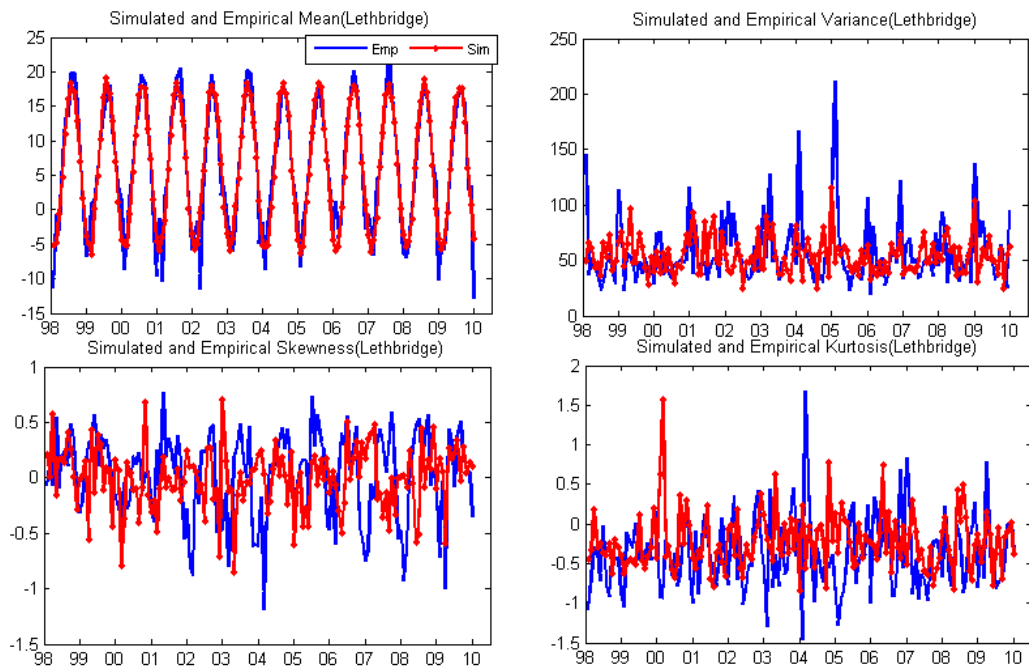


Figure 12 Empirical and Simulated Monthly Moments over 12 Years in Lethbridge

Statistical moments of the four seasons' distributions for Lethbridge are reported numerically and graphically Figure 13 and Tables 5 and 6. From Table 5, the dynamic averages (means) are properly modeled by our simulated data. The variances of the

simulated data are slightly less than those of empirical ones in both Edmonton and Lethbridge, but they are virtually identical to the empirical variance in Calgary. In all three locations the simulated negative skewness is less in absolute value than the measured empirical negative skewness (by about .33 - .36 undervalued), due to the fact that the singular extreme (“snap”) low temperatures in the winter time are not properly statistically modeled. Most undervaluation of the negative skewness exists in Edmonton. These extreme singularities in low temperatures are, mathematically speaking, like dirac deltas and are clearly identifiable in the wavelet scalograms of the empirical data. They tend to occur in the November-December and January-March months and are interrupted by the warm (“snow-eating”) Chinook winds in January.

Also, in all three locations the simulated (platy-) kurtosis (with values less than 3 = normal Gaussian value) is less than the empirical kurtosis. Lethbridge shows empirically even a slight leptokurtosis (with a value greater than 3). Again, this is likely due to the fact that the empirical data incorporate the extreme singular negative winter temperatures. The kurtosis is mostly undervalued in Lethbridge (by .68) and the least in Calgary (by .39), which suggest that the extremely dry and bright skies above Lethbridge make for nonlinear diurnal temperature (radiation) reversions, contributing to leptokurtosis (= more than normal temperature reversions around their seasonally (annually and diurnally) fluctuating mean).

