DEVELOPING CONCRETENESS THROUGH CONNECTIONS WITH MATHEMATICAL REPRESENTATIONS

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DEDICATION

Firstly, Jason, thank you for being my steady partner in this journey. You often carried the weight of two parents, proofread more papers than anyone should, and listened patiently as I rambled about theories and frameworks you never signed up to learn. Dexter and Greyston, thank you for being so understanding when I couldn't come out to play. Your hugs and laughter kept me grounded, and your patience meant more than you'll ever know.

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ABSTRACT

This paper presents a new framework called the Concretion-Based Learning Framework (CBLF). It is a tool created to support teachers in planning learning environments that promote conceptual understanding through the interactions that students have with multiple representations. Drawing from my teaching experiences, this project identifies gaps in existing models, such as the CRA model and the ACT model, specifically with how conceptual understandings are explicitly planned for. Drawing on research from Wilensky (1991), Hattie et al. (2017), and Wiggins and McTighe (2005), the project challenges previous understandings of concrete and abstract knowledge in mathematics and suggests that the process of concretion is the missing component when developing conceptual understandings. The CBLF challenges a more traditional linear instructional approach and suggests a cyclical planning process where goals, assessments, and instruction are intentionally aligned so that we can monitor the connections that are being made by students. In response to this need, a Teacher Clarity Tool was developed to guide the instructional design process to promote alignment to achieve the goal of developing concreteness and mathematical proficiency.

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CHAPTER 1: INTRODUCTION

Situate myself

My teaching experience has been a unique journey but it has allowed me to gain insights into mathematical learning and teaching from a variety of perspectives. My early teaching experience was in a rural school where I was the only math teacher for grades 5 through 12. Students would complete their numeracy education with me and it was my job to ensure that they were exposed to a variety of representations to support their individual path towards becoming numerate. A benefit of this process was that I was building the foundations for their prior knowledge that I could connect with new concepts. Multi-graded classrooms also facilitated rich classroom discourse around similar concepts that were being explored at different levels. To survive in a thriving multi-graded classroom, it was important to take the time to align concepts purposefully to ensure that these conversations could span beyond specific grade level outcomes. My students began to take on a few of the characteristics that I modelled in my teaching and thinking, including understanding the importance of being exposed to multiple representations. I still remember a moment after a lesson on factoring trinomials where a student commented, "that is an interesting way to do it, but I'm not sure about it yet. What else you got?"

Fast-forward to my current teaching experience where I have 4 different classes of grade 8 math with students who have a variety of socio-economic and cultural backgrounds. Initially, the instructional planning seemed easier since it was only one curriculum. However, it ended up becoming just as challenging because students had a variety of experiences and foundational knowledge that needed to be discovered. Teachers were organized by grade level rather than by subject, making it challenging to vertically align concepts or discuss the different representations that students were exposed to. A benefit of this structure was that my planning was built around exploring and strengthening foundational knowledge before introducing new concepts. Additionally, having only one curriculum to plan for allowed me to focused on developing conceptual understandings that I knew would support student achievement with mathematical proficiency.

Another perspective that I had was from the district level when I was the instructional coach for two years. I was able to work with a variety of teachers in different contexts and it gave me a fuller picture of common instruction practices in math classrooms throughout our division. Teachers were not seeing the results that they hoped that they would and they would bring their specific concerns to various committees that I was part of. The school division would then start looking for other strategies to target these specific concerns, causing a revolving door of strategies that are exhausting for teachers to keep up with. Furthermore, teachers begin to resist new instructional models because they are not confident of its "shelf-life" before the next new teaching strategy is brought out. Typically, the shift between strategies reflected a pendulum swing from a focus on procedural skills to critical thinking. Rather than replacing past strategies and models with something new, teachers need a supporting framework for the learning process, where previous proven methods can work together to achieve a desired goal.

Issues that I noticed

From these experiences in my teaching career, I have noticed issues of concern in student learning, the Concrete-Representational-Abstract (CRA) model, and various resources. While teaching in a larger school, student performance in mathematics was an initial concern. Representations that I had used successfully in the past were met with blank stares instead of curiosity and intrigue. Perhaps the students were shown different representations of the concepts that I was introducing but not the threads that connect them into a conceptual web. A teacher might hear students say "I was never taught this" (which I assume they have forgotten since it is in the previous curriculum) or "this is not the way we were taught how to do this" (which I assume they spent more time mastering a process than understanding the foundations). These responses from students raise two concerns: retention of mathematical concepts and depth of understanding of representations in mathematics. While these may present as two distinct issues, they are related and influenced by each other.

When I started evaluating the CRA approach, common themes emerged; like the effective use of it with developing conceptual understanding and with facilitating meaningful interventions. However, as I read through various research with the CRA approach, I felt more confused than I had been before. Milton et. al. (2018) described the CRA sequence and summed up the final stage where "students complete operations using numbers only, associating previously formed representations with symbols" (p. 33). Just when I thought I was about to find the answer to how these association were made, the topic changed. I heard myself (not so subtly) yelling at my screen asking "But how?". Perhaps, like Wiggins and McTighe (2005) says, teachers put "faith in learning through osmosis" (p. 21) and assume that it will happen. I have experienced situations where students have made meaningful connections without guidance but how can a teacher ensure that the instructional methods are not leaving this to chance and that all students have the opportunity for this lightbulb moment? This is why focusing on the opportunities for creating connections should be the focus for teachers, rather than stressing about the instructional strategies that are chosen. There are many great resources and tools out there for instruction, and the CRA approach is one of them, but time is needed to intentionally plan for deeper learning.

Teachers are able to access a number of resources that focus on instructional and assessment strategies but it is much more challenging to find resources that focus on connecting different representations of mathematics for students. Unfortunately, strategies or models that lack clarity around a connection process run the risk of having teachers adopt a model that is "proven" to work without having all of the pieces for the puzzle. Another consequence is that teachers will view a textbook resource as a guide for effective instruction. This realization prompted me to take a closer look at a textbook, and I saw a few things that stood out to me that can influence a narrow understanding of effective instructional strategies. First, when reading the front matter of the textbook, it highlighted the importance of exposing students to multiple representations, however it was clear from the table of contents that manipulatives, visuals, and symbolic representations were explored separately. Additionally, manipulatives and visual representations were used during exploring concepts but they were not integrated into lessons that focused on processes with algorithms. From my experience, students often found visual modelling (especially with fraction division) to be more abstract to them and required a higher cognitive demand than the procedural skill connected to it. The textbook also missed the opportunity to have students engage in a variety of manipulatives and visual methods that might be contextually meaningful to different students. Practice problems in the textbook often structed the visual modelling questions at the beginning of the assignment, using the assumption that this would be surface level knowledge and be a good indicator if a student understood the concept or not. The issues that I was noticing caused me to have more questions than answers but there was one that I kept coming back to: "What is the goal?" If the goal is to have students master an algorithm symbolically, then what purpose do manipulatives or visuals have in the learning progression? If the goal is for students to achieve deep conceptual understanding, then instructional planning must be aligned to the practices that address it.

