Stability and hierarchy problems in string inspired braneworld scenarios

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We generalise the RS braneworld model by taking into account a general stringy bulk containing the scalar dilaton field and the two-form Kalb-Ramond field, apart from gravity. Assuming small fluctuations around a RS background, the back-reacted warp factor is obtained. It is shown that the fine tuning problem in connection with the Higgs mass reappears in a new guise and the effective modular potential fails to stabilise the braneworld.

In recent years, braneworld models with extra spatial dimension(s) have become popular as viable alternatives to supersymmetry as a means of resolving the fine tuning problem (in connection with the large radiative correction to the Higgs mass) in the Standard Model of elementary particles [1, 2, 3]. In the model proposed by Randall and Sundrum [2], one considers a 5 dimensional anti de sitter spacetime with the extra spatial dimension orbifolded as S_1/Z_2. Two $(3+1)$- dimensional branes, known as the visible (TeV) brane and hidden (Planck) brane are placed at the two orbifold fixed points, with the following bulk metric ansatz:

$$ds^2 = \exp{(-A)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \ , \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$ (i.e. the visible coordinates) and $y = r\phi$ is the extra spatial orbifolded coordinate. r measures the distance between the two branes and ϕ is the angular coordinate. ($\eta_{\mu\nu}$ is the usual 4-dimensional Minkowski metric, whereas G_{MN} etc will denote the full five-dimensional metric). In the original RS model, the Standard Model fields (open string excitations attached to the brane) are assumed to be localized on the visible brane, whereas gravity (a closed string excitation) propagates in the bulk. As a result of the warped background geometry, all mass scales in the theory get exponentially warped to the TeV scale, thereby resolving the hierarchy issue. However, the stabilisation of this braneworld with a radius of compactification of the order of Plank length has never been satisfactorily established. The stabilising model using a bulk scalar field with interactions localised on the branes [4] does not take into account the back-reaction of this bulk field on the background geometry. Other pieces of work which aimed at including back-reaction [5] dealt with very special scenarios.

In a string-inspired scenario, it is well known that apart from gravity, the two other massless closed string modes, namely scalar dilaton and the two-form Kalb Ramond (KR) field, can also propagate in the bulk [6, 7]. Our present work aims to investigate both the fine tuning as well as the modulus stabilisation issues in presence of these bulk fields in a back reacted S_1/Z_2 orbifolded geometry. We determine an expression for the modified warp factor with small fluctuations around the RS background. Such fluctuations result from the backreaction of the bulk dilaton as well as the KR field, within the perturbative regime. It turns out that the hierarchy problem in connection with the scalar masses problem can be resolved only if the background energy density of the KR field is fine tuned to an unnaturally small value. This in a sense brings back the fine tuning problem in a new guise. Moreover we show that, despite the presence of a bulk scalar in the form of dilaton, the braneworld modulus is intrinsically unstable.

In suitable units and in the Einstein frame, we begin with the RS metric ansatz (1), and the action

$$S = S_{Gravity} + S_{vis} + S_{hid} + S_{KR} + S_{dilaton} \ , \tag{2}$$

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$$\text{where,} \quad S_{Gravity} = \int d^4x \; dy \; \sqrt{G} \; [2M^3 R + \Lambda] \tag{3}$$

$$S_{vis} = \int d^4x \sqrt{-g_{vis}} \; [L_{vis} - V_{vis}] \tag{4}$$

$$S_{hid} = \int d^4x \sqrt{-g_{hid}} \; [L_{hid} - V_{hid}] \tag{5}$$

$$S_{KR} = \int d^4x \; dy\sqrt{G} \; \exp(\Phi/M^{3/2}) \; [-2H_{MNL}H^{MNL}] \tag{6}$$

$$S_{dilaton} = \int d^4x \; dy\sqrt{G} \; \frac{1}{2}[\partial^M\Phi\partial_M\Phi - m^2 \; \Phi^2] \; . \tag{7}$$

Here Λ is the five dimensional cosmological constant, V_{vis}, V_{hid} are the visible and hidden brane tensions. $H_{MNL} = \partial_{[M}B_{NL]}$ is the third rank antisymmetric field strength corresponding to the two-form KR field B_{MN} [8]. Φ is the scalar dilaton field present in the bulk.