Table 5 Four Moments of Empirical and Simulated Temperature Distributions

	Edmonton		Calgary		Lethbridge	
	EM	SM	EM	SM	EM	SM
Mean	2.75	2.72	4.66	4.65	6.08	6.09
Variance	156.70	145.54	124.47	124.22	135.30	126.42
Skewness	-0.37	-0.01	-0.37	-0.04	-0.39	-0.05
Kurtosis	2.78	2.22	2.95	2.56	3.22	2.54

Note. EM = Moments using 12 years of empirical data; SM = Moments using 12 years of simulated data

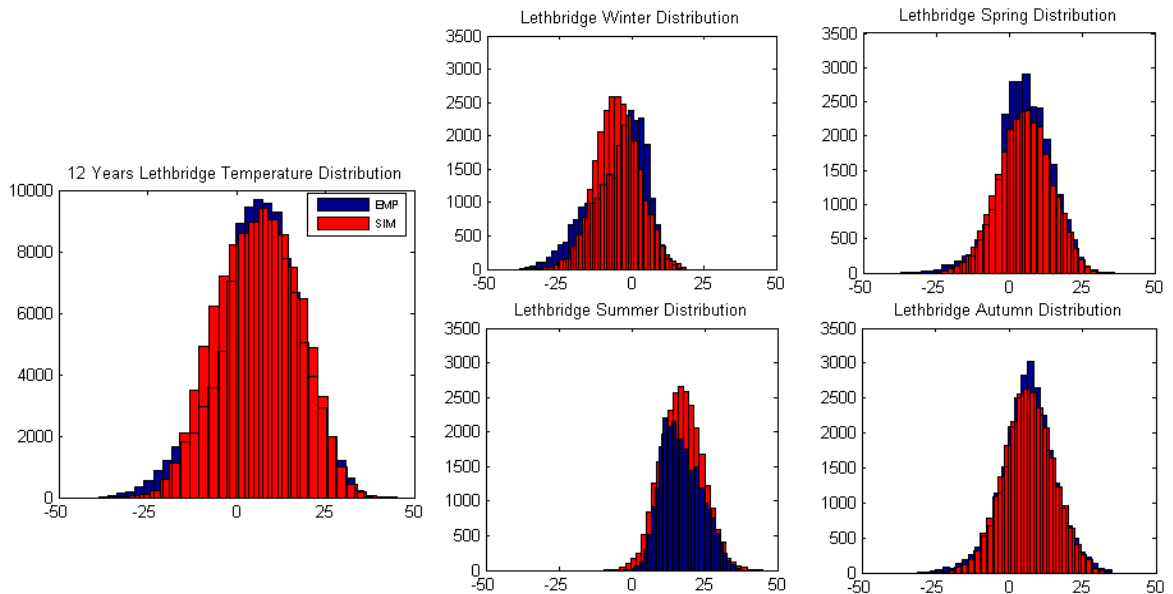


Figure 13 Comparisons of Empirical and Simulated 12 Years of Temperature Distributions and the Four Seasons Temperature Distributions (Lethbridge)

Table 6 Four Moments of Empirical and Simulated Four Seasons Distributions

	Edmonton Empirical Seasonal Moments				Edmonton Simulated Seasonal Moments			
	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
Mean	2.91	15.33	3.26	-10.77	3.04	15.88	2.08	-10.37
Variance	98.83	33.74	75.53	78.61	75.04	40.69	78.10	43.90
Skewness	-0.45	0.26	-0.09	-0.48	0.05	-0.11	-0.009	0.008
Kurtosis	3.76	2.61	3.51	2.86	2.71	3.06	2.72	3.18

	Calgary Empirical Seasonal Moments				Calgary Simulated Seasonal Moments			
	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
Mean	3.93	15.55	5.10	-6.15	4.13	15.51	5.01	-6.26
Variance	78.95	33.61	70.30	79.69	74.07	52.78	74.11	58.96
Skewness	-0.38	0.35	-0.13	-0.42	-0.007	-0.03	0.03	-0.02
Kurtosis	3.52	2.63	3.56	2.65	2.79	2.98	2.83	3.24

	Lethbridge Empirical Seasonal Moments				Lethbridge Simulated Seasonal Moments			
	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter
Mean	5.35	17.09	6.61	-4.92	5.30	17.13	6.68	-4.95
Variance	81.56	44.78	79.73	93.01	78.51	50.14	75.23	57.37
Skewness	-0.38	0.32	-0.19	-0.63	-0.08	0.07	-0.06	-0.02
Kurtosis	3.65	2.51	3.66	2.85	2.83	2.93	3.02	3.10

Figure 13 and Table 6 exhibit a more dynamical picture of our temperature data.

It remains clear that the winter variances are high and the absolute values of the negative skewnesses are too low for the winter periods in all three locations. But the modeled kurtosis is a bit too high. How to reconcile this with our overall annual moments in Table 5?