My initial plan for this project was to focus on developing a unit plan that supported conceptual understanding and positively influence student retention of the topic. Consequently, I found myself (unintentionally) avoiding the metaphorical elephant in the room: concrete connections. I understood that exposing students to multiple representations were important and I also knew that I needed to connect these representations together. Simply highlighting connections between concrete, representations, and abstract concepts were easy enough to demonstrate but the students were not given the time or environment to solidify these connections as concrete to their own understandings. I could see the connections clearly, as the expert in the room, but this is because I have been exposed to many different mathematical concepts. My instructional practices were strong and I could tell that my conceptual understanding was becoming deeper each year. Yes, my understanding, the teacher's understanding, not the students'. And so, I returned to my unanswered question, "What is the goal?" If deep conceptual understanding is the goal for students, then the focus of my project is to develop a framework that facilitates an environment that allows students to explore and create their own connections in mathematics.

CHAPTER 2: CLARITY

Defining and redefining concrete and abstract

What is concrete? What is abstract? My previous understanding was that concrete meant that it was tangible, specifically in a kinesthetic way. Furthermore, abstract meant that the math was presented in a way that could only be understood if the concrete foundation was built, specifically with using symbolic representations. In other words, students would be able to take their understanding and transfer it to something new. Using this definition implied that the learning progression needed to start with concrete experiences and then end with the target of solving abstract, or symbolic problems. My understanding of these terms was challenged when I read Wilensky's (1991) definition of concreteness as "not a property of an object but rather a property of a person's relationship to an object" (p. 4). Additionally, "objects of thought which are given solely by definition, and operations given only by simple rules, are abstract in this sense" (Wilensky, 1991, p. 5). If new knowledge is presented to students in a singular experience, regardless of it being a visual, physical, or symbolic representation, it would be considered an abstract concept.

Wilensky (1991) furthered my understanding of this paradigm by defining concretion as "the process of the new knowledge coming into relationship with itself and with prior knowledge, and thus becoming concrete" (p. 6). This was the moment where I realized that students do not naturally progress from new knowledge to having a conceptual understanding without explicit opportunities to engage in meaningful discourse. The teacher is tasked with designing instruction that allow students to experience concepts through multiple representations as well as opportunities to explore the connections that will develop concreteness. I found that my own teaching practice was not consistently supporting students with conceptual understanding until I started to intentionally focus on building connections and metacognitive skills into learning experiences.

Who are the connections for?

Rather than striving for students to demonstrate a singular abstract understanding of a concept, teachers can encourage them to gain a general concrete understanding, where they have interacted with and made connections between different representations. Gravemeijer (2011) acknowledged the importance of connection making when building concrete knowledge with students, specifically with whose knowledge is involved in this process:

> In mathematics education, we use so-called manipulatives—either in the form of tactile objects or as visual representations--to help students to make connections with what *we* know. While, when giving an example that the others will be familiar with, in a conversation, we try to make a connection with what *they* know (emphasis from original, p. 2)

If this is natural to do in our social interactions then why would teachers move away from it during instruction? I find that when I explore a strategy with manipulatives, I get really excited when I discover a new connection and I want my students to feel that moment too.

Despite my best intentions, the lightbulb moment does not often happen for my students and Gravemeijer (2011) suggested that the reason for this is that abstract mathematical knowledge belonging to the teacher is being used as the foundational knowledge that they are expecting students to make connections with; the knowledge gap is too large to make the leap. Hattie et al. (2017) used the term “the expert blind spot” (p. 40) from Nathan and Petrosino (2003) to describe the scenario when the teacher makes the leap before giving students the tools to construct their own bridge to make the connection. This disconnect, between the owner of the prior knowledge and the one creating connections, was what I was observing in my practice but I did not have the knowledge of how to articulate it. The process of removing teacher knowledge out of the learning experiences will require intentional planning. Additionally, activities that foster connections will initially take up more class time but it will save time when the concepts are revisited because the foundation is already in place. Understanding this potential blind spot allows us to integrate planning strategies that focus on allowing students the time and experiences to connect their prior knowledge to new concepts.

Clements (2000) furthered my understanding of concrete by describing two distinct categories: sensory-concrete (physical objects that students interact with) and integrated-concrete (concepts that are solidified in our minds due to their connections with our other knowledge). The tension between definitions of concrete and abstract are important to understand so that we can vocalize what our goals are for our students and consequently design an effective learning environment to meet those goals. A more concise clarifying moment occurred to me when considering the representations that I would include in my own planning. I looked at the diagram of multiple representations in mathematics that was presented by NCTM (2014), shown in figure 1, and saw that a strong learning environment involves both sensory and integrated definitions of concrete working together.

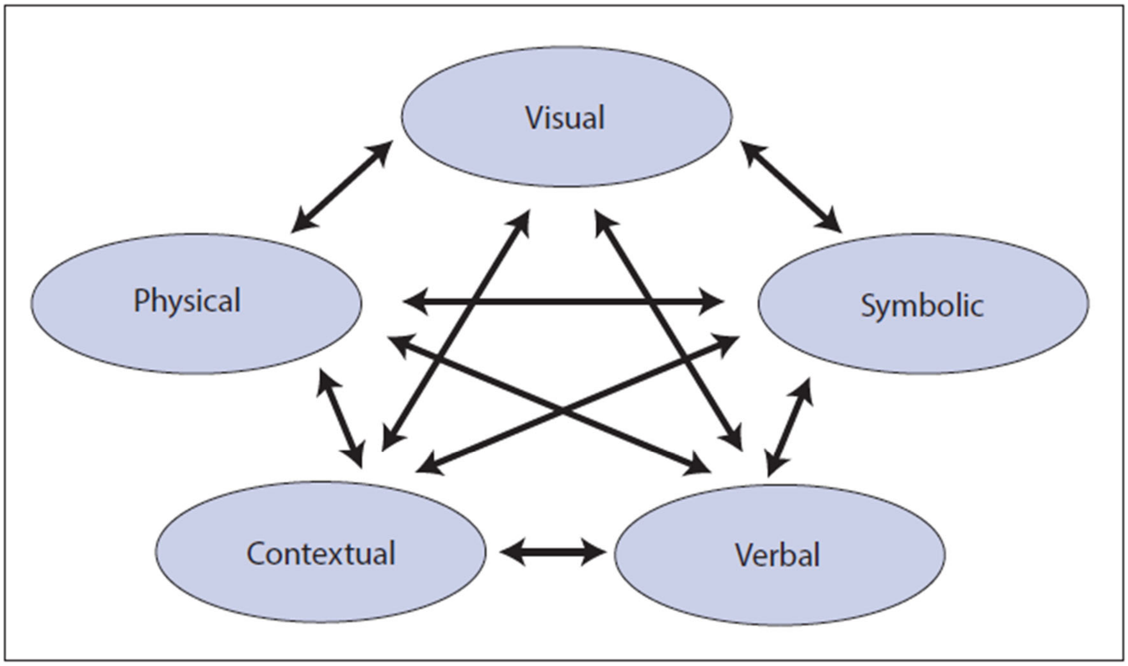

Figure 1. Connections among mathematical representations (NCTM, 2014, p. 25)

NCTM (2014) referred to Tripathi's (2008) depiction of using multiple representations as opportunities to see a concept from a variety of views so a complete picture is visible. If we are to think of mathematical learning as becoming visible for students, we must also consider strengthening the relationship between external representations and internal representations. Goldin and Shteingold (2001) described external representations as being the mathematical concepts that are presented to a student and the learning environment that it occurs in. Further, internal representations are the meanings and connections that students create in relation to what they are exposed to. Hattie et al. (2017) mentioned that instructional strategies become more effective when students are given the chance to interact with multiple representations and respond to them in an individual way. This clarifies that there is a need for teachers to purposely plan for experiences that give students the autonomy to explore connections.

