

Quantum aether and an invariant Planck scale

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We argue that a quantum aether is consistent with the principle of relativity and can provide an economical way of having an invariant quantum gravity or Planck scale. We also show that it may change the effective scale at which quantum gravity effects may be observable.

Predictions of certain theories of quantum gravity, such as the discreteness of space, and requirements of certain others, such as the fundamental building blocks being strings, are expected to be revealed near the Planck scale (or the related string scale), characterized by the following length, mass and time ¹: $\ell_{Pl} = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35} m$, $m_{Pl} = \sqrt{\frac{\hbar c}{G}} \approx 10^{-8} kg$, $t_{Pl} = \sqrt{\frac{G\hbar}{c^5}} \approx 10^{-44} s$. But since the above quantities are not Lorentz invariant, the scale of fundamental structures or building blocks would appear to be different even in different inertial frames in flat spacetimes and in vacuum. Thus one is faced with the following dilemma, at least classically, that either

(i) Lorentz transformations are approximate and a suitable modification near the Planck scale will make these frame independent, or

(ii) Planck quantities have the above values in a special frame.

The first possibility has been explored in the so-called Doubly Special Relativity (DSR) Theories, where by realizing a particular non-linear version of the Lorentz group, it was shown that both c and m_{Pl} can be made frame independent [2] ². However, incorporating the new kinematics in a consistent dynamical theory has not been possible so far, and problems of constructing macroscopic bodies from elementary particles satisfying energy momentum conservation laws have proven difficult. The second possibility on the other hand brings back the notion of luminiferous aether, which as we know is fraught with problems, including contradictions with known experiments.

However, an early interesting proposal by Dirac in [3, 4] suggests that there could yet be a third possibility, that of a quantum mechanical aether which does not violate

Lorentz symmetry. As we argue later, this in fact can also lend a precise meaning to an invariant Planck scale. Such an aether would presumably be light and made up of tiny constituents, (as otherwise we would have detected its obvious presence) and hence subject to the laws of quantum mechanics and the uncertainty principle. Thus it is natural to expect that the ground state of such an aether (at any spacetime point) would be a uniform superposition of all eigenstates of four-velocity v^μ within the light cone at that point. This is analogous (in three dimensions) to the $l = 0$, or ground state of the hydrogen atom (without spin), which being a uniform superposition of all possible position eigenstates, is itself a *spherically symmetric* state. Successive position measurements on such identically prepared states would yield a probability distribution which is also spherically symmetric. Similarly, the aforementioned state of aether, not associated with any specific velocity, would be *Lorentz symmetric*! More concretely, if p^μ is the four-momentum of the aether at any point (which equal mv^μ if the constituents have a non-zero rest mass m), then the wavefunction of aether at a point can be written as as

$$|\Psi\rangle = N \int d\Omega_p |\Psi_p\rangle \quad (1)$$

where the integral is performed over a suitable Lorentz invariant measure $d\Omega_p$ over all momenta within the light cone, $|\Psi_p\rangle$ represents a state of the aether with definite momentum p^μ , and N is the normalization constant. This can be thought of as a four-dimensional generalization of the s -state, and demonstrates (just like the hydrogen atom in a spherically symmetric state) how in general quantum mechanics can produce an enhanced symmetry in certain states, symmetry which is not present in the system's classical description. This is a consequence of the superposition and uncertainty principles. Note that as in the case of the Hydrogen atom, and singlet states in various scenarios in particle physics, the ground state of the aether is expected to be the perfectly symmetric state (1), and if perturbed by small amounts, will return to this state rapidly. Such small fluctuations can have observational consequences however, and provide a

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¹ These were first defined by Planck himself in [1].