From Table 6 it is also clear that the empirical spring temperatures show more variance (energy), more absolute value skewness than the simulated data, like the winter temperatures, and that spring kurtosis, in all three locations is higher than what is modeled; i.e., again there are more diurnal temperature (radiation) reversions than what is modeled and simulated. There appear also to be more diurnal temperature reversions in the autumn. From personal observations, we know that the air in the spring and autumn seasons in Alberta is unusually clear (more than 100km visibility, etc.), leading to sharper than average diurnal temperature (radiation) reversions than elsewhere.

Finally, there may be a very banal governmental reason why in Figures 10 and 11 we find often temperature singularities in both November and March in all three locations

in Alberta: the official seasonal time-change and thus the one-hour jump in the temperature measurement, captured by the phrase (See Figure 14): “fall back one hour in the Fall and spring forward one hour in the Spring.” In most of Canada, Daylight Savings Time begins at 2:00am (in the cold winter night) on the second Sunday in March (so, suddenly, there is a discontinuity in the temperature series). On the first Sunday in November areas on Daylight Saving Time return to Standard Time at 2:00am (so, another annual discontinuity occurs). These temperature discontinuities are noticed as singularities in the temperature records and can increase the volatility of those time series records. However, it appears that the multitude of singular temperature discontinuities is larger than these two annual discontinuities and is thus probably due to the adiabatic Chinook winds.

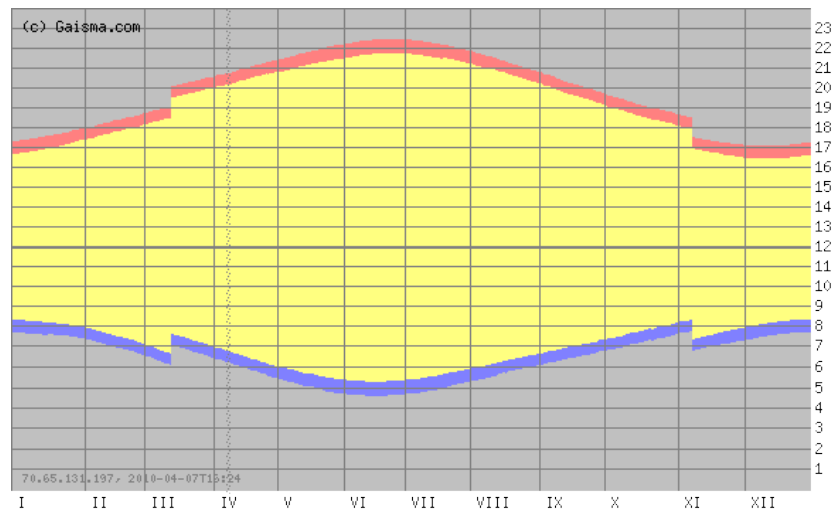


Figure 14 Annual-Diurnal Sun path Chart of Darkness, Dawn, Daylight and Dusk in Lethbridge, Alberta, taking account of Daylight Savings Time Changes in March and November

Note. Grey= darkness; blue=dawn; yellow=sunshine; orange=dusk

In section 3.II, we discussed the dynamic characteristics of temperature moments. We asserted that statistical moments like the original temperature series are not static but time and spatially dependent. Based upon our analysis, we decide to group the simulated data into four seasons to see if the simulated distributions are similar to the non-stationary empirical distributions. Again, the lower-order moments (average and variance) are much easier to identify than the higher-order moments. The simulated means of all four seasons in all three cities are very close to the empirical ones. The variances in both spring and autumn are properly simulated (because the spring and autumn temperature distributions are closer to the normal or Gaussian shape), but not in summer or winter.

The simulated summer variability of the temperature is too high, while the simulated winter variability is too low. Spring, autumn and winter all have negative skewness. Even though the simulated data captures the negative sign, the magnitudes are far less than the empirical ones. The empirical summers in Alberta tend to be a bit warmer or positively skewed (by about 0.3 degrees Celsius) and the empirical Alberta winters tend to be a bit colder than modeled (again by about 0.3 degrees Celsius), mostly because of extreme jumps in temperature (positive jumps in the summer and negative jumps in the winter). The kurtosis tends to be a bit higher in the spring and autumn (= more diurnal temperature reversions, due to less cloud cover) and lower in the summer and winter (= less diurnal temperature reversions, due to more cloud cover).