What environments foster connections?

Kilpatrick et. al. (2001) suggested five strands for mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition), each being tightly woven together and essential for students to become proficient. This woven structure, as shown in figure 2, is a necessary component in a learning environment that fosters connection-making. A list of given strands can present itself as a linear check-list for teachers but the connectivity is where the true power lies. If interwoven strands are essential for mathematical proficiency, I find it concerning that there is a lack of focus on the process that will actually weave it together. Kilpatrick et. al. (2001) further clarified the importance of creating a balanced, or aligned, instructional plan by explaining that planning

> should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn, on one hand, solely by internalizing what a teacher or book says or, on the other hand, solely by inventing mathematics on their own (p. 11).

After an instructional plan has been created, it is important to reflect on the alignment and ensure that each component is supporting the pathway to the learning goal. Intentional reflection can also ensure that the pendulum swing between instructional strategies is not perpetuating a disconnected approach to developing conceptual understandings.

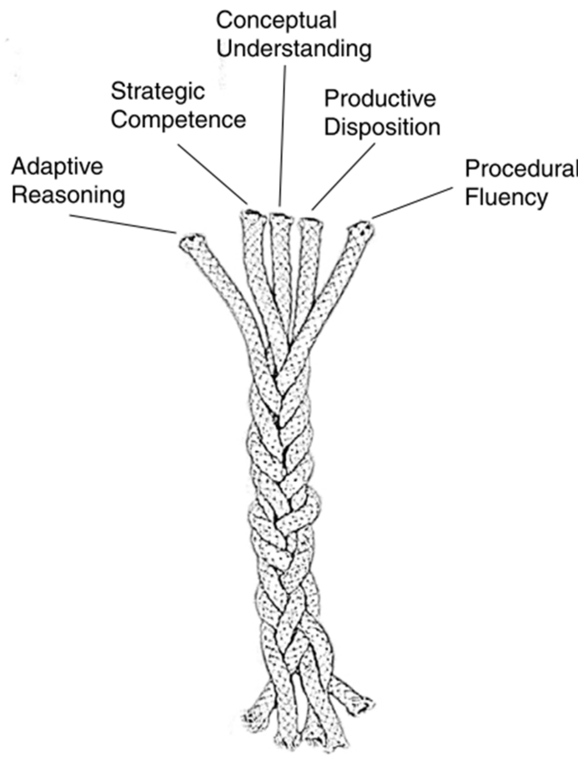

Figure 2. Intertwined strands of mathematical proficiency (Kilpatrick et. al., 2001, p. 5)

Even with an instructional plan that is thoughtfully aligned, instructional strategies need to focus on ensuring that students have developed connections before moving on. Instead of teachers crossing their fingers for these activities to result in students making connections, specific strategies that intentionally target this outcome will need to be planned for and monitored through assessment and feedback. This process, that Wilensky (1991) called concretion, is the missing link when planning for deep conceptual understanding and it will need to be included into the planning process to strengthen the alignment if concreteness is the goal.

CHAPTER 3: LITERATURE REVIEW

The CRA model

The Concrete-Representational-Abstract (CRA) framework has been one of the instructional models that I have encountered the most in my teaching practice. It is frequently referenced in numeracy frameworks, resources, and even in the program of studies as an effective model for developing procedural skills and conceptual learning (Milton et. al., 2018). The CRA is a linear model with three phases where students explore manipulatives (concrete), then pictorial models (representational), and finally use symbolic methods (abstract). Other variations of the CRA model exist but one that was of interest to me was the integrated CRA (CRA-I) approach. The CRA components were the same, however, rather than moving through the phases linearly, all three representations would be integrated simultaneously at the beginning of the learning experience and then gradually move away from concrete and pictorial representations as students showed mastery.

Even with an integrated approach, both of these models involve moving students away from concrete representations and towards using symbolic notations (Ebner et al., 2024). In contrast, the NCTM (2014) described student responses on higher-level demanding questions as responses that include the use of multiple representations (visuals, charts, verbal) that communicate their thinking. This suggests that, rather than explicitly or implicitly communicating that manipulatives and visuals are basic or surface level knowledge, it is with an understanding of the connectivity and uses of a variety of representations that demonstrates a students' concreteness of a concept.

The vast amount of research on the effectiveness of this model is a reason that CRA model is commonly used, especially with the research showing positive impact on student learning. From my teaching experience, simply having students engage in the three phases of the CRA model did not produce strong results for conceptual understanding for my students. For the purpose of my project (and to satisfying my teacher gut instincts) a deeper look into the results of some of these studies was needed. A meta-analysis of 30 studies, by Ebner et al. (2024) concluded that "CRA is an effective intervention for increasing math skills across varying operations, types of intervention, grades, and students with and without disabilities" (p. 38). Interestingly, the CRA-I intervention model was found to be less effective overall but there were some influencing factors mentioned such as it being a new model and with the higher cognitive concepts in those studies. While the CRA model has demonstrated positive outcomes for strengthening mathematical skills, there was no conclusive evidence on its impact on developing deep conceptual understandings. Clements (2000) argued that manipulatives and visuals do not automatically result in integrated-concrete knowledge. Instead, teachers need to intentionally design instruction, feedback, and reflective practices for the different representations for the student to see the connections.

The ACT model

The ACT (Acquire, Connect, Transfer) model focuses on developing a plan for students to go through a series of phases that promotes conceptual understanding and transfer of knowledge. In the first phase of the ACT model, acquire, students work towards building vocabulary, procedural skills, and foundational understandings of a concept. Teachers are able to cover their curriculum while in this stage but this is only the beginning of gaining a concrete understanding of a concept (Stern et al., 2021). In the connection phase, teachers design activities that guide students to link factual knowledge to broader concepts that help them make sense of the content in a variety of contexts. Stern et al. (2021) emphasize the importance of intentional planning so that “not all information is shared at once but is deliberately paced through a variety of contexts to engage students in deeper, more complex thinking” (p. 236). Further more, Stern et al. (2021) suggested that conceptual tools such as essential questions, graphic organizers, and concept maps, can be used to facilitate this process by supporting students with tools to help them analyze patterns and test out connections. The final phase, and the ultimate goal of the ACT model, is called transfer. In this phase, students are expected to apply their conceptual understanding to problems and contexts that are outside of their discipline. This model demonstrates that it is not a spontaneous occurrence that a student will be able to transfer their knowledge but rather it is an explicit and detailed plan by the teacher that guides the student through to this desired outcome.

To summarize, smaller concepts are taught at a foundational level. Then, connections are created between these concepts in similar and dissimilar contexts. Finally, the student will have such a concrete understanding of the larger concept that they are able to transfer it to a problem outside of its discipline. The model addresses the need for concretion in order to develop conceptual understanding but for the ACT model, the purpose is for students to transfer their understanding to topics outside of their discipline. With this as the goal, the pace of the ACT model progresses quickly, sometimes before concreteness has been created within the subject discipline. More time would be needed at the acquire stage to first allow students to build a concrete understanding of each concept that the ACT model would introduce during this stage. If the ACT model was followed as designed, the concepts that were introduced during the acquire stage would remain abstract knowledge for the students because each concept would be explored with one representation. If the goal is to achieve concreteness of mathematical concepts, then the

ACT model lacks the alignment that can slow down the pace and scale of the learning environment to achieve the goal.

