METACOGNITION'S ROLE IN IMPROVING THE NUMBER SENSE DEVELOPMENT IN STUDENTS

TRISTAN RONALD ILKO
Bachelor of Education, Concordia University of Edmonton, 2011
Bachelor of Science, Concordia University of Edmonton, 2010

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DEDICATION

To my mom and dad -- thank you for the encouragement to pursue this journey and the constant pestering to finish.

To my daughters Lillian and Angela -- Above all else be true to yourself, you three make me want to get better with each day.

To my wife Lauren -- your unwavering support, belief in me and frequent editing are the reasons this project is done today. The completion of this masters is due to you and your ability to never let me give up even when I doubted myself. Thank you for everything and I owe you many years of reimbursement for your patience through this whole process.

ABSTRACT

Number sense, as related to mathematics, can be defined in many ways. This seemingly innocent phrase continues to be an aspect of mathematical education that students are not always able to achieve. Number sense is an essential and fundamental skill that holds impacts beyond just the classroom. The purpose of this project is to explore the relationship between number sense development and metacognitive strategies. The focus has been narrowed by using the McIntosh, et al. (1992) number sense framework as well as Flavell's (1979) metacognition model as the essential frameworks in the design of this project. By the end of this paper we will have explored some of the factors that influence the development of number sense, specifically the role metacognition has on assisting in this venture. As well as how it aids in the development of transfer of the mathematical skills that are being developed.

ACKNOWLEDGMENTS

When I first started this journey, I did not know what to expect. But looking back now I cannot believe how much I have learned and grown through this process. What I did not expect is to be challenged in ways that I never thought I would be challenged and have new worlds of thought brought forward. Thank you to my amazing cohort, you all have been a blessing to me during this process. Since our first Wednesday together, you all have provided laughter, reasoning, questions and support during this time, and for that I am grateful to have met and walked through this process with every one of you. To the amazing instructors at the University of Lethbridge: Dr. Slomp and Dr. von Heyking, thank you for your dedication, passion, humor and wealth of knowledge that you brought to this program. Your drive made us all better educators and will continue to drive that passion in us. Dr. Grigg, Dr. Burleigh, Dr. Pelech, and Dr. McCormick, thank you for allowing me the opportunity to slow down and appreciate the diversity and complexity of the world around me. Too often we get caught up in our own bubbles, and your classes allowed that wide perspective to be seen. To Dr. Marynowski, thank you for taking on this opportunity and sharing advice, laughter, and new perspectives. To this day the wonderful memories and new values from the 2017 C&A cohort drive my passion for education.

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> Mathematics is not about numbers, equations, computations, or algorithms: it is about understanding
>
> - William Paul Thurston (Patterson, 2017, para. 12)

1.1 DEFINITION

The term number sense, as we know it today, holds many interpretations, and there is no one set example that truly defines it. The challenge that educators face is what definition we go by when we are working towards this goal with our students, and with so much variety, how do we find one that rings true? Alberta Education (2016) references number sense as the ability of "students [to] connect numbers to their own real-life experiences and … [results in students who are] computationally fluent and flexible with numbers" (p. 7). Researchers such as McIntosh, et al. (1992) defined number sense as the ability to handle the demands of mathematics based on the current time one is in. Others such as Greenes, et al. (1993) describe number sense as the ability to understand the relationships between numbers and how to use them in the proper context. Baroody (2006) saw number sense as more than just a collection of numeral facts, but a "web of richly interconnected ideas" (p. 26) that fostered a conceptual understanding as well as emphasized strategic thinking and productive disposition. National Council of Teachers of Mathematics (NCTM) (2014) speaks about productive disposition as one's ability to see sense in the mathematics that are in front of them, perceive its usefulness, and believe that there is value in learning mathematics to apply to the problems that lie before them.

From these definitions, there are a variety of constructs, focuses, and beliefs. However, one common element is that number sense can be defined as a cognitive process that allows an individual to use their knowledge and skills in regard to numbers and operations in a flexible manner and allows transfer from problem situations into a real-life application (Boaler, et al., 2015; Çekirdekci et al., 2018; Greeno, 1991). Also, there is research that supports the idea that number sense is more than a simple understanding of numbers but an innate and intuitive sense of numbers, where students make connections to their everyday lives (Alberta Education, 2016; Hidayat et al., 2018; Olkunet al., 2017; Shumway & Moyer-Packenhem, 2019; Toll et al., 2016).

Regardless of which definition is adopted or deemed as correct, the one thing regarding number sense that echoes through all of the definitions is the individual and their unique personal context regarding number sense development. No matter what we as educators impart onto students, be it lessons, concepts, algorithms, processes or theories, at the heart of it all are the students and how they make sense of numbers. This section explores various ways students view, interact and understand the numbers they are exposed to on a daily basis.

1.2 AN INDIVIDUAL'S INTUITION OF NUMBERS

It is suggested that intuition of numbers is a part of our daily lives starting much earlier than formal schooling (Baroody et al., 2005; Jordan et al., 2010; Olkun et al., 2017) and is "anchored deep into our brains" (Dehaene, 1997, p. 245). Our innate understanding of numbers is as much a part of us as is our ability to see the colour of a flower, smell the fresh cut grass or hear the wind rustle leaves in the fall. We all came into school with some forms of intuitive sense of what numbers are and how we understood them. This informal understanding served as the basis for our number sense as we moved into formal schooling (Carpenter et al., 1996). Numbers are more than symbolic representations of values, but a sensory understanding, ingrained in the fabric of our brain that we simply cannot avoid quantifying or comparing. We are born with the desire to process these concepts (Shumway & Moyer-Peckenham, 2019), and we define these categories based on how we experience the world through mathematics (Dehaene, 1997). So how does this intuitive ability apply to the development of number sense in students as they reach schooling age?

Çekirdekci, et al., 2018 stated that “intuition about number relationships helps children to evaluate the logical appropriateness of mathematical computational results and support the solution of numerical problems” (p. 2467). This statement speaks of the ability to see, recognize and act upon patterns that are identified in a numerical problem, but also goes on to challenge a student's ability to select appropriate strategies as well as review the solution for suitability. Now, if number sense is intuitive in children, why is there such a discrepancy between the students that show up in our classrooms? One theory, as will be explored further into this paper, is the lack of attention that has been paid to the use of metacognition and students' interpretation as a means to supplement current number sense pedagogy.

Dehaene (1997) argued one of the main causes of the lack of number sense development in students is when teachers emphasize calculation over conceptual understanding. This tendency is seen in formal school settings; a child’s experiences, understanding, and interpretations up to the point of formal schooling are replaced by the instruction of the teacher. However, through metacognitive practices in a classroom setting and individual mathematical interpretation “kids discover strategies that make math meaningful. Unfortunately, however, kids are often taught that these strategies are wrong” (Dewar, n.d.). By approaching math through the lens of student-based strategies, and conceptual understanding we can bridge the gap between a student's personal strategies and the teacher’s instruction.

1.3 EVOLUTION OF NUMBER SENSE UNDERSTANDING

Before we can get into how to develop number sense, we need to explore how we came to this point in math education. The origin of the phase “number sense” is not known, but it is believed that the rise of the phrase was to replace the term “numeracy”, which had been dominating for much of the 20th century (McIntosh et al., 1992). Number sense, in essence, is a student's intuitive ability to demonstrate flexibility, the magnitude of numbers, estimation, and efficiency (Baroody et al., 2005; Greenes et al., 1993; Greeno, 1991; Jordan et al., 2010; Sowder, 1990). The progression towards our understanding of number sense began with a pessimistic view of what children were capable of. James (1890) referenced by Baroody et al. (2005) talks about children’s mathematical understanding as being “a great, blooming, buzzing confusion” (p. 189) with little true understanding to prepare themselves for the world outside the classroom. Thorndike (1922) built upon this view and believed that children came into school with no prior mathematical understanding or knowledge. He believed that, before the second grade, a child had little ability to grasp arithmetic concepts. However, through rote memorization, those children could grasp basic facts in grade 1 for preparation for the following year. This view dominated much of the century until Piaget’s work in the 1960’s and 1970’s which highlighted that “mathematical thinking and knowledge do not simply blossom in the school age children as a result of formal instruction” (Baroody et al., 2005, p. 189), but emphasized that a constructivist process and gradual steps via experiences provide an essential foundation for mathematical development.

From the pessimistic, we ventured into the overly optimistic views of what children *can do.* Wynn (1995) emphasizes the fact that children from an incredibly early age are able to identify, distinguish, and represent small numbers of visual objects (dots, household items, etc) and perform mathematical operations without any formal schooling teaching them the processes. Children are drawn to numbers, almost instinctively, and demonstrate abilities beyond what early century theorists thought they were capable of without formal instruction. This intuitive ability to interact with numbers demonstrates how the pessimistic views of mathematical ability have been holding us back from allowing students to respond to the numbers that they are experiencing on a daily basis. Regardless of the reforms, beliefs, or methods that have been put in place, students continue to struggle in grasping mathematics and many fail this subject early into their mathematical careers (Lemoyne & Favreau, 1981). This clearly shows that there is a missing piece to the puzzle that not only will allow students to develop number sense, but also to develop a greater appreciation for the world of mathematics around them.

However, the highly optimistic views were considered by many as too extreme and a call for a more balanced approach to number sense development was brought forward (Baroody et al., 2005). In this balanced approach, two models came to the forefront of the number sense development world: *mental models* - which explored the evolution of *how* numbers are represented and *progressive abstraction models* - which looked into the evolution of *what* children represent moving from concrete to abstract (Baroody et al., 2005).