Following [4] we also include interaction terms corresponding to the scalar dilaton at the boundary.

$$S_{int} = -\int d^4x\sqrt{-g_{SM}}\lambda_s(\Phi^2(y_\pi) - v_s^2)^2 - \int d^4x\sqrt{-g_{Pl}}\lambda_p(\Phi^2(0) - v_p^2)^2 \tag{8}$$

The 5 dimensional Einstein equations are as follows (where $' \equiv d/dy$):

$$\frac{3}{2}A'^2 = -\frac{\Lambda}{4M^3} - \frac{1}{2M^3}[3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\exp(-\frac{\Phi}{M^{3/2}}) + \frac{1}{4}(\Phi'^2 - m^2\Phi^2)] \tag{9}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3} - \frac{1}{2M^3}[-12\eta^{\lambda\gamma}H_{y0\lambda}H_{y0\gamma} + 3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\eta 00]\exp(-\frac{\Phi}{M^{3/2}}) + \frac{1}{8M^3}[\Phi'^2 + m^2\Phi^2] \tag{10}$$

$$\frac{3}{2}(A'^2 - A'') = -\frac{\Lambda}{4M^3} - \frac{1}{2M^3}[-12\eta^{\lambda\gamma}H_{yi\lambda}H_{yi\gamma} + 3G^{\nu\beta}G^{\lambda\gamma}H_{y\nu\lambda}H_{y\beta\gamma}\eta ii]\exp(-\frac{\Phi}{M^{3/2}}) + \frac{1}{8M^3}[\Phi'^2 + m^2\Phi^2] \tag{11}$$

In Eq.(11), the index i on the right hand side runs over 1,2 and 3, i.e. three spatial components x, y, z, and there is no sum over i. Also, $\eta^{ij} \equiv g^{im}g^{jn}\eta_{mn}$. Adding Eq.(10) and the x, y, z components of Eq.(11),we get,

$$\frac{3}{2}[A'^2 - A''] = -\frac{\Lambda}{4M^3} + \frac{1}{8M^3}[\Phi'^2 + m^2\Phi^2] \tag{12}$$

Subtracting Equ.(12) from Equ.(9),we have,

$$\frac{3}{2}A'' = -\frac{1}{4M^3}\Phi'^2 - \frac{3}{2M^3}G^{\mu\nu}G^{\alpha\beta}H_{y\mu\alpha}H_{y\nu\beta}\exp\left(-\Phi/M^{3/2}\right) \tag{13}$$

The equation satisfied by the y-dependent VEV of the KR fields is given as [12] ,

$$G^{\mu\alpha}G^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta} = bM^5\exp\left(2A(y)\right)\exp\left(2\Phi/M^{3/2}\right) \tag{14}$$

where $bM^5 = \eta_{\mu\alpha}\eta_{\nu\beta}k^{\mu\nu}k^{\alpha\beta}$, $k^{\mu\nu}$ is a constant antisymmetric tensor, independent of y, and b is a dimensionless parameter measuring the energy density of the KR field. It can be shown that the solution for $H_{\mu\nu\lambda}$ (or $B_{\mu\nu}$), derived from the equation of motion for the KR field satisfies Eq.(14). The proof (without the dilaton field) follows from our earlier paper [9]. The proof including the dilaton field follows along similar lines.

Similarly, the classical equation of motion satisfied by the dilaton field is given as,

$$\Phi'' - 2A'\Phi' - m^2\Phi^2 + \frac{6\exp\left(-\Phi\right)}{M^{\frac{3}{2}}}(G^{\mu\alpha}G^{\nu\beta}H_{y\mu\nu}H_{y\alpha\beta}) = 0 \tag{15}$$

Now using Equ.(14) in Equ.(15),we have,

$$\Phi'' - 2A'\Phi' - m^2\Phi^2 + 6M^{\frac{7}{2}}b\exp\left(2A\right)\exp\left(\Phi/M^{3/2}\right) = 0 \tag{16}$$

Also using Equ.(14) in Equ.(13), we have,

$$A'' = -\frac{1}{6M^3}\Phi'^2 - bM^2\exp\left(2A\right)\exp\left(\Phi/M^{3/2}\right) \tag{17}$$

We linearise Eq.(16) and obtain the solution for Φ as a power series in the dimensionless parameter b, which is defined in Eq.(14). To leading order, the solution reads [13] :

$$\Phi(\phi) = \Phi_0(\phi) + b\Phi_1(\phi) \tag{18}$$

$$= \Phi_0 \exp\left[2kr(1-\nu)\phi\right] + b\sum_{n=0}^{\infty} \frac{6M^{7/2}}{k^2} \frac{\left(\Phi_0/M^{3/2}\right)^n}{n!} \frac{\exp\left[kr\phi(2n(1-\nu)+4)\right]}{(\omega_n^2 + 4\omega_n - m^2/k^2)} \tag{19}$$