After taking a critical look in the CRA model and the ACT model, two overarching issues stood out to me: achieving deep conceptual understanding, and creating a realistic pathway for the intended goal.

Issue 1: Not achieving deep conceptual understanding

The Understanding by Design (UbD) framework, developed by Wiggins and McTighe (2005), is a model for curriculum design that focuses on developing conceptual understanding. The backward design process encourages teachers to first identify the learning outcomes and success criteria before planning the instructional activities that they will use to achieve their desired goals. This design process challenges the more traditional approaches that start with activities or textbooks that are later connected to learning outcomes.

Wiggins and McTighe (2005) suggested that using a strong design will help teachers avoid the "twin sins" (p. 16) of a traditional design: (1) activity – oriented teaching where students might be engaged but it lacks alignment to the learning goal, and (2) coverage-focused instruction where students need to complete a certain number of tasks regardless of their understanding. These two approaches can lead to a disconnect in the learning or a surface level understanding of concepts. In contrast, UbD prioritizes the development of "enduring understandings" (Wiggins & McTighe, 2005, p. 342) or the concepts that become concrete knowledge for students to build onto in the future.

During the instructional planning stage, the facets of understanding can offer a lens through which teachers can design learning experiences that are aligned with the goals and assessments (Wiggins & McTighe. 2005). Instruction can be designed to move students towards deeper understanding rather than become derailed by either of the two sins. Activities and lessons can be designed to develop connections between the multiple representations and additional representations can be selected based on the chosen learning experiences.

A deep level of conceptual understanding would have, as described by Wilensky (1991), a strong level of concreteness for the individual. This type of understanding occurs when formal instruction interacts with other representations and experiences that help us make sense of new concepts. In the development of mathematical proficiency, the process of sense making (through justifying, explaining, and reflecting) is referred to as adaptive reasoning (Kilpatrick et. al., 2001). Imbedding adaptive reasoning into the concretion phase of the instructional design will support students in developing conceptual understanding. Wilensky (1991) suggested that a constructionist approach to instruction would be the appropriate model to implement during the concretion phase. This is not to say that a constructionist approach will guarantee concreteness, instead the approach involves components that allow students to explore concepts. Regardless of the approach that is chosen, it is essential that the instructional design is created with alignment to an intended goal so that teachers are not leaving conceptual understanding to chance (Wiggins & McTighe, 2005)

According to Hattie et al. (2017), the goal for teachers is to structure a learning environment that fosters students to move from surface level knowledge, engage in deep learning, and then transfer their knowledge to new situations. Hattie et al. (2017) described surface learning as "the initiation to new ideas. It begins with development of conceptual understanding, and then, at the right time, labels and procedures are explicitly introduced to give structure to concepts" (p. 29). An important aspect of surface learning is that it is not without meaningful substance, in fact it requires rich experiences that lay the foundation for their learning. This level of learning can include a variety of representations found in NCTM (2014) diagram of multiple representations, shown in figure 1; allowing students to view a concept from many perspectives and explore their own conjectures that they begin to make. Furthermore, activities can specifically target vocabulary, fluency, or metacognitive skills, depending on what goals have been articulated by the teacher. If teachers have not intentionally planned an exit to this stage of learning, there is a risk of students getting stuck in this stage and missing the opportunity to build onto their understanding (Hattie et al., 2017). If the definition of abstract knowledge (new knowledge taught in isolation) by Wilensky (1991) is used, alongside with the definition of surface learning (introduction of new learning) described by Hattie e. al. (2017), we can see a common idea of how learning can progress from the abstract to the concrete.

The deep learning stage is where students begin to recognize connections between the surface level conceptual understandings that they have built. A strong foundation is key before engaging in deep learning and a good indicator that students have this foundation is that they are able to identify similarities and differences between representations (Hattie et. al., 2017). Stern et al. (2021) suggested that the integration of graphic organizers, as a tool to help students reflect on the attributes and characteristics of each representation, can support this process. Organizational tools should be present during all levels of learning with opportunities to reflect and revise their thinking. A common theme in the strategies that Hattie et al. (2017) presented was on how essential mathematical discourse and metacognitive activities are during the deep learning stage. Students need the opportunities to talk through different representations and thoughts, allowing them to agree and disagree with their peers and even their own reasoning.

As stated earlier, concretion is the process where new knowledge and prior knowledge interact and connections are explored. Hattie et al. (2017) referred to this process as deep learning and emphasizes that this is a process and not an end goal. Additionally, "students move to deep learning when they plan, investigate, and elaborate on their conceptual understandings, and then begin to make generalizations" (Hattie et al., 2017, p. 31). The final stage of learning, according to Hattie et al. (2017), is called transfer learning and this is when students can take what they know and apply it to new situations. Initially I thought that this conflicted with my new understanding of how abstract learning occurred at the beginning of the learning process, specifically because the word abstract and generalizations are usually associated together. Conversely, if there is a concrete understanding of a concept, it will now be possible to use this knowledge in a general way with a new abstract idea that can be connected to it; implying that this is part of a cyclical process of moving from abstract knowledge to concrete while gathering new concepts along the way. Developing a cyclical learning environment can be a complex process and teachers need to be aware of what the success criteria looks like so that they can design appropriate tasks which requires specific attention to whether surface or deep learning is required (Hattie et al., 2017). The relationship between surface, deep, and transfer learning was visually shown by Hattie et al. (2017) with the spiral shown in figure 3 from which I took inspiration from for developing my own visual for my framework. I liked how it started from an initial point (prior knowledge) and then gradually expanded as it progressed through the learning process. Also, by including an arrow at the end of the cycle implied that even though students have reached transfer learning, the process is never considered finished.

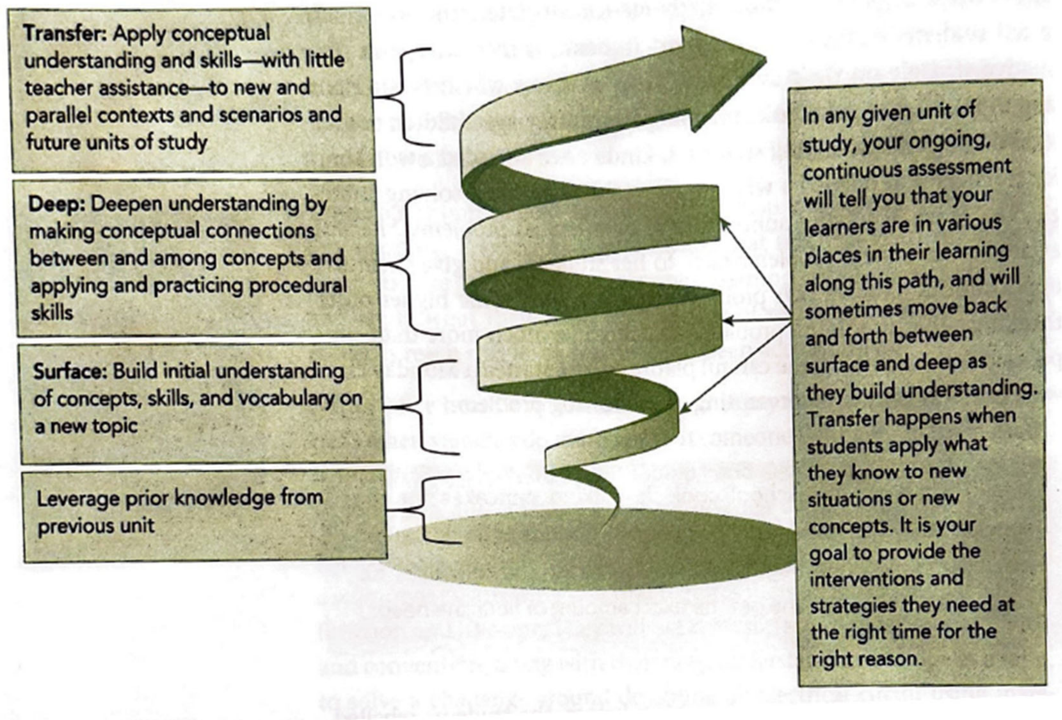

Figure 3. The relationship between surface, deep, and transfer learning (Hattie et al., 2017, p. 34)

