Randall-Sundrum with Kalb-Ramond field: return of the hierarchy problem?

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Abstract. We show that when the antisymmetric Kalb-Ramond field is included in the Randall-Sundrum scenario, although the hierarchy problem can be solved, it requires an extreme fine tuning of the Kalb-Ramond field (about 1 part in 10^{62}). We interpret this as the return of the problem in disguise. Further, we show that the Kalb-Ramond field induces a small negative cosmological constant on the visible brane.

1. Introduction

The difference of about sixteen orders of magnitude between the electroweak scale ($\approx 1\ TeV$) and the Planck scale ($\approx 10^{16}\ TeV$), is known as the hierarchy problem. While theoretically there seems to be nothing which can rule out such a difference, it certainly seems a strange thing to be. Figure 1 demonstrates this fact.

Of the many attempts to explain the hierarchy problem, two most recent ones deserve special attention, the proposals themselves being quite simple in themselves. Collectively known as the *Brane World Scenarios*, they assume the existence of one or more spatial dimensions in our universe, in addition to the four spacetimes that we observe. In other words, if there are a total
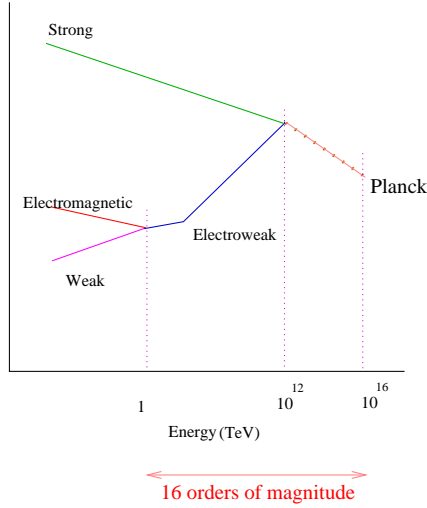
Figure 1. Difference between the electroweak and Planck scale

of d spacetime dimensions, it can be decomposed as: $d = 4(Observed) + (d-4)(Unobserved)$, as shown diagrammatically below:
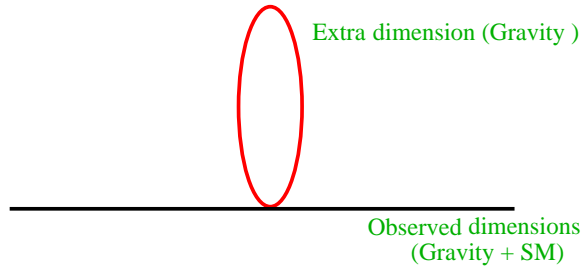

Figure 2. Horizontal line = observed universe. Circle = unobserved universe.

Here it is assumed that both Standard Model (SM) and gravity are present in the observable part of the universe ('the brane'), while gravity alone is present in the bulk.

1.1. ADD scanerio

The first brane world scenario, known as the Arkani-Hamed-Dimopoulos-Dvali (ADD) model, requires at least 2 (possibly more) extra dimensions [1]. One may start with the Einstein action in d-spacetime dimensions, $\mathcal{R}_d$ and G_d being the d-dimensional curvature scalar and gravitational constant respectively :

$$S = \frac{c^3}{16\pi G_d} \int d^d x \sqrt{-g_d} \; \mathcal{R}_d \tag{1}$$

and substitute in it a d-dimensional metric ansatz of the form:

$$ds_d{}^2 = ds_4^2 - dy_I dy^I, \tag{2}$$

where the two terms represent the observed 4-dimensional and the hidden $(d-4)$-dimensional parts respectively (index $I = 1, \ldots, d-4$). Integrating over the unobserved dimensions, one obtains the *effective* 4-dimensional action:

$$S = \frac{c^3 V_{d-4}}{16\pi G_d} \int d^4x \sqrt{-g_4} \; \mathcal{R}_4 \tag{3}$$

$$\equiv \frac{c^3}{16\pi G_4} \int d^4x \sqrt{-g_4} \; \mathcal{R}_4 \tag{4}$$

where the 4-dimensional gravitational constant is given by:

$$G_4 = \frac{G_d}{V_{d-4}} \; . \tag{5}$$

Correspondingly, the d-dimensional and 4-dimensional Planck masses are related as:

$$M_{Pl(d)}^{d-2} = \frac{\hbar^{d-3}}{c^{d-5} G_d} = \frac{\hbar^{d-3}}{c^{d-5} V_{d-4} G_4} = \left(\frac{\hbar}{cL}\right)^{d-4} M_{Pl(4)}^2 \; , \tag{6}$$