1.4 NUMBER SENSE DEVELOPMENT IN STUDENTS

Numbers are embedded in our lives. They are seen on spreadsheets, scores on video games, credit cards, sporting statistics, taxes, and virtually every component of our social lives (Dehaene, 1997; Turkel & Newman, 1988). If this is true, then it is important for a mathematically literate individual to have an understanding of what these numbers mean and how they can be used by us on a daily basis, because “many have no appreciation of number magnitudes - no grasp of very large numbers and little understanding of small ones” (Turkel &

Newman, 1988, p. 53). Number sense development at an early age has been reported as a critical indicator of mathematical success and achievement in later years (Jordan, et al., 2010; Shumway & Moyer-Peckenham, 2019; Turkel & Newman, 1988). However, the challenge has been to determine what does development of number sense looks like in students with varying ages, skills, experiences, and interpretations to concepts being taught. How do we develop number sense in students who were not successful in grasping the concepts prior to, or during, formal schooling years? This is an important issue because as teachers our goal is to support all of our students and since all students come to our classrooms with varying levels of experience in mathematics, we need to ensure we can provide them with experiences that support their experiences. Along with that, how do we enhance those who already have the skills and are thirsty for more? One solution is that we do not force mathematical procedures on students but provide them with opportunities to create understanding for themselves via their own processes and strategies.

McIntosh et al. (1992) compiled a breakdown of some of the most agreed-upon components of number sense and demonstrated their general findings into Figure 1.

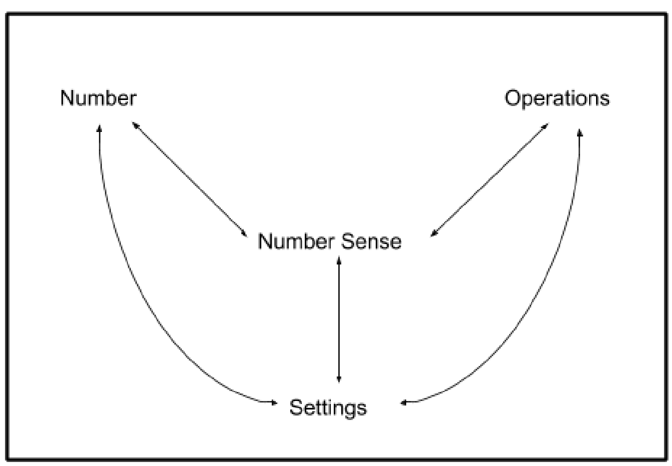

Figure 1: Interconnections of Major Components of Number Sense (McIntosh et al., 1992, p. 5)

Figure 1 illustrates that number sense is an interwoven construct that relies on several interlocked components to be developed in order to achieve mastery. Additionally, this figure incorporates a highly individualized aspect of number sense because “number sense exhibits itself in various ways as the learner engages in mathematical thinking” (McIntosh et al., 1992, p. 3). Drawing deeper we can argue that number sense to an individual is a unique experience that cannot be forced onto someone but students must be provided opportunities to engage with numbers. Çekirdekci et al. (2018) illustrated how number sense is “a way of thinking, [and] skills such as perception, attention, flexible thinking, strategy development are required” (p. 2472) to be successful all the while being personal in nature. This was not always our thinking when it came to number sense development in the students in classrooms across the world. Baroody (2006) explains how conventional wisdom in teaching number sense saw little value to the use of strategies that assisted students in counting and reasoning and emphasized the rote memorization of facts as the precursor to understanding. The idea of scientific efficiency came into play during this time, where students were seen as nothing more than vessels that could be filled versus individuals that had thoughts, processes and understandings beyond our own. The belief was that a student was unable to move onto more advanced mathematics until memorization of prior foundational facts was mastered (Thorndike, 1922).

Earlier, Brownell (1928) published work that argued for a form of arithmetic that contained both mathematical and social aims, he aptly named it *meaning theory*. Meaning theory, according to Brownell (1945), stated that understanding was created through developing meaning, and “meaning is to be sought in the structure, the organization, [and] the inner relationships of the subject itself” (p. 481). The mathematics that students should be participating in are more than calculations and problems, however, most experience the opposite as the excessive emphasis on rote memorization processes tend to reign supreme in number sense teaching (Carr, 2010). Brownell's (1945) research explored the connection that students have with the mathematics they are interacting with, and further probed how, in order to develop number sense, the student needs to participate in mathematics that is meaningful to them at the moment and built through experiences (Brownell, 1945; Higgins & Wiest, 2006). Higgins and Wiest (2006) referenced how "students should both make sense of the mathematics itself and know how it applies to the real world" (p. 26), a concept that we are still struggling with creating in most classrooms today. From the discussed research, we can argue that developing number sense in students can be achieved by addressing three main areas in the classroom: process is not understanding; explanation of thinking; and mathematical environments.

1.4.1 PROCESS IS NOT UNDERSTANDING

Looking back at our own experiences in the mathematical classroom it is safe to assume that many of us were brought up with an emphasis on algorithmic methods. These classrooms emphasized that students follow the algorithm that was provided by the teacher, because the teacher knew best. These algorithms were step-by-step, always worked (as long as you followed the steps), and was a process that had to be memorized because this was the way to do it (Carr, 2010). Some common algorithms looked like the following (se figure 2 below):

$$\begin{array}{r} {}^{1}1\,{}^{1}4\;5 \\ +\;\underline{5\;7\;7} \\ 7\;2\;2 \end{array} \qquad\qquad \begin{array}{r} {}^{2}\cancel{3}\,{}^{14}\cancel{5}\;5 \\ -\;\underline{1\;\;7\;\;2} \\ 1\;\;7\;\;3 \end{array} \qquad\qquad \begin{array}{r} {}^{1}\cancel{{}^{2}}33 \\ \text{x}\;\underline{47} \\ 231 \\ +\underline{1320} \\ 1551 \end{array}$$

Figure 2: Traditional Algorithm Examples

However, research shows that fluency, specifically procedural fluency, is not established simply by following a process step by step or by a blanket approach (Boaler et al., 2015; Parrish, 2011), in fact step by step processes accomplish the opposite. The algorithmic fact based process advocated by Thorndike (1922) stifles the creativity and interest in learning and "results in students failing to understand how procedure can be altered and transferred" (Carr, 2010, p. 182). This is evident when children with weak number sense start to move into more challenging problems that involve several digits or processes that are not identical to the procedural problems they have solved prior. As mentioned in definitions previous, number sense is an intuitive understanding of numbers that children develop from a very young age. Children have to make sense of the mathematics they are engaging in; they need to view it as useful but also a worthwhile venture (Baroody, 2006; NCTM, 2014). Baroody (2006) further explained that it is easier to understand material when it is put into a meaningful manner that students can get behind and invest in. However, at some point during their educational tenure "children suddenly shift from an intuitive understanding of numerical quantities, supported by simple counting strategies, to a rote learning of arithmetic" (Dehaene, 1997, p. 126). As a result of this students fail to grasp how the algorithm, which they have been told to use, can be modified, altered or transferred into situations that are beyond controlled classroom examples (Boaler, 2015; Boaler, 2020; Carr, 2010). Bisanz, referenced in Dewar (2008), provides an example of where the algorithm takes more precedence than mathematical thinking:

> American six year olds and nine year olds [were given] this problem: 5 + 3 – 3 = ? The 6-year olds tended to solve this without doing any calculations. They just observed that the positive 3 and the negative 3 cancel each other out. However, the 9-year olds (who had

> learned from their teachers what the "right" approach was) were more likely to take the long route to the answer:
>
> 5 + 3 = 8
>
> 8 - 3 = 5
>
> In other words, 9 year olds had learned that they should follow the teacher's procedure first, and think later. (paras. 5 - 8)

This demonstrates issues surrounding the dependency students create due to their reliance on pre-set strategies. The six year-olds in the example above demonstrate their intuitive sense of numbers by recognizing that the 3's will cancel each other out, while the nine year olds (the more experienced in mathematics) have foregone all of those understandings for the reliance of the step-by-step algorithmic approach. Yes, each gets the same result in the end but number sense is not defined on the ability to solely accomplish a correct answer, but developing an understanding on why and how the relationships between those numbers result in the solution achieved by the students. Universal strategies have the inherent flaw of removing the understanding of how those procedures work in all forms. The students are not investing in why this understanding is important and because the algorithm is pre-set for them there is no meaning or connection to that strategy, it is simply *the right way*.