² In the rest of the paper, we will use $\hbar = 1 = c$ units.

mechanism of its detection³. Now the velocity of aether at any point is completely indeterminate *per se*, and if a measurement of its velocity is made, one of the infinite possibilities will be picked out at random. This can be used to locally (in space and time) define a rest-frame in which the aether is at rest. We go one step further and propose that if such a measurement is made at a spacetime point, then the aforementioned Planck scales are in *that frame*. This is similar in spirit to the potential re-appearance of absolute time and absolute simultaneity in the presence of a quantum aether, albeit in a small neighborhood and quasi-instantaneously, in accordance with the uncertainty principle, and described statistically [4]. Furthermore, just as observing an electron in a hydrogen atom initially in its *s*-state at a particular point in space does not violate the spherical symmetry of the Hamiltonian, here too the random appearance of a preferred frame does not violate the Lorentz symmetry observed in nature. And as mentioned before, it would rapidly return to its quantum ground state. And even if no such observation is made for a long time, one is still certain that if and when it is made, it will result in a short-lived preferred frame. Consequently, the existence of a fundamental scale (or more than one scale) is not incompatible with it. Furthermore, the result would be the same when viewed from any inertial frame! Thus, as observed in [4], it may constitute a *perfect vacuum*. It is interesting to note that quantum mechanics plays a crucial role in the definition of the Planck scale (which contains \hbar) as well as in the above proposed resolution. We further note that the fundamental postulate of relativity, and in particular the relativistic velocity addition formula ensures that whatever random velocity of aether is measured at any spacetime point (say by a mechanical experiment), this will not affect the velocity of light in vacuum, and hence will have no effect on interference fringes in Michelson-Morley type of experiments (which aimed to measure aether velocity)⁴. In other words, there is no contradiction with known experiments. Finally, we would like to point out that decoherence may play a very important role in the collapse of wavefunction (1), to a velocity (or momentum) eigenstate during a ‘measurement’. This, as pointed out in [6], may be the outcome of an interaction of the aether with its en-

vironment, and also of certain superselection rules, if the wavefunction has additional symmetries. However, one would require more information, or need to make further assumptions about the aether and its wavefunction to make a more detailed study. We leave this to a future publication.

What about the normalizability of the aether wavefunction? As noted in [3] the wavefunction described by (1) is ordinarily not normalizable. It can be seen for example by substituting for $d\Omega_p$ a relativistically invariant measure such as $d^3p/(2\pi)^3 2E_p$ and evaluating

$$\langle \Psi | \Psi \rangle = \frac{|N|^2}{\pi} \int_0^\infty \frac{p^2 dp}{\sqrt{p^2 + m^2}} = |N|^2 \times \infty, \quad (2)$$

which diverges for any non-zero N (where we have used $\langle \Psi_{p'} | \Psi_p \rangle = 2E_p (2\pi)^3 \delta(p' - p)$ [7], and $p = |\vec{p}|$). Dirac argued that since the state (1) was after all an idealization, similar to plane waves (which too are not normalizable), it can be approached indefinitely close, but can never be attained in nature exactly. However, as being argued here, if there is an invariant scale, one may also consider it as an upper cut-off Λ (with or without an invariant lower cut-off), and then for the choice $N = \sqrt{a\pi}/\Lambda$, and $N = (3m\pi/\Lambda^3)^{1/2}$ for $\Lambda \gtrsim m$ and $\Lambda \ll m$ respectively (again, m being the mass of the aether quanta, or that of its fundamental constituents, if thought of as a fluid) the integral in (2), which go as Λ^2 and Λ^3/m in these limits, is finite⁵. And with a normalizable wavefunction, the aether state of perfect vacuum could well be attainable in nature. Note that within the current interpretation, Λ being Lorentz invariant, the integral can be regarded as Lorentz invariant as well. Note also that this cut-off could be the Planck scale. But another intermediate scale can exist by the same token [8]. In any case, such a fundamental scale is expected to arise from a correct theory of quantum gravity or of yet unknown physics beyond the electroweak energy scale. This does not seem to be the case for DSR theories.

Finally, we ask the question: since the aether’s motion is random at any point, what would be the effective Planck scale at which quantum gravity effects would be expected? First we note, following Lorentz length contraction, time dilation etc, that the numbers presented at the beginning provide strict lower bounds (for Planck mass and Planck time) and an upper bound (for Planck length). One can then compute the average of the ‘observed’ Planck mass, namely $M_{Pl} = \gamma m_{Pl}$ with

³ The above notion of aether may in fact be compatible with, and a concrete realization of Einstein’s efforts to re-introduce it, in which context he mentions [5] “... the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether.”.