Issue 2: Realistic pathway from beginning to intended goal

Wiggins and McTighe (2005) created a three-stage model, shown in Figure 4, to support teachers with using the UbD framework. In the first stage, teachers define the learning goals of what students should understand and know how to do. Hattie et al. (2017) referred to this process as establishing teacher clarity but regardless of the name, they agree on the importance of starting here in the design process. Stage 1 includes the provincial outcomes as well as the conceptual understandings that the teacher can craft into essential questions that will help guide the process. In the second stage, teachers identify the types of assessment that will provide them with the evidence of student understanding. Finally, in the third stage, lessons and activities are designed to ensure that there is alignment between the three stages (Wiggins & McTighe, 2005). The attention to alignment was an aspect that was missing in the CRA and ACT model, however, each model recognizes the importance of structuring the learning environment in a way that allows students to transfer their learning.

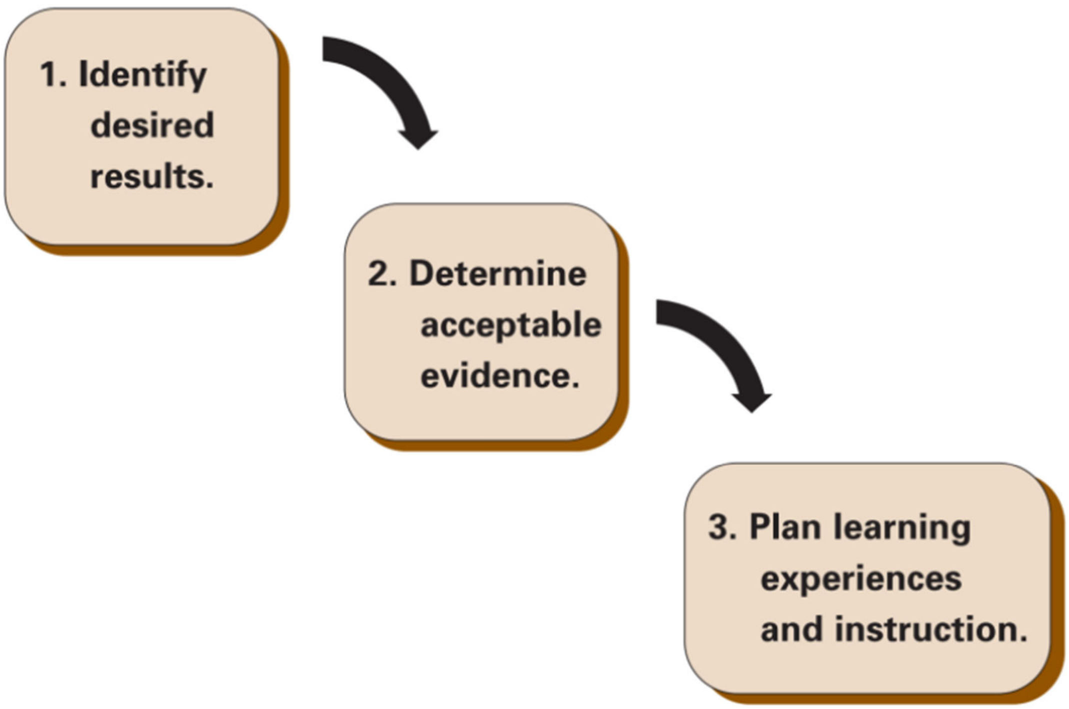

Figure 4. Stages of backwards design (Wiggins & McTighe, 2005, p. 18)

Determining acceptable evidence, the second stage in the UbD framework, ensures that our assessment practices are purposeful and designed to move the learning towards the intended goal. If time is taken to find common ideas on what evidence of learning is needed for the success criteria, then it can lead to higher levels of reliability in our classroom assessment (Wiggins & McTighe, 2005). Rather than assessing learning only at the end of instruction, UbD allows for teachers to see assessment as a tool that can make student thinking visible so that adjustments can be made to the instructional design.

Wiggins and McTighe (2005) described a six-sided view of the concept of understanding (explain, interpret, apply, perspective, empathize, self-knowledge) that should be reflected in our assessment and instructional design. A variety of question stems can be used in the essential questions to ensure that each facet of understanding is being developed and the success criteria can be aligned to the targeted facet. This is especially important when designing a learning experience with connecting multiple representations since each might require a different facet of understanding that our assessment should be aware of.

Effective mathematics assessment has progressed away from simply measuring correct answers and, when applied purposefully, it can reveal and move learning forward towards conceptual understanding (Hattie et al., 2017). When considering the approach where multiple representations are integrated into all stages of instruction, assessment strategies need to be designed to monitor the progression through the concretion process. To promote metacognition and internalize learning, a common theme in the research was the importance of involving students in the assessment process. Brown and Harris (2013) emphasized that self-assessment should be viewed as a reflection on the quality of one's own work rather than as a checklist. If students are aware of the learning goals and the success criteria, then this type of reflection can help students gauge where they are in their learning progression. Assessment tools, like journals and portfolios, can support this process, especially when students are in control of selecting their own evidence that they include. When students have regular opportunities to reflect, revise, and assess their work, they develop higher levels of metacognition and this is an important skill when engaging in classroom discourse around multiple representations (Belgrad, 2013). The format of the reflective practice is less important than the quality and purpose of the reflective prompt that the teacher creates. Even a short journal entry or exit slip given at the end of a lesson has the potential to give a teacher insights about the students' internal representations and the connections that they are making between concepts (Brown & Harris, 2013). Hattie et al. (2017) emphasized that it is not only about having strong assessment practice, more importantly, it is about intentionally aligning instructional practices to show teachers and students "how their conceptual understanding links to more efficient and flexible ways of thinking about the concept" (p. 31).

Success criteria, as defined by Hattie et al. (2017), are statements that are created to describe the evidence needed to meet the learning goal. If criteria are used during different stages in the instructional design, then the success criteria can support formative assessment practices that move student learning forward. Additionally, if learning goals along with the success criteria are communicated with students, it will have a positive impact on their metacognitive and reflective practices (Hattie et al., 2017). Conversations are a necessary component of formative assessment, especially if you are interested in metacognition. Black (2013) referred to this as interactive regulation, where the teacher uses student responses to make student learning visible. Assessment for and as learning ensures that students are constantly evaluating their learning and rethinking their understanding of the learning goal.