If we are to develop number sense, we must move beyond just looking to provide students with a way to get the correct answer and focus on fluency. Bay-Williams and Kling (2019) spoke of developing number sense through five fundamental areas, with the most foundational being the development of procedural fluency. Procedural fluency is exhibited when a student is demonstrating accuracy, efficiency, flexibility and selecting appropriate strategies with one's calculations (Bay-Williams & Kling, 2019; Hattie et al., 2017; Parrish, 2011), as presented in Figure 3:

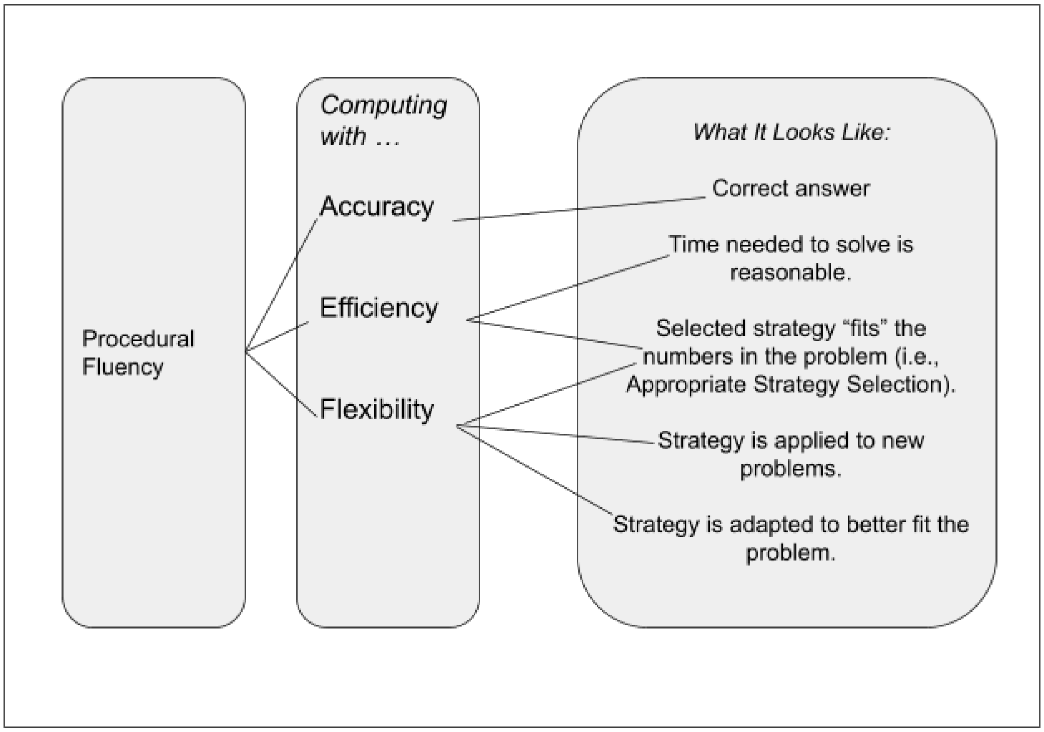

Figure 3: What Procedural Fluency Is and What It Looks Like (Bay-Williams & Kling, 2019, pg. 2)

Accuracy is achieving the correct solution, efficiency is solving a problem in a reasonable amount of time with a selected strategy and flexibility, explored in more detail later on, is the ability to transfer the knowledge to new experiences (Bay-Williams & Kling, 2019; Pintrich, 2002). By developing procedural fluency in students, teachers can attempt to move a child away from a reliance on procedures that are pre-established and into self-development and understanding of new strategies that help define what numbers represent. An example of this is "when children begin double or triple digit arithmetic they often develop buggy algorithms...as a result of poor understanding of place value" (Carr, 2010, p. 182). It is common practice in classrooms to base a student's understanding on their ability to achieve the correct answer.

Accuracy through a prescribed algorithm does not reflect that a student understands why or when to use it in various situations, nor allows for understanding in how the relationships of the numbers mesh together to result in an appropriate answer (Carr, 2010; Pintrich, 2002). This allows students to impact all operations that are being explored, not simply basic facts and allow new interrelated pathways to be explored. An example where procedural fluency is at the forefront is a program called Number Talks. This method of teaching number sense explores how a "five- to fifteen-minute minute conversation around … computational problems…[allows students] to combine the essential processes and habits of mind doing math" (Parrish, 2011, p. 199). Number Talks, in its essence, focuses on allowing students the opportunity to achieve accuracy, efficiency and flexibility through mental math processes drilled in conversation and experience.

1.4.2 EXPLANATION OF THINKING

How often have students in the classroom surprised teachers with low marks on tests or quizzes and a teacher thinks "what happened? They knew this just the other day"? The reality is they may not know what knowledge and understandings the students possess if we rely on the traditional tests and quizzes alone. There is a tendency for students to not follow through with explaining the process they used to arrive at an answer. This is due to the fact that many classrooms focus on the importance of a correct answer, which is an important piece of developing number sense, but the understanding of what that answer means and how to get there can be overlooked in the process (Carr, 2010). The use of self-explanation has been found to promote understanding and conceptual change and holds the opportunity for us as teachers to get a glimpse into the thinking of the students in our classroom, how they perceive numbers, how they arrived at their final solution, and are there areas of misunderstanding or misconception

(Schoenfeld, 1992). Integrating explanation into one's lessons allows students the opportunity to realize the state of their own knowledge of the topic at hand as well as the effectiveness of the strategies they have selected to implement (Pintrich, 2002). Pulling a student aside and asking them “how did they …” is not enough. It is a start in the right direction, but we need to allow the students to share their process or strategy with not only the teacher, but also with those around them. Not only will this opportunity to express themselves highlight the success or failures of the students, but paint a better picture for us as teachers. This ties well into the elementary and middle school levels because research shows us that when students can “describe when and how to use strategies [they] are more likely to be successful” (Carr, 2010, p. 181). The ability to explain one's work gives the students the opportunity to share ideas, clarify understanding, and construct convincing arguments on how and why things work based within their context. After all, if a student is unable to explain their process and they rely solely on rote procedures we run the risk of having students develop misconceptions in their learning without a place to interject and guide them onto the correct path. Carr (2010) built on this point further by explaining how even if they are incorrect in process and understanding, explanation allows for those misconceptions to be brought to the surface. If children are unable to verbalize their reasoning’s the conceptual and procedural understandings are prevented until those misconceptions are corrected.

Talking about thinking strategies, or thinking aloud is important because, through discussion, it develops the vocabulary students need for explaining how they are working through problems, and in turn it seems to produce more thinking in students (Carr, 2010; Djudin, 2014; Louca, 2003). This discussion in turn supports the necessary skills required in developing a greater ability to apply these skills in various settings and contexts. Students’ knowledge of what they believe to be true must be challenged in order to provide opportunities to expand their thinking. Explaining one's thought process to others provides such an opportunity. It is suggested that students solidify their understanding more effectively when they are allowed to explain their choices, strategies, and reasoning to an audience. "Students are active members of the discourse community as they explain their reasoning and consider the mathematical explanations and strategies of their classmates" (NCTM, 2014, p. 35). As students share their answers with the class, they are provided with the chance to alter their understanding of a concept/strategy they once believed to be true. By challenging their understanding, students are provided with new knowledge that can support what they currently know, provide a correction to a misconception, or provide new avenues to pursue (Carr, 2010; Louca, 2003; NCTM, 2014; Parrish, 2011). This understanding emphasizes how students, who are defending their solution, need to draw on multiple areas of knowledge (ex. language, relationships, processes, etc.) in order to demonstrate why their solution is correct, and be prepared to justify such a process (Bonnett et al., 2017).

As much as explanation assists students on an individual level it also provides support for others in the mathematics classroom. Explaining one's process allows peers to hear and see how others approach a common solution which can cause students to compare strategies, recognize relational pathways and make judgements about the effectiveness of other strategies versus their own (Pintrich, 2002). By emphasizing self-explanation in one's classroom students are allowed the opportunity to construct understandings through the implementation of strategies, such as Number Talks, think-alouds, think-paired techniques, and reciprocal teaching. As one defends their strategy they are able to develop a much deeper and richer understanding of how to "base judgments about accuracy of their conclusions and success of [their chosen strategies]" (Carr, 2010, p. 180).

1.4.3 MATHEMATICAL ENVIRONMENTS

A math classroom is more than just a place where algorithms, processes, and strategies are taught and learned, but also a haven where understandings are generated. Schoenfeld (1992) talks about how "learning is a social act, taking place in a social context" (p. 347); a place where "teacher and learners interact over the curriculum" (NCTM, 2014, p. 8) and where students are able to draw on, learn from, and justify reasoning with those around them. It is a place where mathematics is present at every turn and students are comfortable enough with their peers to take risks, explore, collaborate, reflect, and have meaningful discussions regarding the problems they are facing collectively (Bonnett et al., 2017; Parrish, 2010; Schoenfeld, 1992). It is fruitless to expect that the motivation, desire, and sense making will appear without structures in places that promote such learning. As teachers, it is our responsibility to surround our students with the foundation that not only provides instruction, but opportunity for further growth. In order to accomplish that, we need to be constantly mindful of this environment as "some researchers are concerned that an overly-regimented approach to [mathematical] education could backfire … develop[ing] negative attitudes" (Dewer, n.d., para 62) about the numbers that students are interacting with.

The environment of the classroom needs to surround the students with the opportunity to immerse themselves with numbers on a constant basis in a place where they feel safe to make mistakes, try new procedures, and be safe enough where they can be vulnerable for critique. Students should feel comfortable in offering responses for discussion, questioning themselves and their peers, and investigating new strategies. The culture of the classroom should be one of acceptance based on a common quest for learning and understanding (Parrish, 2010). The challenge that we face is that creating a classroom culture of understanding and mutual support is not something that happens overnight. This process, known as enculturation, is central for the development of knowledge, as it provides perspective for students to acquire traits, habits and understandings that the community or culture of the classroom deems important (Schoenfeld, 1992). This takes time to become a routine in one's classroom and requires constant attention in order to maintain the environment for all students. The pressures we face from outside forces to cover curriculum, score well on large-scale tests, meet deadlines, and accurately assess students all chip away at the environment's stability long-term. However, without a safe, open mathematical classroom environment, students may not be able to grasp the ability to learn from those around them and understanding falls to the individual compared to a collective venture.