In summary, our simulation is able to successfully and accurately identify the lower-order moments of the temperature distribution, but less successfully the higher-order moments of skewness and kurtosis. Additionally, when modeling such non-

stationary time series, we find that a standard static distribution analysis of the whole series may cause the loss of important information. By grouping the series into different time intervals or windows (in our case, into four seasons), we were able to identify the distinct distributional characteristics of Alberta temperatures in three locations within each time window (each season).

7. Practical Importance of the Alberta Temperature Model: Natural Gas Futures

Since NYMEX first offered Henry Hub gas future contracts in April 1990, natural gas has become one of the most heavily traded commodities in the futures market. The underlying asset of one contract is 10,000 million British thermal units (MMBtu) of natural gas delivered at Henry Hub, Louisiana. The main consumption of natural gas in North America is derived from space heating in residential and commercial sectors, and from the electric power sector. The supply conditions of natural gas are mainly influenced by pipeline capacity, stock levels of gas in storage and operational difficulties from suppliers (cited in Mu, 2007). In general, natural gas is stored during spring and summer to meet the high demand starting in the late fall and winter seasons. Thus, the inventory level, driven by a highly seasonal demand, displays also a strong seasonal pattern: an increasing (high) inventory level from April to October and a decreasing (low) inventory level from November to March.

Without any injection of natural gas in the winter, the supply of natural gas is said to be fairly inelastic. The price movement of natural gas, therefore, is a perfect indication of seasonal demand-supply conditions. Strong seasonality in demand and storage also implies highly nonlinear volatility dynamics. The sufficient natural gas inventory intended for all winter months buffers the price volatility of the early winter contracts; however, with an ongoing huge demand and tight inventory level in the later winter, even a small magnitude weather shock can lead to substantial price fluctuations and significantly increase the futures price volatility (market risk). Thus, prices of options on

those natural gas futures, which reflect their volatility levels, can be significantly affected.

Recently increasing interest has emerged in studying the price dynamics and volatility of the natural gas futures market. However, much of empirical literature generally focuses on examining the role of the convenience yield of storage of natural gas - a marginal benefit from holding a commodity, and its impact on natural gas spot and futures prices and volatility (Geman & Ohana, 2009; Pindyck, 2004). Little effort has been devoted to examine the importance of fundamental factors such as the weather, in particular, temperature, in studying the determinants of price volatility in natural gas futures market even though most people know that the price of natural gas and its volatility are mostly, if not completely driven by temperature.

There are two very recent exceptions that are worth noting. Both Mu (2007) and Chan, Wang, and Yang (2009) have incorporated weather as a key factor into the modeling of natural gas futures price and volatility. Mu examines how weather shocks, storage surprises along with some other minor factors impact the price dynamics in the US natural gas market. The author employs the standard degree days (DD) method – counting the deviation of temperature on a given day from its normal level - to measure weather shocks and storage surprise, the two most predominant factors in the proposed natural gas futures returns and volatility. Not surprisingly, the results confirm the significant effect of weather shocks on both the conditional mean and volatility of natural gas futures returns. Mu (2007) also found that, by adding the weather shock and storage surprise variables in the GARCH model, the price volatility persistence is reduced by

about 40%. Similarly, Chan et al. (2009) incorporate an extremely low temperature variable and the inventory surprise variable into their jump dynamics model to explain the price spikes and volatility in US natural gas futures and spot markets.

Thus, both papers have acknowledged the important impact of fundamentals such as weather on modeling natural gas futures prices and volatility. But they simultaneously focus too much of their attention on the modeling of spot and futures prices per se, without fully understanding the recurring physical temperature patterns. Modeling natural gas futures price dynamics should not just focus on a few spot and futures prices. Instead, a complete, detailed and concrete analysis of the most basic but the most vital factor – temperature - must be conducted before the analysis of natural gas prices can take place. Weather risk, in particular temperature risk, has many aspects, represented by not only the variance, but also by the higher-order moments.