A central component of assessment for learning is the use of effective feedback, especially when it is mutually beneficial to both the teacher and the student (Heritage, 2013). A student response can give the teacher useful information regarding external representations and, in turn, effective feedback given to the student can clarify misconceptions. Effective feedback should be done in a timely manner and time will be needed for students to act on the feedback (Wiliam, 2013). Timely feedback can also inform the teacher about the effectiveness of certain representations for individual students. With feedback, "the essential point about teacher–student interaction as a source of evidence is that it enables teachers to have access to student thinking so that they can advance from the current state" (Heritage, 2013, p. 182). Wilensky (1991) highlighted the importance of shifting assessment practice away from an outcome as a stand alone objective and instead assess "the modes of interactions and models which a person uses to understand the object" (p. 5). For teachers to make this shift, assessment practices need to be purposefully designed to make student thinking visible.

If the goal is for students to develop deep conceptual understandings, then a framework is needed that supports the research around creating a learning environment that fosters it. To increase the success rate of reaching the goal, the creation of a planning tool that ensures alignment between all components is needed.

CHAPTER 4: PROJECT COMPONENTS

Introduction

I suggest that a smaller framework, focused on the process of concretion, be embedded in existing numeracy frameworks that are already adopted. My framework, which I call the Concretion-Based Learning Framework (CBLF), is a framework to support the intentional planning for concretion. Concretion, as defined by Wilensky (1991), is “the process of the new knowledge coming into relationship with itself and with prior knowledge” (p. 6) and this is the missing link when the goal is deep conceptual understanding. When considering the idea of concretion, I found it challenging to articulate how this learning process might progress. The process is not linear and it does not have a definitive beginning and end and this is why I found it challenging to organize. To support my process, I developed a flowchart (see appendix A) to help me visualize this complex process.

The CBLF is needed so that teachers can design a learning environment that allows each student to create a foundation to build onto, regardless of what prior knowledge they are bringing to the classroom. The focus is less about the grade level curriculum knowledge and more about the concretion process that students experience with the curriculum knowledge. The CBLF is designed to support the instructional method that a teacher might already be using so that they are less likely to resist change. The addition of the concretion stage is a small initiative to integrate into their practice that can make a big impact on student learning and retention.

The focus of the framework created for this project was to establish the strategies that can be used to encourage connections to be made that will promote the development of concreteness. If more opportunities of interacting with a concept are created, and the interactions with them are in a variety of representations, then the chances of it becoming concrete will increase (Wilensky, 1991). Retention of concrete knowledge is more successful since students connect it to their own prior knowledge and organize it with other concepts that they have established relationships with (Kilpatrick et al., 2001). The CBLF challenges the current thinking around learning progressions from a linear process to a cyclical process. It requires teachers to challenge their own thinking and consider a new definition of concrete and abstract and to do this the CBLF needs to be clear and concise. Rasolofoarison and Russell (2024) stated that "theory-communication visuals enhance clarity, memorability and communicability of ideas" (p. 1) so I felt that it was important to create a visual that represented the CBLF. The visual, shown in Figure 5, supports teachers with understanding the process of knowledge starting as abstract and gradually becoming more concrete to us as it interacts with other representations.

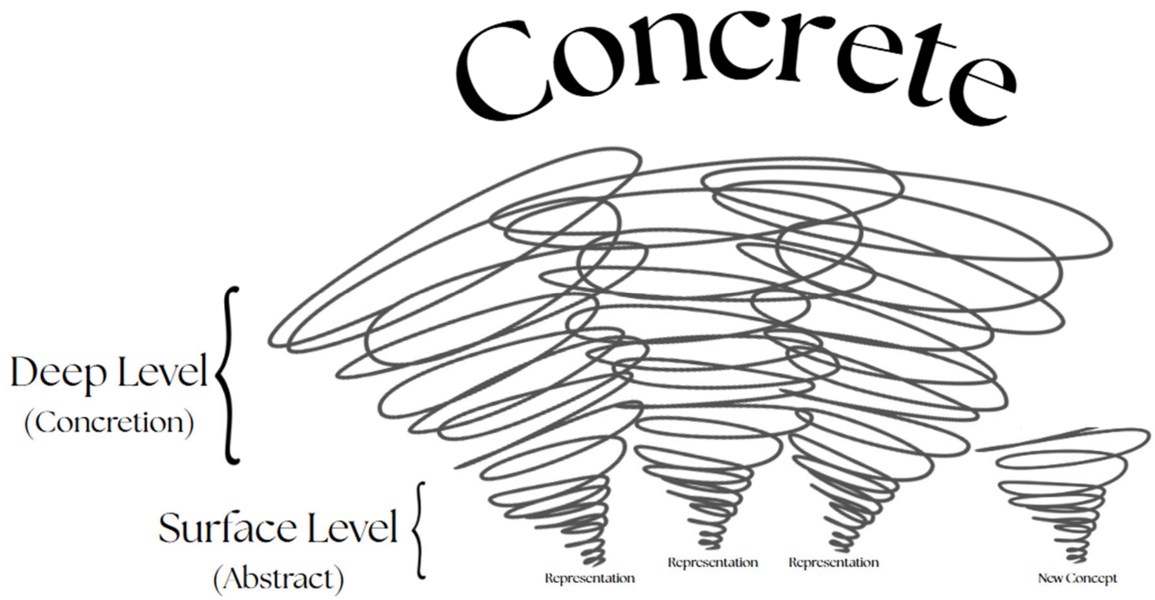

Figure 5. Visualization of the Concretion-Based Learning Framework

Teacher Clarity Tool

In order to achieve the rich discourse that facilitate the process of concretion, tasks and organizational tools need to be planned to facilitate our desired outcomes, and this is why developing teacher clarity is the starting point (Hattie et. al., 2017). The Teacher Clarity Tool (see appendix B) was developed because the CBLF needed to support teachers with the alignment between learning goals, assessment, and instruction. To achieve this alignment, I used a backwards design process inspired by Wiggins and McTighe's (2005) UbD framework. The Teacher Clarity Tool begins with the teacher establishing the curricular outcomes as well as developing an overarching essential question. Assessment practices are embedded throughout the planning process and then finally the instructional representations are purposefully chosen.

I was inspired by the planning tool that Hattie et. al. (2017) adapted from the SOLO model (Biggs & Collis, 1982). This planning tool guided the user to consider different stages in their learning and specifically communicate what the intentions and success criteria could look like for the student. Teachers would be able to map out the progression from surface learning to transfer learning. More importantly, there was a section where deep learning (or concretion) was articulated and the success criteria at that stage needed to be stated. From the UbD approach, the success criteria would be stated before the activities are selected and this ensured that our selection of activity is purposeful for our goal (Wiggins & McTighe, 2005). One missing piece that I felt needed to be added was a section for teachers to plan the type of activity that they would use at this concretion stage to foster the success criteria that they identified. In addition to the activities, I felt that it was important for the teacher to include their formative assessment practices that will provide the necessary feedback in regards to the success criteria.

The Role of Assessment

The CBLF is focused on creating an aligned instructional plan for student learning and assessment is an essential component because it will monitor how effectively the teacher, student, and concepts are interacting (Kilpatrick et. al., 2001). The intention is for it to also be a communication tool between teachers and students so that all participants understand where they are going and how they will get there. Wiggins and McTighe (2005) said that it is important for students to understand the learning goals and criteria so that students can effectively reflect on their own performance. Informal student reflection can be done during various stages in the framework so that all areas of the system can benefit. These opportunities will allow the student to paint a picture of who they are as a mathematics learner and it will also give the teacher a glimpse into the students' dispositions.