⁴ Consider for example, an observer measures the aether velocity at any spacetime point to be \vec{v} , and let the velocity of light measured by an observer who is at rest with respect to the aether at that point be \vec{c} , with $|\vec{c}| = c = 2.9979 \times 10^8$ m/s. Then by the relativistic addition formula, the velocity of light as seen by the first observer is $\vec{c}' = \frac{\vec{v} + \vec{c} \cdot \vec{v} \hat{v} + \sqrt{1 - v^2/c^2} (\vec{c} - \vec{c} \cdot \vec{v} \hat{v})}{1 + \vec{c} \cdot \vec{v}/c^2}$, from which it is easily seen that $|\vec{c}'| = c$.

⁵ $a = 2$ for $\Lambda \gg m$ and $a = \left[1/\sqrt{2} - 1/2 \ln(1 + \sqrt{2})\right]^{-1} \approx 3.7$ for $\Lambda \approx m$.

$\gamma = (1 - v^2/c^2)^{-1/2} = \sqrt{p^2 + m^2}/m$, as

$$\begin{aligned} \langle M_{Pl} \rangle &= \langle \Psi | (m_{Pl} \gamma) | \Psi \rangle = \frac{|N|^2}{\pi} m_{Pl} \int_0^\Lambda \frac{p^2 \gamma dp}{\sqrt{p^2 + m^2}} \quad (3) \\ &= \frac{a m_{Pl} \Lambda}{3 m}, \quad \Lambda \gtrsim m \\ &= m_{Pl}, \quad \Lambda \ll m \end{aligned}$$

which is finite. For the first case, $\Lambda \gtrsim m$, we have at least two possibilities

(i) $\Lambda \gg m$, i.e. an aether which is very light compared to the cutoff scale, the latter being for example, the Planck scale. Then $\langle M_{Pl} \rangle$ would be huge, making it (as well as quantum gravity effects) impossible to observe, and practically irrelevant.

(ii) $\Lambda \approx m$, with Λ being much less than the Planck scale, as otherwise the aether would be heavy. Then $\langle M_{Pl} \rangle \approx m_{Pl}$ and the scale of quantum gravity effects remain unchanged.

For the second case, $\Lambda \ll m$, too, the quantum gravity scale remains unchanged. Although possibility (i) above may appear more natural, in the end, it is for experiments to decide whether one of the above is true. Experiments may also help to obtain bounds on the cut-off or the aether mass, using (3).

To conclude, we have shown here that a quantum mechanical version of an all permeating aether is consistent with the principle of relativity, and may provide a mechanism for having an invariant quantum gravitational scale. Further, depending on the relative magnitudes of the mass of its quanta and this cut-off, the effective Planck scale may acquire very large or small values. There is no need to change the theory of relativity. The simplicity

⁶ Again, according to Einstein [5] “*To deny the ether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view.*” and “*According to the general theory of relativity space without ether is unthinkable;...*”.

of its construction ensures that problems associated with the composition of macroscopic bodies or with additivities of energy and momentum etc do not arise. Neither does it present any obvious problem to existing dynamical theories such as gauge theories and general relativity. Thus it may represent the physical vacuum⁶. A consistent dynamical theory of aether itself would have to be formulated however, which would shed light on the nature of its constituents, and the fate of observable preferred frames in the long run. It is tempting to speculate that this version of aether may have some bearing on the abundance of *Dark Matter* and *Dark Energy* in our universe. One would of course have to compare with their quantitative estimates and also reconcile with the fact that *Dark Energy* is associated with a minuscule vacuum energy density. Such a cosmological connection with an aether which is also treated as a superfluid was proposed in [9]. Aether in the cosmological context has also been recently studied in [10]. Naturally, one must also check the consistency of our current hypothesis with all other relevant physical theories, and more importantly, extract predictions which can potentially be checked in the laboratory or in astrophysical observations. This would be a true test of its existence.

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