Note that in order for this perturbation series to be valid over the entire bulk spacetime, one requires $b \lesssim \exp(-4kr\pi) \approx 10^{-64}$. In other words, the existence of a perturbative solution around RS requires b to be severely fine-tuned. We now observe that in the above expression for Φ, the leading order contribution from the summation in the RHS comes from the term $n = 0$. Substituting for ν with appropriate approximation, the truncated solution (in the variable $y = r\phi$) for Φ is obtained as,

$$\Phi(y) = \Phi_0 \exp[-m^2y/4k] - \frac{6bM^{7/2}}{m^2}\exp[4ky] \tag{20}$$

Using the above solution for Φ we solve Equ.(17) for $A(y)$ to leading order in the perturbation parameter b,

$$A(y) = ky - \frac{\Phi_0}{M^{3/2}}\exp[-m^2y/2k] - 32b\frac{\Phi_0 M^{1/2}k^2}{(16k^2 - m^2)^2}\exp(4k - m^2/4k)y$$
$$- bM^2 \int \left[\int \exp\left[2ky + \frac{\Phi_0}{M^{3/2}}\exp[-m^2y/4k]\right]dy\right]dy \tag{21}$$

Now since $\frac{\Phi_0}{M^{3/2}}\exp[-m^2y/4k] << 2ky$ in the exponent of the last term in the' RHS, we can approximate the integrand and obtain the following expression,

$$A(y) = ky - \frac{\Phi_0}{M^{3/2}}\exp[-m^2y/2k] - 32b\frac{\Phi_0 M^{1/2}k^2}{(16k^2 - m^2)^2}\exp(4k - m^2/4k)y$$
$$- \frac{bM^2\exp(\frac{\Phi_0}{M^{3/2}})}{(2k - m^2/4k)^2}\exp[(2k - m^2/4k)y] \tag{22}$$

Equ.(21) or Equ(22) is the back-reacted expression for the warp factor $A(y)$ where the second and the third terms in the RHS are the contributions from the dilaton and the KR field respectively. It is easy to show that in absence of the dilaton field we get back the expression for the warp factor in a KR-gravity bulk [11].

We now explore whether the KR-dilaton back-reacted warp factor can resolve the hierarchy problem in connection with the mass of the Higgs boson. The scalar mass on the visible brane is given by the warped relation,

$$m = m_0 \exp[-A(y)]_{y=r\pi} \tag{23}$$

where m_0 is the mass scale in the Planck brane.

We now estimate the contribution of mass warping due to the dilaton and KR field induced terms in the warp factor (namely the second, the third and the fourth terms in the RHS of Equ.(22)). Recall that in the original RS scenario k and r are taken near the Planck mass and the Planck length to avoid the introduction of any unknown intermediate scale in the theory. Our solution here [Equ.(22)] describes a perturbative modification of the warp factor over the RS value. Thus taking $kr \sim 12$ with $k \sim M_{Pl}$ and $r \sim l_{Pl}$ along with $\Phi_0 \sim M^{3/2}$, we estimate the warp factor at the visible brane as,

$$[A(y)]_{y=\pi} = 37 - 10^{-16} - b10^{62} - b10^{31} \tag{24}$$

As our perturbative solution is valid for $b \lesssim 10^{-64}$, the exponent $A(y)$ evaluated on the visible brane (including the back-reaction) is always positive, and is very close to the RS value. Thus, the above value of b, for which the perturbative expansion in Φ is valid, results in a small fluctuation in the RS value of $A(y)$ in a self-consistent manner. Therefore, the hierarchy problem can be resolved even in the presence of the dilaton and KR fields, although the parameter b in the theory needs to be severely fine-tuned. We re-emphasise that this fine-tuning arises from the requirement of the existence of perturbative solutions to the equations of motion. The desired warping from Planck to TeV scale therefore can be obtained only if the KR energy density parameter b is fine tuned to 10^{-60}. Thus the fine tuning problem reappears in a new guise. Similar fine tuning was obtained in [11] where only the KR field was considered in the bulk.

To understand the stability issue of this model we observe that even without having to solve the equations for $\Phi(y)$ and $A(y)$ explicitly, it is possible to address the stabilisation issue with some very general assumptions regarding the solution. For the stabilisation analysis,we write down the complete form of the stabilising potential, which is a function of y_π (i.e the value of the extra coordinate at the location of the visible brane),

$$V_\Phi(y_\pi) = \int_0^{y_\pi} dy \exp{-2A(y)} \exp(-\Phi)[-2H_{MNL}H^{MNL}] + \exp{-2A(0)}\lambda_p(\Phi^2(0) - v_p^2)^2$$

$$+ \int_0^{y_\pi} dy \exp{-2A(y)}[\Phi'^2 + m^2\Phi^2] + \exp{-2A(y_\pi)}\lambda_s(\Phi^2(y_\pi) - v_s^2)^2 \tag{25}$$