Observing internal representations is not always possible but inferences can be made through the interactions that students have in the learning environment (Goldin & Shteingold, 2001). The CBLF can be used as a tool to ensure that a variety of formative assessments are used so that students have received the feedback needed to progress towards mathematical proficiency. The goal with assessment in the CBLF is to make student thinking more visible so that appropriate representations and activities can be used to develop concreteness. The success criteria will help focus the lens of the formative assessment. At the surface level, our formative assessment will be focused on identifying misconceptions in skills or in concept understandings. Evidence of this might be collected from journals, whiteboard answers, or exit slips, just to name a few examples. The implementation of an interview process, where the teacher would use crafted questions that probe the student's thinking, can further enhance the feedback loop (Black, 2013).

During the concretion stage, formative assessment might take many forms but it is essential at this stage. Each activity that is selected should include some form of formative assessment that is embedded in the process. Student discourse is the main objective at this phase and peer feedback might be more appropriate for some activities. Effective peer-assessment will result in both students gaining insights of the learning during the activity (Topping, 2013). In my discussions with peers during assessment planning, adaptive reasoning is more commonly used on summative assessments to measure a student's depth of knowledge. If this is the case, it can be a missed opportunity for students to develop their own understanding of the relationships they encounter and this can have a negative impact on their dispositions towards mathematics. Teachers can be guided to include adaptive reasoning before or during instruction and they can plan for optimal uses that will support the development of mathematical proficiency through concretion (Kilpatrick et. al., 2001). Students should also be given opportunities to self-reflect at this stage, along with feedback from other sources, so that they have a clear understanding of their learning progression.

In the final stage, concrete knowledge, teachers will design tasks for an assessment of learning. Students should be able to demonstrate an understanding of the success criteria as well as the overarching essential question. This does not mean that the learning experience is over. The intention is to answer "Is the concept concrete?" If so, this is now a tool that can be used to help students acquire new abstract knowledge. If it is not concrete yet, perhaps there is a different representation that can support students with the concretion process.

Essential Questions

Essential questions are a component in the UbD framework that are designed to spark inquiry, reveal assumptions, and guide students to make connections between concepts. Furthermore, Wiggins and McTighe (2005) described six characteristics of an essential question, one of which speaks to their role in creating meaningful connections with prior knowledge and individual experiences. The UbD framework encourages teachers to build in activities that allow students to revisit and revise their thinking of the essential questions so that they can deepen their understanding of abstract concepts (Wiggins & McTighe, 2005). A framework, like the UbD, can be used when considering the design of connecting multiple representations of a concept and essential questions can guide students during a concretion phase. Questions like "How can we represent the same relationship in different ways?" or "What does a model reveal that a number sentence might hide?" can guide students to use metacognitive thinking in their reflections. Essential questions can also help students transfer their understandings across disciplines and grade levels which can have a positive impact on their retention of knowledge (Wiggins & McTighe, 2005). Furthermore, Stern et al. (2021) suggest that the use of "conceptual relational question stems are the most useful way to direct student attention to the connections between and among concepts" (p. 236). With intentional planning, teachers can craft rich essential questions that guide students towards the connections that are articulated in the learning goals and success criteria. Furthermore, essential questions can be categorized as topical (unit or topic specific) or overarching (general or big ideas). Wiggins and McTighe (2005) noted that it can sometimes require the exploration of multiple topical questions before students are ready to attempt overarching questions. In the CBLF, an overarching essential question is designed to address the concept that students will develop a concrete understanding of. Topical essential questions are connected to each representation that students explore and students will revisit these questions during the concretion process to strengthen their responses as well as develop connections between them. The purpose of essential questions in the CBLF is to support the teacher with focusing their plan to avoid the two-sins as well as ensure a strong alignment between goals, instruction, and assessment.

Concretion

The Teacher Clarity Tool is designed to draw attention to the activities that are chosen and evaluate if they link back to the success criteria that the teacher is wanting the students to achieve. Activities should also be designed to give students information that can help them answer essential questions. Using an organizational tool, such as the CLICK Thinking Tool created by Stern et al. (2021), can support students with collecting evidence that will help them answer essential questions throughout the learning experience.

Hattie et al. (2017) described the tasks that support concretion as ones that require students to find connections and relationships with the concepts from their surface learning. The design for these tasks should be focused on open-ended questions or questions that have multiple answers or solution pathways. This does not mean that the teacher should be crafting an exhaustive number of open-ended questions and rich tasks for students to answer. In fact, creating activities that encourage students to ask each other questions and exchange ideas is much more productive for fostering concretion (Hattie et al., 2017). When using discourse, either as a whole class or with small groups, norms should be established and modelled by the teacher to support productive conversations that are focused on the success criteria (Hattie et al., 2017). It is important to guide students in this process so that their communication acts as a form of peer and self feedback with regards to their current understanding. Developing general question stems to facilitate the type of language that you are hoping to hear can help guide students in the discourse.

Student responses can reveal multiple strategies as well as misconceptions but these do not need to be hidden from the rest of the students. Displaying student solutions or errors, possibly made on a previous exit slip, can be used as a small group activity where students can find similarities, differences, and errors through discussions. Hattie et al. (2017) suggested a Think-Pair-Share structure to allow students time to consider their own ideas and connections before collaborating with peers. Furthermore, once students are put into small groups, they can use a conversation roundtable method to help students focus on what each person is bringing to the conversation (Hattie et al., 2017).

Multiple Representations

Exposing students to multiple representations should not be only during surface learning. These representations should be purposefully integrated into all aspects of the learning environment and it is the success criteria that dictates the purpose for its use. At the surface level, a variety of representations (seen on Figure 1) can be presented separately to introduce a concept with the essential question and success criteria guiding each experience. Unlike the CRA model, the CBLF views the use of multiple representations as a cyclical process. Since the research on the CRA model has shown a positive impact on developing procedural skills, using physical, visual, and symbolic representations could be a good starting point for introducing concepts if they make sense to do so. Students will interact with these representations again during the concretion stage as they engage in meaningful discourse about the connections that they make with the representations. Students can engage in discussions around the similarities and differences between representations that can begin to reveal deep connections (NCTM, 2014). Finally, in the concrete knowledge stage, students will begin to view different representations as tools that can be used when they are needed. Furthermore, students will be able to select an appropriate representation to help them solve a variety of tasks that demonstrate the communicated success criteria.

Limitations of the CBLF

The CBLF aims to promote the development of concrete knowledge while ensuring that the learning process is strongly aligned. Through the development of this framework, limitations with differentiation and intervention were identified. The Teacher Clarity Tool does not have a section for the teacher to address specific differentiation strategies. However, results from the Ebner et al. (2024) meta-analysis reported that the CRA model had a significant positive impact students with learning disabilities. Furthermore, NCTM (2014) referenced Fuson and Murata (2007) suggesting that "math drawings and other visual supports are of particular importance for English language learners, learners with special needs, or struggling learners, because they allow more students to participate meaningfully in the mathematical discourse in the classroom" (p. 25). This suggests that using multiple representations in the learning environment is a positive differentiation strategy in a mathematics class. Teachers will need to pay close attention to assessment data so that they can address student needs as soon as they are visible.

When the assessment data reveals an area of struggle for a student or a group of students, an intervention practice might be needed. The CBLF does not address interventions specifically but in my flow chart (see appendix A) I do recognize a path if additional instruction is required. Rather than re-teaching previous representations, the teacher will need to consider if other representations are needed to support the concretion process. These limitations, in theory, of the CBLF are areas that might need to be addressed in the future if they persist in instructional practices.