where we have used: $V_{d-4} = L^{d-4}$. Now, the four dimensional (observed) Planck scale is:

$$M_{Pl(4)} c^2 = 10^{19} GeV = 10^{16} TeV \; . \tag{7}$$

Therefore, from (6), the following possibilities (and many more) result:
(i) $d = 6$, $L = 100 \; \mu m \quad \Rightarrow \quad M_{Pl(6)} c^2 = 1 \; TeV$
(ii) $d = 10$, $L = 1 \; Fermi \quad \Rightarrow \quad M_{Pl(10)} c^2 = 1 \; TeV$.
In other words, the 6 or 10-dimensional Planck mass can be as low as a TeV. Moreover, these cannot be ruled, out since inverse-square law of gravity has been tested to 0.1 mm so far [1]. As a result, the hierarchy problem is solved in higher dimensions, where, more precisely, the problem ceases to exist !

1.2. RS scenario
Next, we come to the second or the Randall-Sundrum (RS) brane world scenario, where one again starts with the action (1), but instead of the metric ansatz (2), one uses the following 'warped', or non-factorisable metric [2]:

$$ds_d^2 = e^{-A(y)} ds_4^2 - dy_I dy^I \; . \tag{8}$$

$\exp(-A)$ is known as the warp factor. Now, the effective 4-dimensional action, the gravitational constant and the relation between Planck masses read as:

$$S = \frac{\left[c^3 \int d^{d-4} y \sqrt{g(y)} e^{-A} \right]}{16\pi G_d} \int d^4x \sqrt{-g_d} \; \mathcal{R}_d \tag{9}$$

$$\equiv \frac{c^3}{16\pi G_4} \int d^4x \sqrt{-g_4} \; \mathcal{R}_4 \tag{10}$$

$$G_4 = G_d \left[\int d^{d-4} y \sqrt{g(y)} e^{-A} \right]^{-1} \tag{11}$$

$$M_{Pl(d)}^{d-2} = \left(\frac{\hbar}{c}\right)^{d-4} \frac{M_{Pl(4)}^2}{\int d^{d-4} y \sqrt{g(y)} e^{-A}} \approx \left(\frac{\hbar}{c}\right)^{d-4} k^{d-4} \; M_{Pl(4)}^2 \; . \tag{12}$$

[1] for d=5, $M_{Pl(5)} c^2 = 1 \; TeV \Rightarrow L \gg 1 \; mm$. Therefore it is ruled out.

In the above, a warp factor of the form $A(y) = k\sqrt{y_I y^I}$ has been assumed, which is a solution of the Einstein equations in the RS scenario, as we shall see shortly. Now, if $d = 5$, and our universe ('visible brane') is located at $y = y_0$, the conformal factor of the metric (8) is of the form $\emptyset^2 = e^{-A(y_0)}$. Considering any matter action (such as that for the Higgs field) with a mass parameter m_0, and integrating over the extra dimension with the metric (8) and the above conformal factor results in the following *physical mass*, which is exponentially suppressed:

$$m_H = e^{-A(y_0)} m_0 \ . \tag{13}$$

Thus if $m_0 \, c^2 = 10^{16} TeV$ and $A \approx 12$, then $m_H c^2 = 1 \, TeV$. In other words, a small conformal factor explains hierarchy. The situation is depicted in the figure below, where $y \equiv r\phi$, r being a characteristic length scale, and $y = y_0$ corresponds to $\phi = \pi$. The warp factor on the *Hidden Brane* is unity.
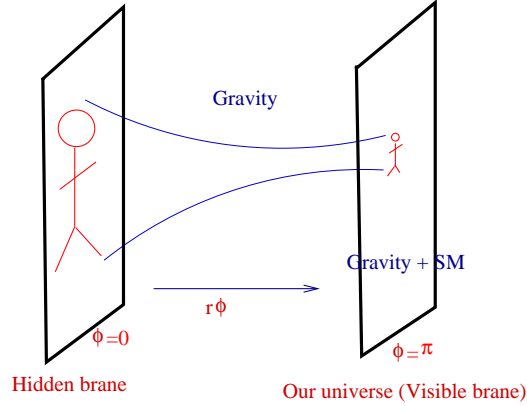