As the environment is established a culture can be created where students are the ones who are leading their learning and utilizing their own understanding in order to defend and support their strategy use and solution. The need is to establish a culture that promotes, accepts, and strives for productive struggles, and not one where rescuing the student at the sign of trouble is the priority (NCTM, 2014). The students become the essence of mathematical practitioners, following in the footsteps of true mathematicians, not just reproducers of procedure, which is why the need for a broad range of exercises need to be offered to the students, from exercises to open-ended questions that spur new thoughts and challenges (Boaler, 2013; Parrish, 2011; Schoenfeld, 1992). Teachers, traditionally, have been the sage of knowledge when it comes to mathematics. We provide the problems, procedures, strategies and answers for the students. As students become the practitioners of mathematics the role of the teacher alters from dictator to facilitator. This change is role allows for situations where students are the ones highlighting each other's mistakes, because “learning is facilitated when a student is able to reflect on the difference between what he/she thinks is true and new information that supports or discounts that belief” (Carr, 2010, p. 177). Schoefeld (1987), referenced by Ebdon et al. (2003), wrote about how an environment that emphasizes a mathematics culture is the best way to develop metacognition. Which is why, in order to create the best results for our students developing number sense, we must create a climate where numbers are at the forefront in as many ways as possible, yet the skills they need to explain and think through their processing are emphasized as well. Through conversation, challenge, and struggle students are able to take positive steps towards deeper understanding of the number relationships and processes (Bonnett et al., 2017).

CHAPTER 2: METACOGNITION

The focus in education used to be about asking questions to get children to give you the right answer. But now it's more about going beyond the answer and helping them think about their thinking

- Sharapan, 2015, para. 2

Metacognition is not a new term in the realm of education, the first roots of it date back as far as Plato and Aristotle, while the majority of what we know today was put forward during the 20th century (Noushad, 2008). Metacognition today is commonly known as the ability of an individual to think about their thinking but what does that mean exactly? Flavell (1976) characterized the term as "refer[ring] to one's knowledge concerning one's own cognitive processes or anything related to them…[and] the active monitoring and consequent regulation...of those processes" (p. 232). Through his work, Flavell created a model of metacognition (Figure 4) that served as the foundation for further work to be done in this area.

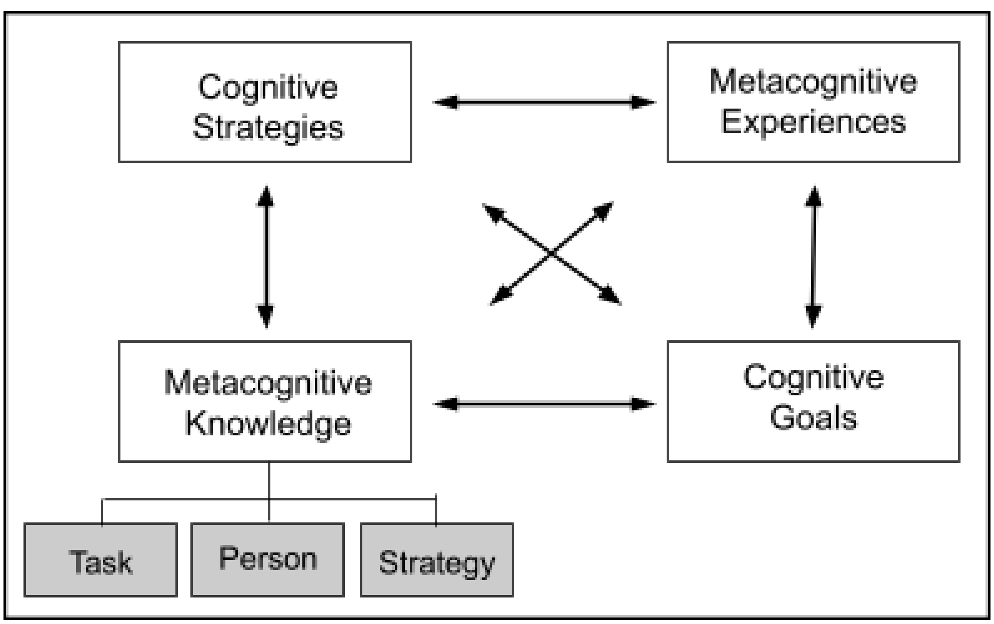

Figure 4: Flavell's (1979) Model of Metacognition (Djudin, 2017, p. 126)

The role metacognition plays should not be underappreciated, as “it has the potential to empower students to take charge of their own learning and increase the meaningfulness of students’ learning” (Mahdavi, 2014, p. 533). This not only is creating meaningfulness, but it is essential in the supplementing of students learning in the realm of number sense with opportunities to connect with the material on a personal level. Çekirdekci et al. (2018) reference the finding that ones’ sense of number is not simply a skill looking at a collection of numbers will suddenly help one make sense of it. It is a way of thinking that one must adopt in order to embrace the interaction between the numbers, which makes it relevant and important. “Metacognition mediates between the learner and their cognition. While cognition can be considered as the way learners’ minds act on the ‘real world’, metacognition is the way that their minds act on their cognition” (Noushad, 2008, p. 6). It was also highlighted that metacognition alone does not have the substance to achieve desired results for number sense development (Çekirdekci et al. 2018, Mahdavi, 2014; Noushad 2008). The strength of metacognition lies in its ability to support the learning process of the child, allowing for students to learn how to respond to a situation when they are not sure what to do next. Metacognition is most commonly known for its work in education circles in relation to reading, “but it is just as relevant for improving students’ mathematical [abilities]” (Ebdon et al., 2003, pp. 490-491). This is what makes metacognition such a vital component to number sense development, as it aids in the learning of numbers, their relationship with one another, and how those numbers can be adapted to new situations for continued growth of the child.

2.1 IMPACT ON LEARNING

Metacognition is the support structure that aids students in their journey through understanding numbers. It is a critical strategy that not only provides students with knowledge and confidence, but is connected to learning results. Problems occur when students lack these general metacognitive skills because they have not been forced to manage their own learning. Students do not see that mathematics should make sense, and until they are provided with the freedom to be curious, vulnerable, and persistent in their ventures, they will not have to make any effort to seek that understanding (Carr, 2010; Hidayat et al., 2018; Paris & Winograd, 1990). Metacognition not only is an essential component in developing number sense, but "plays a central role in learning and achievement" (Djudin, 2017, p. 124). If we choose not to acknowledge the influence that metacognition plays we are setting students up for a limited understanding of the relationship of numbers versus a deep and broad exploration. Through research by Hacker (1998), as found in Noushad (2008), metacognition was further defined into three types of thinking: *knowledge*, *skill*, and *experience*. Metacognitive knowledge speaks to what an individual knows about knowledge, skills are the actions that the individual is currently doing (strategies), and experience is the current emotional state that one is in. Each of these components plays a role in the ability of a student to make the necessary connections to their own knowledge in order to arrive at understanding. No one component is the key to success. Noushad (2008) explained how "it is widely accepted that metacognitive knowledge or regulation is not sufficient" (p. 9) and students must also be willing and eager to use the skill that they have.

Metacognition aids students in the construction and development of understanding on their own and assists in pulling meaning from the concepts that are being taught; it is the process that occurs during problem solving (Noushad, 2008). Constructing understanding or developing comprehension is more than just a cognitive venture that is undertaken by students as they develop number sense. In order for gains to be made the supplementary role of metacognition is a necessity to provide a foundation to allow students to develop a deeper, more meaningful connection to their learning (Djudin, 2017). Students need the understanding that number relationships are more than simply thinking and doing repetitive processes on paper, it requires the metacognitive skills of reflection, analysis, and monitoring to provide that deeper layer of understanding. Students need to train their thinking to "know what mathematical tools the individual has at his or her disposal" (Shoenfeld, 1992, p. 349) and the confidence to pull those tools out when they have identified it is necessary to use them. The ability to draw back into one's memory by identifying what they know allows for students to not only identify their own understanding in relation to that problem but also make meaningful connections to mathematics and in turn provide a strong foundation for them moving forward. Çekirdekci et al. (2018) argue that "since metacognition is part of cognition that controls these skills, it also has the task of controlling number sense" (p. 2472) and without adhering to these skills we are not providing students with the opportunity to become proficient in their understanding of how numbers work together.

2.2 DEVELOPING THE SKILLS

Developing the skills of metacognition in students is a process that requires time and patience before results will be noticed. The expectation that one-year worth of practicing metacognition will turn the tide is narrow-sighted to say the least. Mahdavi (2014) argued that the components that make up metacognition and influence its effectiveness with student learning comes with age. Along with this challenge is the diversity of the students themselves as each individual student will progress at differing rates during their progression (Noushad, 2008). However, this does not mean that we wait until later years to address the skills in students as it is deemed more age-appropriate, the opposite can be argued to be true. Studies have shown that even though older students have a higher level of metacognitive skills when it comes to planning, monitoring, and reflecting, it is the younger students who demonstrate the clearer ability to discuss the process that they went through when solving a problem (Noushad, 2008). If we support the students at a younger age and teach them the skills and strategies the benefits not only in their skills will increase but their self-efficacy as well. This is a crucial point to reach since it has been noticed that "as students become more skilled at using metacognitive strategies they gain confidence and become more strategic, more independent as learners" (Djudin, 2017, p. 124).