CHAPTER 5: CONCLUSION

This CBLF addresses a gap in teacher resources, the concretion process, with the purpose of supporting teachers with a plan to achieve learning goals. Rather than providing teachers with more strategies to promote the individual strands that are necessary for successful mathematical instruction, it will instead focus on what to do along with those strategies to create the connections that students are missing. In larger schools, teachers might only have one semester or one year to work with a group of learners. Teachers want our students to not only learn the curriculum but also take with them the prior knowledge that they can use to engage with abstract concepts and feel confident that they will make it concrete.

The intent of this project is to present an alternative perspective of how concrete and abstract knowledge are viewed in regards to instruction in mathematics. When teachers are presented with alternate perspectives, opportunities for meaningful discourse will naturally arise. Furthermore, teachers will begin to critically evaluate the resources that they integrate into their instructional practice. In the future, I hope to see resources and documents clarify the perspectives and intention of concrete and abstract concepts. The CBLF perspective challenges the CRA approach and reimagines the purpose of multiple representations in the learning progression. With a concentration on connecting multiple representations to the prior knowledge of the students, an increase in mathematical proficiency and retention should be evident. Additionally, students will gain a positive view of multiple representations and with how they can build a toolbox of strategies that are useful in a variety of contexts.

The CBLF offers a purposeful instructional process that will promote conceptual understandings and it defines how assessment plays an important role in achieving this goal. To further the CBLF, a defined construct is needed so that the assessment practices can evaluated on their reliability, validity, and fairness. If the CBLF is focused on building concrete knowledge through relationships, the construct will need to focus on the communication of the connections that students explore and build. Developing a strong construct will allow future work to be done with the CBLF and to evaluate the effectiveness of the framework on the development of conceptual understandings. In my personal teaching practice, the CBLF will be a tool that I will use to enhance the learning for my students. This continued use will allow me to observe the effects of the framework as well as reflect on possible changes that can improve the CBLF. I plan to share the CBLF with my school division and hopefully integrate it into the numeracy framework that is in development. Along with sharing the document, opportunities for leading professional development on the CBLF, as well as through mentorship, will be possible. The overall goal for the CBLF is not to replace existing strategies, but rather to enhance them by drawing attention to facilitating a concretion phase. As I continue to work with and improve this framework, I hope that it will promote deeper conceptual understanding, stronger retention, and meaningful mathematical experiences for both students and teachers.

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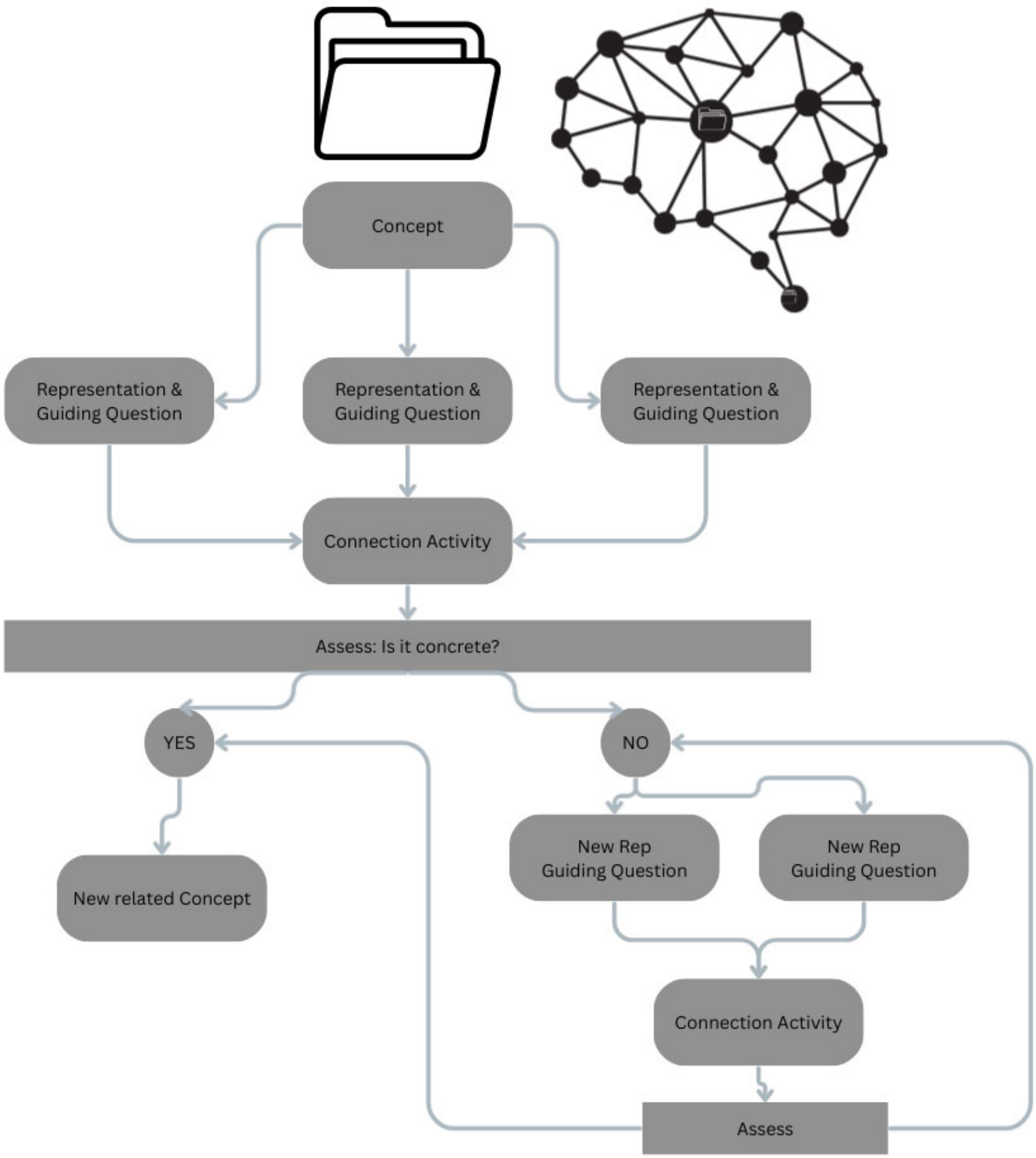
APPENDIX A: FLOW CHART OF MY THINKING



APPENDIX B: TEACHER CLARITY TOOL

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<tr><td colspan="7">Overarching Essential Question:</td></tr>
<tr><th colspan="7">Evidence of Concrete Knowledge</th></tr>
<tr><td>Success Criteria</td><td colspan="6"></td></tr>
<tr><td>Assessment(s)</td><td colspan="6"></td></tr>
<tr><th colspan="7">Deep Level: Concretion</th></tr>
<tr><td>Success Criteria</td><td colspan="6"></td></tr>
<tr><td></td><td colspan="3">Formative Assessment:</td><td colspan="3">Formative Assessment:</td></tr>
<tr><td></td><td colspan="3">Activity 1:</td><td colspan="3">Activity 2:</td></tr>
<tr><th colspan="7">Surface Level: Learning Intentions</th></tr>
<tr><td>Success Criteria</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Topical Essential Questions</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td></td><td colspan="2">Representation 1:</td><td colspan="2">Representation 2:</td><td colspan="2">Representation 3:</td></tr>
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