Figure 3. Hidden and visible branes in the RS scenario

Now, we explicitly compute the warp factor in Eq.(8), which is first written in the following form for $d = 5$:

$$ds^2 = e^{-A(\phi)} \ \eta_{\mu\nu} dx^\mu dx^\nu - r^2 d\phi^2 \ , \tag{14}$$

where the solution for A follows from extremising the following action ($M_{Pl(5)} \equiv M$):

$$S \ = \ S_{Gravity} + S_{vis} + S_{hid} \tag{15}$$

$$\text{where,} \quad S_{Gravity} \ = \ \int d^4x \ r \ d\phi \sqrt{-G} \left[2M^3 R + \Lambda \right] \tag{16}$$

$$S_{vis} \ = \ \int d^4x \sqrt{-g_{vis}} \left[L_{vis} - V_{vis} \right] \tag{17}$$

$$S_{hid} \ = \ \int d^4x \sqrt{-g_{hid}} \left[L_{hid} - V_{hid} \right] \ , \tag{18}$$

Λ being the 5-dimensional cosmological constant, and c and G have been set to unity. The corresponding equations of motion are ($' = d/d\phi$):

$$\frac{3}{2} A'^2 \ = \ -\frac{\Lambda}{4M^3} \ r^2 \ , \tag{19}$$

which has the following solution ($k = \frac{-\Lambda}{24M^3}$)

$$A = 2kr\phi \tag{20}$$

$$V_{hid} = -V_{vis} = 24M^3k \ . \tag{21}$$

Thus from (13), we see that the following suppression of mass occurs:

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = e^{-2kr\pi} \approx (10^{-16})^2 \ , \tag{22}$$

from which, it follows that:

$$kr = \frac{16}{\pi}\ln(10) = 11.6279\ldots \tag{23}$$

We will call this the 'RS value' of the warp factor.

At this point, it is natural to ask as to what happens to the above value if there are other fiels in the bulk. In particular, one can consider the massless NS-NS fields in string theory, which can be written as $\alpha^\mu_{-1}\ \tilde{\alpha}^\nu_{-1}|0;k\rangle$ ($\alpha^\mu_{-1}, \tilde{\alpha}^\mu_{-1}$ are annihiliation/creation operators and $|0;k\rangle$ is the string ground state), whose symmetric, anti-symmetric and trace parts are interpreted as the graviton ($g_{\mu\nu}$), dilaton (ϕ) and the Kalb-Ramond ($B_{\mu\nu}$) fields respectively. The last of these, which gives rise to a 3-form field strength $H_{MNL} = \partial_{[M}B_{NL]}$ will be included here and its effect on the RS scenario studied.

2. RS scenario with Kalb-Ramond field
We once again start with the metric ansatz (14), with the the Kalb-Ramond (KR) action added to the action (15).

$$S = S_{Gravity} + S_{vis} + S_{hid} + S_{KR} \tag{24}$$

$$S_{KR} = \int d^4x\ r\ d\phi\sqrt{-G}\ [-2H_{MNL}H^{MNL}] \ . \tag{25}$$

They give rise to the following equations of motion (which reduce to Eq.(19) when $H_{MNL} = 0$):

$$\frac{3}{2}A'^2 = -\frac{\Lambda}{4M^3}\ r^2 - \frac{3}{2M^3}g^{\nu\beta}g^{\lambda\gamma}H_{\phi\nu\lambda}H_{\phi\beta\gamma} \tag{26}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3}r^2 + \frac{\exp(-2A)}{2M^3}\ \eta^{\lambda\gamma}[-12\eta^{00}H_{\phi0\lambda}H_{\phi0\gamma} + 3\eta^{\nu\beta}H_{\phi\nu\lambda}H_{\phi\beta\gamma}] \tag{27}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3}r^2 + \frac{\exp(-2A)}{2M^3}\ \eta^{\lambda\gamma}[-12\eta^{ii}H_{\phi i\lambda}H_{\phi i\gamma} + 3\eta^{\nu\beta}H_{\phi\nu\lambda}H_{\phi\beta\gamma}] \ . \tag{28}$$

Remarkably, the above set of equations has a unique solution of the form:

$$e^{-A} = \frac{\sqrt{b}}{2kr}\cosh\left(2kr\phi + 2krc\right) \tag{29}$$

$$c = -\frac{1}{2kr}\tanh^{-1}\left(\frac{V_{hid}}{24M^3k}\right) = -\pi + \frac{1}{2kr}\tanh^{-1}\left(\frac{V_{vis}}{24M^3k}\right) \ , \tag{30}$$

$$B^{\mu\nu} = k^{\mu\nu}\frac{2kr}{b}\tanh\left(2kr\phi + 2krc\right) \tag{31}$$

where the parameter b is related to the KR energy density, and $k^{\mu\nu}$ is a constant polarisation tensor. Requiring $A(0) = 1$ on the hidden brane leads to:

$$\frac{2kr}{\sqrt{b}} = \cosh(2krc) \ . \tag{32}$$

Note that the RS limit corresponds to: $b \to 0$, $c \to -\infty$.