If we ignore the opportunity to provide students with the structures and supports of metacognition, this can leave a negative impact for the students as they progress through more advanced mathematics that require a greater understanding of number relationships. Carr (2010) argues that altering levels of metacognitive knowledge are developmentally tied to a student's ability to grasp the procedural knowledge that is necessary to be considered a high achiever for number sense development. Procedural knowledge, as explained previously, is essential to the development of number sense as it addresses the student's ability to understand the pathway to reach their solution compared to the traditional means of rote algorithmic steps. Carr (2010) referenced this point when she wrote about how "metacognitive skills and knowledge are not the only factors that affect learning, but without metacognition, learning becomes difficult and slow, as students rely on others to guide development" (p. 177). Without the emphasis of metacognition skills and development as the primary focus in the goal of increased number sense, the learning is not placed in the hands of the students. They are dependent on those around them, primarily the teacher, to guide their understanding and growth. The issue with that is the students lack the ability to think and explore for themselves; they take what is offered to them as canon and lack the willingness to explore new connections that can spawn from questioning and challenging processes. Students learn how *not* to access the information in their head even though they have it organized in their own way based on their experiences (Schoenfeld, 1992). By not emphasizing these skills, students are missing out on the opportunity to expand their minds and grow into new and diverse thinking. Declarative metacognitive knowledge seems important for newly emerging mathematics strategies as opposed to older, more familiar strategies because it aids students who are beginning to experiment with a new strategy to better select problems on which to use the strategy.

The inclusion of metacognitive skills in a child's education has been shown to support students' exploration of new knowledge. Metacognition does this by allowing students to continue to build from previous experiences and understandings they have previously developed (Carr, 2010; Schoenfeld, 1992). Like many things, metacognition is a skill that must be practiced in order to become more proficient in its impact on a student. As students practice metacognition they develop an awareness of their own cognition. This awareness is an important step for students as it plays an important role in the ability of students to act on this awareness to not only learn better but quicker (Carr, 2010; Mahdavi, 2014; Pintrich, 2002). Dirkes (2010) argued that the metacognitive strategies are beneficial for students by allowing them to: connect new information to former knowledge, selecting thinking strategies deliberately, and plan, monitor, and evaluate thinking processes. This is especially important for the development of number sense as this supports the process that students go through as they are learning new materials and making sense of it based on their own context. The utilization of metacognition in this area provides students with that additional support to provide understanding to a topic by forcing them to look at the concept from, as Schoenfeld (1992) stated, a "mathematical point of view" (p. 344).

2.3 METACOGNITION AND NUMBER SENSE STRATEGY DISCOVERY

The term strategy is a buzz word that has been burning through the field of education like a brush fire and in mathematics commonly refers to a student's personal way of solving a mathematical problem. But what does the term strategy mean and how do students develop these so called strategies to assist them in solving problems and develop number sense? To some a strategy is a teacher giving a tool that aids them in solving a problem, such as a long division algorithm, counting on, doubles +/- 1, and more. Teachers everywhere employ strategies in their classrooms to aid students with tricks to help them remember how to solve problems. Students of any age and level use strategies, however we do not always realize that teacher given strategy use is dependent on the context of the current problem and does not equate to deeper understanding of the mechanics behind the mathematics relating to numbers. Waters and Kunnmann (2010) noticed that students are only able to use strategies as long as all of the conditions are perfect for its implementation. The students need to have the right materials, processing conditions, and set of instructions or there is a good chance that the teacher taught strategy will not be implemented to its desired effect.

It is only when students are exposed to a wide variety of problems in a rich mathematical environment are they able to "broaden their strategy use across different materials and processing conditions" (Waters & Kunnmann, 2010, p. 3). As students acquire a wider range of skills in the area of metacognition, the opportunities to develop, evaluate and implement strategies efficiently is likely to occur. Waters and Kunnmann (2010), wrote about how strategy training (teacher-led strategy implementation) can initially appear to create success in students on the surface, but it is when the teacher steps back and students are left to solve problems on their own do issues arise. Students do not understand what is required of them on the metalevel once the teacher supports are stripped away and the necessity of developing the metacognitive skills, even at an early age, can assist in counteracting this reliance on.

The development of metacognitive skills and awareness of a students' overreliance on one strategy not only adds the benefit of impacting the number sense of all students, but influences potential change of strategy use and the effectiveness of knowledge transfer into other subject areas as well (Carr, 2010; Pintrich, 2002). The challenge that we face is that until these metacognitive skills are developed, which takes time and maturing of the student, we as teachers are forced to continue to provide foundational support that assist in student understanding and in doing so, we put up the very blockages we strive for our students to overcome. While these skills are developing, teachers can utilize a plan of action to assist in the metacognitive development of their students. This development involves embracing a culture that puts the student in charge of their own pathway for learning. Embracing concepts such as think-aloud procedures, think-paired technique, and reciprocal teaching provides opportunity for students to adjust to the new demands placed on them and push into new areas of questioning and understanding (Djudin, 2017). However, we need to be mindful that following the metacognitive practice ideas are not a guarantee to develop these skills in all students equally, and we still need to address the difference in stages that each student is at and provide the patience and support to allow each student to develop at their own pace.

Metacognition goes hand-in-hand with strategy development as it provides the tools for students to look at, evaluate, and move from a strategy recall stage to a point where the students are active and goal-directed in their strategy use. "The ability to use metacognition skills is influenced by the student's state of current conceptual understanding" (Carr, 2010, p. 180) and is a topic that needs to be a primary focus in teachers' minds as they deal with students of differing ages and abilities across an educational system. The challenge lies on how to promote this ability in the younger students since it has been shown that the younger the student, the less prone they are to using metacognition effectively to develop deeper understanding. The key is "strategy-performance connection" (Waters & Kunnmann, 2010, p. 17) where students are able to create a connection to the task based on the strategy that is being implemented. Older students display the ability to be goal-directed in their use of strategies by being able to explain processes, and actions as they work compared to younger ones, however how do we truly know when students are being strategic in their use of strategies versus simple recall?

Waters and Kunnmann (2010) summarized how students "under cognitively demanding circumstances, even though a strategy is implemented, [were] … unlikely to make the strategy-performance connection because of limits on cognitive capacity" (p. 16). They noted that students are more likely to discover strategies that work for them when the learning conditions are relatively easy. It was noted that if the task was simple enough the students would not need to utilize strategic behavior and this prepared them to discover the strategy that is being implemented. The effectiveness of a strategy is hindered on the connection that the student can make via that strategy, if that connection does not occur the chances of that strategy being effective is "essentially zero" (Waters & Kunnmann, 2010, p. 16). This light load requirement allowed for those students to recognize their own understanding of the given task, and able to explain how they were able to figure things out under their own through process. This ability to reflect and monitor one's process enabled the transfer of the learning to more challenging activities. This transfer was not apparent in circumstances that were far more demanding for students due to the high level of cognitive process that was required (Waters & Kunnmann, 2010).

When looking to develop number sense in students, the use of strategies can be an important tool to aid them in this venture, but they need more than just a list of strategies to allow them to develop proficiency and a deeper understanding. The framework presented by McIntosh et al. (1991) earlier in this paper (Figure 1) suggests a monitoring framework which links number sense to metacognition and how "good number sense is thinking about, and reflecting on the [strategies utilized], numbers, operations and results" (p. 5). Strategies hold a key place in the development of number sense. However, strategies cannot be something that is provided by the teacher alone, and the impact in a child's learning occurs when they are able to discover the strategy themselves. Through exploration students are able to connect to a much deeper understanding of the material and in turn manipulate that strategy to fit into more complex scenarios to achieve an answer (Parrish, 2011; Waters & Kunnmann, 2010).

The ability for students to develop their own strategies and have the opportunity to put them into practice, test their sustainability, and critique their process demonstrates the presence of metacognition as a foundational source for student learning and deeper understanding.
To assist students in the development of number sense and strategy implementations surrounding number sense, classroom tasks at every age need to be developed through scaffolded nature: begin with a very simplistic task where students can use their conceptual knowledge to solve a question and utilize metacognition to monitor and reflect on their strategies effectiveness. As students demonstrate competency of the simplistic level, more challenging tasks based on the knowledge of the simplistic task can be implemented allowing for transfer to be tested (Parrish, 2011; Waters & Kunnmann, 2010). When simple understandings are challenged with more

complex tasks students are forced to think in different ways, but are able to explore the numerical relationships of the numbers instead of just recalling a process or trick, “in other words, metacognition trumps the more incremental, steady progress associated with practice and automaticity” (Waters & Kunnmann, 2010, p. 12).

Once a strategy is developed by the student and utilized, the challenge is ensuring that the strategy can be carried forward beyond that single context. In order to accomplish this, students must develop metacognitive awareness, which allows students to recognize when and how to use their strategy in various ways. The ability to employ metacognitive awareness at a high level is rare in young children without experience, but by no means is it absent. In many classrooms these skills are being developed, however, teachers are not always aware of this development occurring. The key is to prepare students to “note the connections between what they do and outcomes will generalize this metacognitive mindset to contexts outside the original classroom with different instructors” (Waters & Kunnmann, 2010, p. 20). As educators, the process must begin in a simplified environment, initially, where all students can explain their process. As students become more comfortable with the process, emphasis can be placed on being strategic in order to aid them as they embark on more complex tasks. However, throughout this whole process, students still need the opportunity to think about what they do, explain their process and provide insights during classroom discussions (Parrish, 2010, Waters & Kunnmann, 2010).