Another reason which makes Alberta temperature special for energy modellers is derived from the fierceness of its nature. Alberta temperature volatility is largely affected by the famous Chinook föhn winds in January. Interestingly, even though Chinooks bring extremely warm temperature to southern Alberta, these same winds may lead to extreme temperature drops in the central Canada and the United States. An Alberta Clipper (Wikipedia, n.d.) is a fast moving snowstorm originating from the Chinooks. The Clipper is formed when the Chinooks are entangled with the cold air mass over the Canadian prairies. Then the storm, like a clipper sailing ship, sails southeastward under the push of a northwesterly jet stream into the Upper Midwest and the Great Lakes regions of the United States toward the Atlantic Coast. The Alberta Clipper is generally weaker than a

regular storm and brings less snowfall because of its severe lack of moisture and its fast speed. However, as soon as the storm travels close to the Great Lakes regions and the cold dry air absorbs moisture from the lakes, the snowfall amount can be substantially increased while the temperature remains very low. Therefore, weather in Alberta can serve as an early warning signal to the Great Lakes, in particular, Chicago energy market in the United States.

Strong Chinooks in Alberta can result in an unpredicted natural gas price swings in the immediately following days. Energy traders and energy companies can have extra time to adjust their trading strategies accordingly rather than be beaten by unpredicted shocks in the energy demand. Potentially interesting future research could examine what if any direct, and perhaps modellable, relationship exists between the non-stationary, time-varying Albertan temperature distributions and the natural gas price fluctuations in the United States.

8. Conclusion

This paper makes the following contributions:

1) Exploration of the critical characteristics of the fundamental temperature factor in order to help groups of people such as farmers, energy companies, ski resort managers, insurance companies in Alberta and, potentially the fertile Great Lakes areas, to better understand weather risks they are facing.

2) Application of advanced signal processing methodology of wavelet multi-resolution analysis (MRA) to identify the degrees of long range dependence, or Hurst exponents for non-stationary, self-similar data.

3) Employment of a fast wavelet based synthesis of fractional Brownian motion to simulate the dynamic statistical distribution moments and to verify the quality of their modeling.

4) Strong motivation to research in detail the relationship between the temperature data in Alberta and the lagged natural gas futures prices in the great lakes area, since the Alberta temperatures may form an early warning signal for lagged heightened volatility and risk in the natural gas markets of the Midwest.

5) Motivation for the design of a possible Chinook Option contract that may function as an enhanced temperature risk insurance contract for use by energy companies, farmers and ski resorts in Alberta.

This paper analyzes, models, and statistically simulates the detailed time-varying statistical moments of the non-stationary temperature data at three locations in Alberta –

Edmonton, Calgary and Lethbridge. Hourly data are used to study the high-order statistical moments of temperature distribution. In contrast to the assumptions of conventional static distribution theory, our findings show that the first four moments of temperature distribution are indeed dynamic, seasonal and time-dependent, showing Long Memory.

A simple temperature model consisting of a bi-sinusoidal temperature model - one representing the annual cycle and the other the diurnal cycle - and a fractal residual model is proposed. We implement the advanced signal processing methodology of wavelet multi-resolution analysis (MRA) to both original temperature data and to the de-seasonalized data to identify the multi-fractal degrees of global dependence. The computed Hurst exponents differ between the various time scales, indicating multi-fractality. We observe that both the original temperature and the temperature residual have persistent pattern at intra-day time scales, i.e., within 24 hours. At the extra-day-within-one-year time scales, both temperature and temperature residual series exhibit anti-persistence, meaning that temperatures mean-revert or are possibly chaotic, depending on the value of the Hurst exponent. At extra-year time scales they are completely random.

There is a discernible large energy (measured by residual variance) gap between summer and winter temperatures, caused by the Chinook föhn winds. Temperature volatilities in the winter seasons, particularly in the November-December and February-March periods, are much larger than those in summer. Thus, in practice, we advocate that those energy modelers must be cautious if they assume constant or stationary volatilities

in, for example, futures pricing models. To avoid the risk of underestimating the volatilities, we suggest that the volatility valuation of any weather related financial products must be differentiated over different time horizons. Facing seasonal dependent volatility, weather derivatives modelers would be prudent to identify the initial weather index value and the volatility level of the temperatures if they employ the degree-day index to price the call and put weather options.

A wavelet-based synthesis of fractional Brownian motion is implemented to reconstruct the fractional temperature residuals series and time-dependent dynamic moment system and to test the quality of our dynamic moment model identification. Our simulation can accurately reproduce the low order statistical moments for non-stationary data distribution, and, unfortunately, less accurately, not the higher-order moments. We indicate possible physical reasons for this to happen. Future research on modeling dynamic, time- and spatially- varying higher-order moment systems is urgently needed.

9. References

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