The counterpart of Eq.(22) is now:

$$\left(\frac{m_H}{m_0}\right)^2 = e^{-2A}|_{\phi=\pi} = \frac{\sqrt{b}}{2kr} \cosh\left[2kr\pi + \cosh^{-1}\frac{2kr}{\sqrt{b}}\right] \tag{33}$$

$$= \left[\cosh\left(2kr\pi\right) - \sinh\left(2kr\pi\right)\sqrt{1 - \frac{b}{(2kr)^2}}\right] \tag{34}$$

$$\approx (10^{-16})^2 \ , \tag{35}$$

inverting which, we get:

$$b = (2kr)^2 \left[1 - \left(\coth(2kr\pi) - (m_H/m_0)^2\text{cosech}(2kr\pi)\right)^2\right] \ .$$

Note that, the RS value of kr corresponds to $b = 0$ (as expected). For $kr >$ RS value, one gets $b > 0$, whereas $kr <$ RS value corresponds to $b < 0$. The last possibility is unphysical however, since it corresponds to an imaginary metric and warp-factor, as can be seen from Eqs.(29) and (33). Now let us examine the range of (positive) values of b, which solve the hierarchy problem in this case. Figure 4 shows the plot of $\log|b|$ vs kr. The kink corresponds to the RS value of kr, for which $b = 0$. The LHS of the kink corresponds to $b < 0$ (unphysical sector), whereas the RHS corresponds to $b > 0$ (physical sector). Note that b rises to a maximum of $\approx 10^{-62}$ at $kr \approx 11.8$ and then falls back to zero. Thus, we see that although b has to be non-zero, for any finite value of kr, it is extremely fine-tuned. It is interesting to note that such a small value of the KR field was also predicted in a somewhat different context in [3]. The hierarchy problems appears to come back in disguise. This is our main result, which first appeared in [4].

Next, we compute the induced 4-dim cosmological constant on the visible brane, which is given by [5]:

$$\lambda = \frac{1}{2}\left(kV_{vis} + \Lambda\right) \ . \tag{36}$$

Using (30), we get:

$$\lambda = -12M^3k\left[\tanh(2krc) + 1\right] \approx -24M^3k\,\frac{b}{(4kr)^2} \approx -10^{-63} \tag{37}$$

where in the last step, we have used the (small) value of b derived earlier. This is contrary to the currently accepted value of about $\lambda = +10^{-123}$ in Planck units.
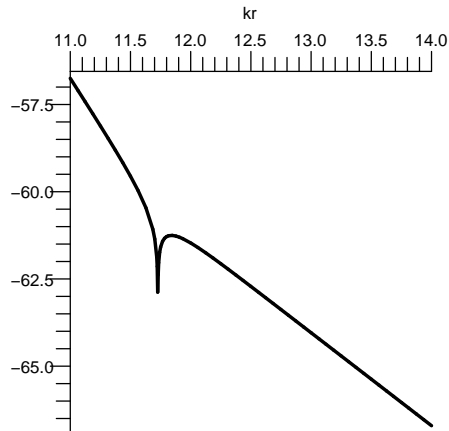
Figure 4. Plot of $\log|b|$ vs kr

3. Summary and discussions:

In this article, we have shown that on inclusion of the anti-symmetric Kalb-Ramond field, the RS brane world scenario continues to provide a solution to the hierarchy problem, albeit with an extremely fine-tuned value of the KR field. In our opinion, this can be interpreted as the re-appearance of the problem in another guise. Furthermore, the KR field induces a (small) negative cosmological constant in the visible universe, which is in variance with the currently accepted (small) positive value of the cosmological constant.

It would be interesting to probe further phenomenological implications of the inclusion of the KR field, as well as those of the dilaton field. It would also be interesting to see whether brane worlds are stabilised against perturbations when these fields are present [6]. While we hope to report on these issues elsewhere [7], here we certainly seem to be faced with the general question: if RS brane world the answer to the hierarchy problem?

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