2.4 ROLE OF METACOGNITION IN WORKING MEMORY

We as humans are information processors by our very nature and we react to the stimuli that we come up against on a daily basis, be it light, sound, smell or any other form of stimulus. Working (or short-term) memory is the portion of our cognition where the thinking gets done, and does so by receiving information from two sources: our sensory buffer (registers visual, auditory, and tactile stimuli) and their long-term memory (Schoenfeld, 1992). Figure 5 demonstrates this pathway as one attempts to solve a mathematical problem.

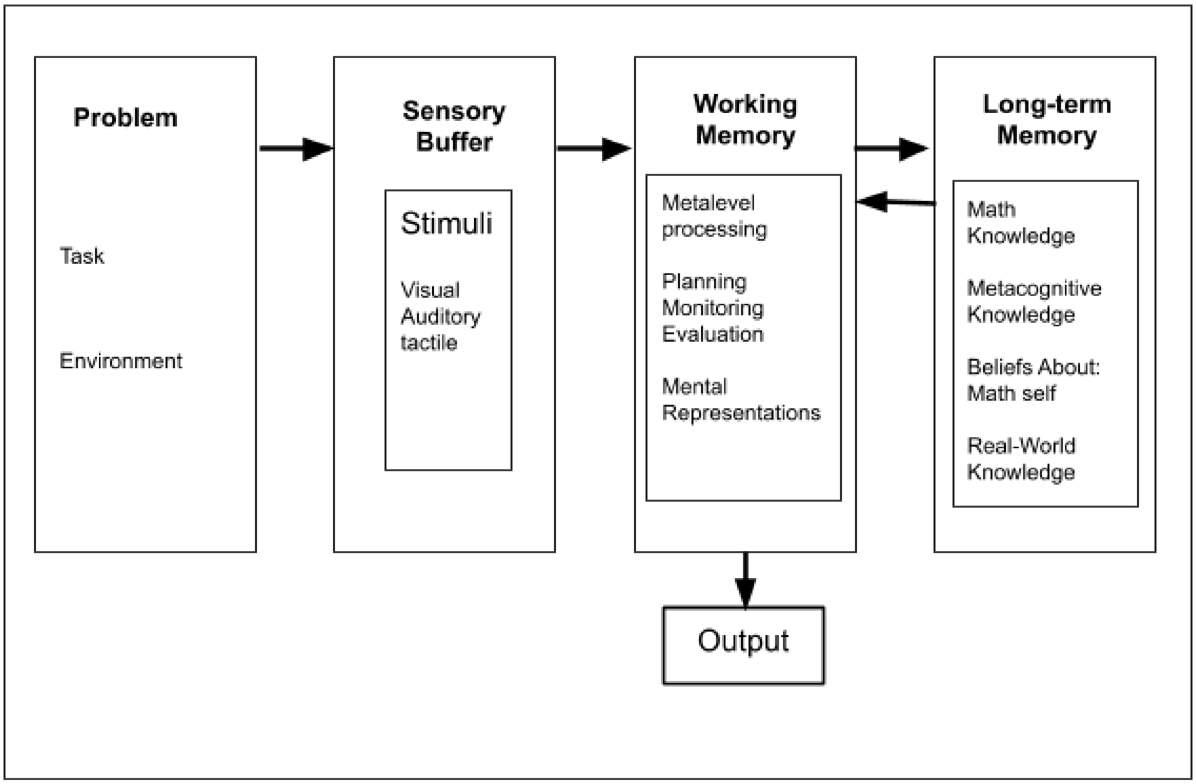

Figure 5: The Structure of Memory (Schoenfeld, 1992, pg. 351)

Working memory has repeatedly connected to math achievement and development and has the ability to store and manipulate information during a task. Due to this working memory has the potential to be a vital component to the development of number sense (Toll et al., 2016).

The issue we face is that "despite the huge amount of information humans can remember in general, they can only keep and operate on about seven 'chunks' of information in short-term memory" (Schoenfeld, 1992, p. 350). However, Doolittle (2013) suggested that through new research utilizing magnetic resonance imaging it is actually closer to four. So as a necessity, we need to capture the most out of one's working memory as possible which means we need to be efficient to ensure that each chunk is not wasted with unessential tasks. A chunk is any stimuli (auditory, visual or other) that we can recognize with little to no effort. For example, "spoken and printed words are chunks" (Schoenfeld, 1992, p. 350) as would a string of basic numerical facts.

Whitebread (1999) identified that interactions between metacognition and working memory occurred when learners were faced with the construction, selection, and execution of a strategy. This interaction is a common occurrence, especially in mathematics, as learners come to develop declarative knowledge in regard to basic computations all the way up to problems of higher complexity. Once learners reach higher levels more thinking is demanded in order to achieve a solution and students who have not developed a declarative knowledge base in basic computations hinder their ability to maximize their working memory (Carr, 2010). This is because as more thinking is done, there is less space in working memory for other components to play a part. Learners, who have limited working memory capacity, tend to be prone to display more errors and misconceptions as they attempt to work through problems (Ashcraft & Krause, 2007). So how can we assist students in ensuring their working memory is not littered with unnecessary steps and are utilizing it the most efficient way? This is where metacognition plays an essential role. As Adam and Vogel (2017) suggested "metacognitive monitoring may be key to working memory success" (p. 1506). Metacognition provides students with a way of tracking how they are utilizing automatic facts from their long-term memory. The recall automaticity that a learner develops critically impacts the available space within one's working memory and Whitebread (1999) identified this by recognizing that:

> task[s] will initially show little or no "intelligent self-regulation". Then, as the task and its subprocesses become more familiar, processes of automatization lead to the freeing up

> of working memory capacity, and an increasingly metacognitively active period of monitoring and self-regulation. Finally, as the necessary subprocesses and their co-ordination become overlearned, expertise is achieved and performance on the task becomes relatively automatic. (p. 490)

As working memory spaces are freed up, metacognitive skills aid the effectiveness of a learners working memory by providing students with the ability to think over their mathematical knowledge, select the fact, strategy, or procedure that is most appropriate, and evaluate if that selection is beneficial or should be abandoned (Mahdavi, 2014). Through this metacognitive process, students are able to filter through the vast amount of knowledge around them and create, select, and alter strategies to make the most effective use of their working memory capacity so working memory can carry out that selected strategy on the problem at hand (Whitebread, 1999). Baroody (2006) noted that students with good working memory are able to recall facts with little to no effort and once this factual knowledge is established, it is "easier to retain in memory and to transfer to learning other new but related facts" (p. 25). The challenge that we face is when students move into more demanding problems that require multiple steps, each aspect that they need to think about takes up a working memory chunk (Komori, 2016; Waters & Kunnmann, 2010).

As mentioned previously, we are only able to handle a certain amount of information at one time and the utilization of metacognitive skills takes up space in one's working memory when trying to solve problems. Whitebread (1999) highlighted the fact that, although they are effective, "the metacognitive processes can only occur … when all of an individual's working memory capacity is not taken up by carrying out the actual task" (p. 490). But if working memory is provided with space metacognition not only benefits the students learning it supports and strengthens students’ working memory as well. Through metacognitive skills, learners are able to reflect on their strategies and select the ones that are of the greatest benefit for the problem in that context and limits, the extra processes that are not necessary. This in turn assists in the development of memory, by allowing the essential information to be emphasized and distractions limited in order to get the most out of each student (Adam & Vogel, 2017; Mahdavi, 2014).

CHAPTER 3: TRANSFER OF UNDERSTANDING THROUGH METACOGNITIVE PRACTICE

> Knowing the answers can be a trap; learning mathematics is about looking at what you thought you understood and seeing that there's deeper mystery there than you realised
>
> — Dan Finkel (Gaskins, n.d., para. 2)

The transfer of information is an intricate part of number sense and is not a component that appears out of thin air. Transfer requires individuals to acquire "knowledge, skills and attitudes and then apply these capabilities to other contexts" (Ford et al., 1998, p. 218) and not something achieved after practicing facts a certain number of times. To develop mastery in any discipline requires more than simple practice (Baer, 2014); it is an understanding based on foundational work and strategy development that is unique to each individual as they progress through mathematical experiences (Bay-Williams & Kling, 2019). In order for students to see the deeper level, the incorporation of metacognitive strategies must be implemented in order to allow them to see beyond procedure and answer and into relationships as to how numbers can be utilized and manipulated. Mahdavi (2014) identified that "it is the range and combinations of all strategies that ineffective learners lack, [but] it is the metacognitive strategies which seem to be the strategy types most lacking in the arsenal of less successful learners" (p. 529). We have provided our students with strategies to solve mathematical problems, however a major component is missing: the metacognitive supports that allow students to push beyond the believed tried and true methods and into new areas of understanding that will provide them with the ability to apply set strategies to new experiences. Pintrich (2002) stated that:

> metacognitive knowledge of all these different strategies seems to be related to the transfer of learning; that is, the ability to use knowledge gained in one setting or situation in another...students are often confronted with new tasks that require knowledge and

> skills they have not yet learned...they cannot rely solely on their specific prior knowledge or skills to help them, on the new task. (p. 222)

In order to ensure we provide students with as much opportunity to create transfer we need to explore the impact metacognition has on: student's mastery vs performance orientation, the learners disposition, and the development of flexibility. Each component provides a critical piece of information to aid learners in transferring their knowledge from one setting to another.