Now,for the ground state configuration of Φ we take,

$$\Phi(0) = v_p \tag{26}$$

$$\Phi(y_\pi) = v_s \tag{27}$$

Note that the solution $\Phi(\phi)$ contains only one constant Φ_0 . Eliminating Φ_0 from Equ.(26) and Equ.(27), we obtain y_π as a function of v_s and v_p. Using this relation in Equ.(27), and using Equ.(26) to eliminate v_p, we obtain Φ_0 purely as a function of v_s. This implies that the solution $\Phi(\phi)$ has an explicit dependence only on v_s and none on y_π. Thus, the dependence of $V_\Phi(y_\pi)$ on y_π comes solely from the upper limit of the integrals over the extra dimension.

In this form, the first and the second derivatives of the potential can be readily obtained, as follows:

$$V_\Phi'(y_\pi) = \exp(-2A(y_\pi) \exp(-\Phi(y_\pi)[-2H_{MNL}H^{MNL}]_{y=y_\pi} + \frac{1}{2}\exp(-2A(y_\pi))[\Phi'^2 + m^2\Phi^2]_{y=y_\pi}$$

$$= 6\kappa \exp(\Phi(y_\pi)) + \frac{1}{2}\exp{-2A(y_\pi)}[\Phi'^2 + m^2\Phi^2]_{y=y_\pi} \tag{28}$$

The second equality uses (14) with $\kappa = bM^5$.The condition of an extremum requires $V_\Phi'(y_\pi) = 0$,the solution to which gives the value of y_π at which the braneworld is stabilised,viz.

$$-6\kappa \exp(\Phi(y_\pi)) = \frac{1}{2}\exp(-2A(y_\pi))[\Phi'^2 + m^2\Phi^2]_{y=y_\pi} \tag{29}$$

The second derivative, on using the equations of motion and Equ.(29), gives,

$$V_\Phi''(y_\pi) = \exp(-2A(y_\pi))\Phi'(y_\pi)[4k\Phi'(y_\pi) + 2m^2\Phi(y_\pi)] + 12\kappa \exp\Phi(y_\pi)A'(y_\pi) \tag{30}$$

The above expression, whose sign determines the nature of the extremum, can be calculated without resorting to the full solution of the equations of motion, and using only the boundary values of Φ' and A'. Near the boundary at $y = y_\pi$ only the delta-function dependent terms in the equations of motion become important which could be integrated to obtain the expressions of the first derivatives at $y = y_\pi$. Thus one obtains,

$$[\Phi'(y)]_{y_\pi} = -2\lambda_s\Phi(y_\pi)(\Phi^2(y_\pi) - v_s^2) = 0 \tag{31}$$

$$[A'(y)]_{y_\pi} = -\frac{1}{12M^3}V_{vis} \tag{32}$$

Using Equ.(31) and Equ.(32) in Equ.(30),we find,

$$V_\Phi''(y_\pi) = -\frac{\kappa}{M^3}\exp\Phi(y_\pi)V_{vis} \tag{33}$$

As V_{vis} is negative, the stability condition (i.e $V'' \geq 0$) can be achieved only if κ is positive. On the contrary, κ is negative from Equ.(29), if there exists a stationary point for the potential. Thus, the presence of the stringy bulk fields back-reacts on the geometry in a way which evidently jeopardises the stability of the braneworld. However, it may be noted that if we consider the kinetic terms for the bulk dilaton or KR field with an opposite sign (i.e a phantom like field) then κ will be positive leading to a possible stability of the resulting braneworld.

To summarise, we have shown that the requirement of fine tuning the Higgs mass by one part in 10^{16} can be avoided at the expense of even more fine tuning of the KR field energy density by one part in 10^{64}. Furthermore, inclusion of the dilaton and KR fields in the bulk results in an effective modular potential which clearly does not have any minimum. Thus stabilisation of the modulus cannot be achieved in presence of these stringy bulk fields. The modulus can however be stabilised when phantom-like scalar fields are included in the bulk. This work therefore raises questions about

the efficacy of the string inspired braneworld models to resolve the gauge hierarchy and the modulus stabilisation problems.

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[12] It may appear that the solution for $H_{\mu\nu\lambda}$ in reference [10] (Eq.(7)) differs from the one we have here. In that paper, $H_{\mu\nu\lambda}$ was expressed as a dual of a vector field (Eq.(5)). Using this along with Eq.(8), it can be shown however that the two solutions are consistent with each other.

[13] In principle there could be non-perturbative solutions for which the conclusions might be different. However in this paper, we examine the small fluctuations produced by the KR and dilaton fields (which may always be present), and how robust the resolution of the hierarchy problem is under such fluctuations.