3.1 MASTERY VS PERFORMANCE

Learning mathematics is an empowering force that allows students the opportunity to take control of their own learning (Schoenfeld, 1992), but why do some learners pick up mathematics so much more quickly than others given the same instruction? There are differing perspectives on how many learner orientations there are. For example, Hidayat et al. (2018) wrote about four: mastery, performance, mastery-avoidance and performance-avoidance. While others like Ford et al. (1998), Heirdsfield and Cooper (2002) and Schoenfeld (1992) wrote about only two: mastery and performance. For the purposes of this project the focus will be on the two main types of learner orientations that occur in classrooms: 1) mastery and 2) performance.

Mastery orientation provides the greatest opportunity of learning by exhibiting "educational attributes such as greater engagement, requesting appropriate help, and seeking conceptual understanding" (Bonnett et. al, 2017, p. 3). These are learners who go above and beyond and believe that they should "aim for understanding and flexibility" (Heirdsfield & Cooper, 2002, p. 3) through effort. They devote their effort in developing a deeper understanding of why things work the way they do and believe that one's ability is malleable through the incorporation of strategies such as reflection, planning, and monitoring. It is suggested that transfer occurs when students are asked to be reflective in their mathematical work surrounding number sense (Carr, 2010; Fuchs et al., 2003).

Performance learners tend to be driven primarily by extrinsic motivators such as achieving a higher grade than someone on an assignment, passing specific tests, or completing tasks as quickly as possible with very limited time spent on reflection or understanding (Bonnett et al., 2017; Ford et al., 1998; NCTM, 2014). Learners who portray performance orientation are not vested in the development of deeper understanding, nor have the desire to put the effort in to achieve that. These learners see succeeding with limited effort as the sign of ability and see failure in themselves when others achieve higher than they do or others understand concepts when they struggle, as understanding should come naturally. If this failure is felt these learners will avoid tasks that present any challenge to them to avoid being seen as low achieving, they would rather be seen as lacking desire or lazy (Bonnett et al, 2002; Ford et al., 1998; NCTM, 2014).

Carr (2010) noted that "a significant difference between expert and novice mathematicians and high and low-performing students is in their use of [metacognition] when dealing with numbers and how those numbers interact with each other" (p. 180). Students with a mastery orientation seem to demonstrate a greater sense of number and flexibility and were able to demonstrate a more consistent use of metacognitive skills when trying to solve mathematical problems. These learners demonstrate a more persistent ability to think through issues themselves and in examining the arguments put forth by others, predict outcomes, select and manipulate strategies, recognize when they are going off track, and alter their behaviors when they deemed necessary in their computations compared to their less skilled peers (Carr, 2010; Djudin, 2017; Schoenfeld, 1992). Schoenfeld (1992) also noted that students "get good at what [they] practice and there isn't much transfer" (p. 346). Metacognitive instruction assists in the learner's ability to transfer their knowledge to unique tasks (Carr, 2010; Fuchs et al., 2003; Kramarski, 2003) unless students are able to put that practice to use in various situations and understand the relationship between the numbers being used. This lack of transfer, by emphasizing one procedure versus exploration and development, is highlighted as students move to higher levels where more demand is placed upon their computational abilities.

Learners who have focused on one strategy or algorithm, the metaphorical all eggs in one basket, have missed out on the opportunity to develop the metacognitive skills that allow them to "monitor [their] progress as [they] learn and [understand that] making changes and adapting" (Djudin, 2017, p. 125) their strategies is a natural process towards understanding. Their beliefs are often challenged when they are unable to use the believed tried and true processes with the consistency they have become accustomed to. Students have demonstrated that once they find a strategy that works, they tend to utilize that strategy with a blind trust that it will always work because teachers have only provided students with examples that allowed that process to work, students have not been pushed to expand upon their strategies, and the teacher-taught method was emphasized during class time. Carr (2010) points out that "there is no evidence that one strategy is better than another" (p. 183), yet an emphasis on teacher promoted algorithms still leads the charge in many of our classrooms today.

Schoenfeld (1992) and Carr (2010) suggest that novice mathematicians (performance orientation) tend to steer towards using methods that they are comfortable with such as trial and error, procedural algorithms and, if struggling, will eventually guess at an answer and move on. Once an answer is found, whether right or wrong, there is little to no reflection by the learner in verifying if the solution is plausible, it is accepted as correct and the problem ends. Students with this mentality have the tendency to “fail to reflect on the adequacy of procedures used for solving, why they are selected and whether the outcome makes sense” (Carr, 2010, p. 183). These learners do not challenge their understanding on why the solution is or is not plausible, because the priority is not on understanding, it is simply completion.

Experts (mastery orientation) on the other hand will analyze their selected strategy as it is being used, monitor progress and effectiveness, and check their results to verify if it makes sense based on the demands of the problem. If the experts are not happy with the result they will reflect and select a new strategy that may provide a better result (Schoenfeld, 1992). Individuals with a mastery orientation demonstrate the use of strategies that are required for deeper processing. This can suggest that these learners will attempt tasks at differing levels of complexity and learn skills that help understand the nature of the task (Ford et al, 1998). A simple example of mastery orientation can be seen in algebra when students insert the unknown variable back into the equation to ensure that they have equal values on both sides. Students who utilize this method are able to take another look at the problem, reflect on their process, and justify if their solution is correct or not.

The primary difference between these two orientations is the emphasis on the learning going on at the moment. Mastery orientation is present in learners when “the focus is on learning, rather than peer comparisons” (Bonnett et al., 2017, p. 16). It is not about being motivated to do better than others in the class, but being motivated to better one’s self in the mathematics or any discipline. Hope is not lost on learners who are at the performance orientation, as Paris and Winograd (1990) note, because “students can enhance their learning by becoming aware of their own thinking” (p. 7) and because “the plasticity of the brain: ability and intelligence grow with effort and practice” (Boaler, 2013, p. 150). Unfortunately, it seems that teachers are "waiting hopefully and expecting confidently for learners to automatically 'go meta' and self-regulate their own learning. This expectation] seems quite impossible and unrealistic" (Mahdavi, 2014, p. 530) without the skills at the learner's disposal to provide the motivation to look deeper.

3.2 LEARNER DISPOSITIONS

It can be hypothesized that learners with mastery orientation demonstrate a different disposition than those who are of a performance level, as Heirdsfield and Cooper (2002) suggest, learners' dispositions could prompt how they respond to the achievement in mathematics. Research has shown that students who develop a greater self-efficacy of their abilities are able to be much more resilient as they face problems that are more challenging, but how can self-efficacy be developed to change the disposition of students? Paris and Winograd (1990) wrote how self-appraisal is a common aspect of children's learning that is overlooked and can lead to how students view themselves in mathematics. Self-appraisal includes "personal reflections about one's knowledge states and abilit[y]" (p. 8) and forces that learner to judge themselves in facilitating or failing at a certain problem. However, the environment that students are in must be conducive to the disposition of the child. If it is not, the risk of developing a student with a performance orientation is possible.

Boaler (2013) states "when students believe that everybody's ability can grow, their achievement improves significantly" (p. 150) and metacognition can aid in this dispositional work because it impacts the self-appraisal of learners by giving them the opportunity to reflect on their abilities. Metacognition has the potential to emphasize the appraisal on a personal level and help alter how students view themselves and their abilities. It focuses on learners on an individual level and addresses the uniqueness in how each of them learn (Paris & Winograd, 1990). The biggest challenge that is faced is ensuring that what occurs in the classroom is allowing the students to view themselves as successful and capable to achieve the goal they are working towards. It has been shown that students who have the belief that they are capable of learning regardless of the difficulty they are faced with are seen to be more: effective, willing to embrace challenge, resilient, and consistent at achieving a higher standards compared to those who believe that they are either smart or dumb at math or don't have a math brain (Boaler, 2013).

By promoting metacognition skills in our students we, as teachers, are able to not only convert the responsibility of monitoring to the learner themselves from us, but also increase the self-efficacy and motivation in our students (Paris & Winograd, 1990; Pintrich, 2002). This is a necessity because students are commonly mistaken in their own abilities unless given ample opportunity and exposure to metacognitive skills. It is noted that many students, especially those at a young age, are not strong at metacognitive (or self-appraisal) skills as they are learning and due to this, they can get caught into creating an illusion of understanding prior to any truly taking place (Paris & Winograd, 1990). We need to move away from the traditional views that students cannot adapt their learning to be successful in mathematics. Boaler (2013) explains how:

> research shows [that] the plasticity of the brain and the ability of students to develop smartness [can be developed] through hard work and challenge, some schools bombard students with the messages that ability is fixed and that some students have talent and intelligence while others do not. This chasm between research evidence and practice is most clearly reflected in the ability grouping practices used in schools that communicate to students that their ability is fixed, initiating the harmful beliefs that research has shown detract from students' learning opportunities throughout life. (p. 145)

By focusing on the metacognitive skills that allow students to reflect on their abilities, they are empowered to challenge the preconceived notions of what they thought they were capable of and help propel them into new areas of understanding and self-enlightenment.

3.3 FLEXIBILITY AND INFLEXIBILITY

In order to achieve transfer of one concept to another, one must develop flexibility within their understanding and be able to use a variety of efficient strategies based on the problem (Heirdsfield & Cooper, 2002). However, in order to aid in the development of understanding, metacognitive skills need to be implemented. Through the development of metacognitive skills, opportunities are made available to learners who are able to alter their ability from simply following directions, to opening their minds to new ideas and processes. This in turn allows for students to alter their disposition towards mathematics, build resiliency when faced with a challenge, and aid in the transfer of learned skills to different tasks (Bonnett et al., 2017; Djudin, 2017; Schoenfeld, 1992; Waters & Kummann, 2010). If we look at students over the years, we can see a trend of students who are more than capable of getting an answer correct when a question is presented to them, so long as the question is similar to something they have seen before and have practiced. An argument can be made that the learner is demonstrating understanding and flexibility because they were able to take what was taught to them in the classroom and use it to solve a different problem.

To fight the this is the way to solve it, we must not only promote construction of coherent understanding of both new and old knowledge, but also demonstrate the ability to select an appropriate strategy that can expedite the process of solving the problem, believe in that strategy, and utilize prior knowledge to attempt to solve new mathematical challenges (Parrish, 2011). “The research in mathematics education suggests that many of the difficulties [that students face]

... are results of conflicts between current beliefs and new info" (Carr, 2010, p. 178) and without metacognition to allow students the chance to reflect on their own abilities, they will continue to face this dilemma. As a result, we can see that flexibility is not an ability that some have that can be developed, but a way of thinking that allows people to view numbers and strategies in different ways.

When looking deeper at flexibility (Figure 6) by Bay-Williams and Kling (2019) it can be noticed that flexibility is derived from procedural fluency as a primary pillar in development. However, it is also interconnected to various other foundational processes that need to be achieved in order to develop flexible number sense specifically.

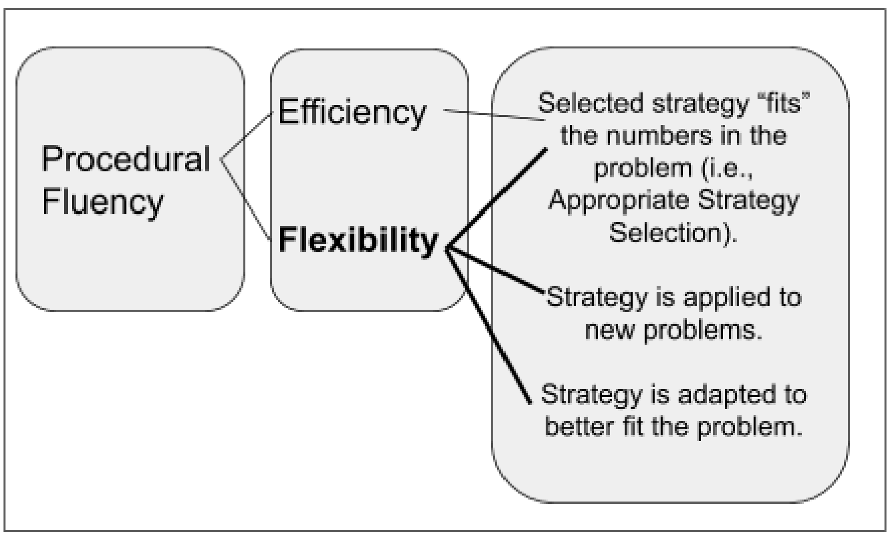

Figure 6: Close-up on Flexibility (Bay-Williams & Kling, 2019, pg. 2)

Developing one's flexibility with how they can apply numbers to different situations is not a simple task and is one that is dependent on the students themselves and the environment that teachers provide for them. This is where the effects of metacognition can significantly impact the ability for students to develop flexibility in themselves, because, Volet (1991) referenced by Ford et al. (1998), "found that … students taught metacognitive activities received better course grades and were better at applying their knowledge to solving new problems" (p.

220). Students who incorporated metacognitive skills into their mathematics were able to not only develop understanding to do well on assessments, but were able to take that knowledge and apply it to problems that were new and unique.

Heirdsfield and Cooper (2002) recounted their findings of two students who demonstrated flexibility on opposite ends of the spectrum. The first student, Claire, utilized metacognitive skills of reflection, monitoring, and evaluation as she worked through problems and found alternative ways than the teacher-taught method to solve questions that were posed to her by the interviewer. These alternative methods at times were more effective for her than previous methods and she demonstrated "strong self-efficacy and… confidence in her own strategies" (p. 7). This goes to show that if left to their own devices, kids develop strategies that make math meaningful. Unfortunately, however, kids are often taught that these strategies are wrong (Dewar, 2008). The second student, Mandy, on the other hand was convinced "of the importance of speed and accuracy, and her lack of understanding of many aspects of number resulted in her resorting to an automatic procedure for both number facts and mental computation" (p. 10). Her confidence in the teacher-taught method prevented her from developing any understanding on why the procedure worked or if it was even the most efficient one. Unfortunately, Mandy's case is a common practice in traditional classrooms. Students memorize facts or universal algorithms and regurgitate them on timed tests to demonstrate competence in those areas. Good number sense is noted in learners who "are flexible, that is, … they must be aware of a variety of strategies and have confidence to use them...in different situations depending on numbers and context because they are disposed to making sense of mathematics" (Heirdsfield & Cooper, 2002, p. 3). Mathematical learners should not be following on blind faith, but thinking about the numbers and reflecting on how they achieved a solution, if that solution makes sense and why it makes sense (Çekirdekci et al., 2018).

Thinking about one's answers, metacognition, is the primary difference between a student's ability to develop deeper understandings of number relationships that go beyond classroom examples. If these metacognitive skills are not promoted in a classroom, students can become dependent on the memorized or teacher taught procedures (pencil/paper algorithms) as these are the only foundational pieces that they have to work from. These processes, presented as the best way, can become the go to choice for students to solve problems and because they can be executed with limited thinking the illusion of understanding is presented (McIntosh et al., 1992). This reliance is why the emphasis on metacognition is so vital for extending the understanding of learnings in the realm of mathematics.

Metacognitive skills are needed when habitual responses are not successful and nurtures thinkers and lifelong learners who are independent and display the ability to productively struggle with new situations. After all, "every time we learn something we develop new pathways and strengthen those connections" (Boaler, 2020) in the brain. The flexible learner must be able to select appropriate strategies when required, but willing to adapt based on different tasks that present themselves. They are not locked into one specific strategy but utilize a plethora of strategies based on effectiveness. These learners understand how to learn and continue to learn throughout their lifespan in this rapidly evolving pace of life (Baroody et al., 2005; Djudin, 2017; Mahdavi, 2014; Pintrich, 2002; Waters & Kunnmann, 2010).

CHAPTER 4: ASSESSMENT'S ROLE IN NUMBER SENSE GROWTH

> The activities and assessments traditionally associated with learning basic facts (such as drill, flash cards, and timed testing) exclusively focus on students' accuracy and one part of efficiency (speed), neglecting strategy development
>
> - Bay-Williams, J. & Kling, G. (2019, p. 4)

Throughout this paper I have explored some of the factors that play an important role in developing the number sense of learners of various ages and how metacognition is a fundamental aspect that contributes greatly to student success. Traditionally, assessments in the classroom have been used to emphasize the importance of student evaluation, and this perception has dominated the culture of North America since the beginning of schooling. Unfortunately, it continues to be more pronounced as we enter the new decade through timed tests, quizzes and the emergence (or re-emergence) of some standardized assessments (Gareis & Grant, 2015; NCTM, 2014). In this section, the focus will be on the research behind why assessment is a vital component that, when embraced, can provide teachers with essential information about their students that can assist in their learning (Gareis & Grant, 2015). More information on some assessments that incorporate the traits being explored in this section will be provided on the website that was developed for those interested in including them into their classrooms.

In many school districts, pen-and-paper, timed, or standardized assessments have become the main component in measuring students' understanding in mathematics (Boaler, 2014; Brookhart, 2013). However, if we look back at what the roots of assessment means we come to a very different understanding of what it is and how it should be used. Assessment is derived from the Latin word *assidere* which means to "sit beside". In an educational context, it is "the process of observing learning; describing, collecting, recording, scoring, and interpreting information about a student's or one's own learning" ("A Short", 2018). If we delve into that definition, we can see a wide variety of sources to which we can gather information; timed tests or provincewide assessments are but a small part of those sources. Yet, the opportunity to sit beside our students present itself where we, as educators, are able to witness and aid students in understanding their learning. It becomes more than a transfer of knowledge, but a mutual partnership towards growth; after all, “the primary purpose of assessment is not to *measure* [emphasis added] but to *further* [emphasis added] learning” (Bonner, 2013, p. 97).

4.1 STUDENTS AT THE CENTER

Currently, there is no easy way for educators to assess students’ levels of number sense or metacognitive ability. These are not qualities that can be exposed by a simple paper test. However, through proper structuring of assessment activities and observation, we are able to develop a quality picture of the student’s ability and understanding. Teaching relies on a teacher’s ability to accurately determine the level of understanding that each of their students have, as well as their ability to determine a student’s degree of learning, what they are *not* learning, and what to teach next, this is no easy task (Gareis & Grant, 2015).

Research highlights that any effective plan has the student as the focal point of the assessment process and gives the student ample opportunity to be a major player in how they can learn from the feedback provided by educators (Andrade, 2013; Brookhart, 2013; NCTM, 2014). The reason the student’s role in the assessment process is critical is reflected by the three legged stool example presented by Gareis and Grant (2015), represented by Figure 7 below.

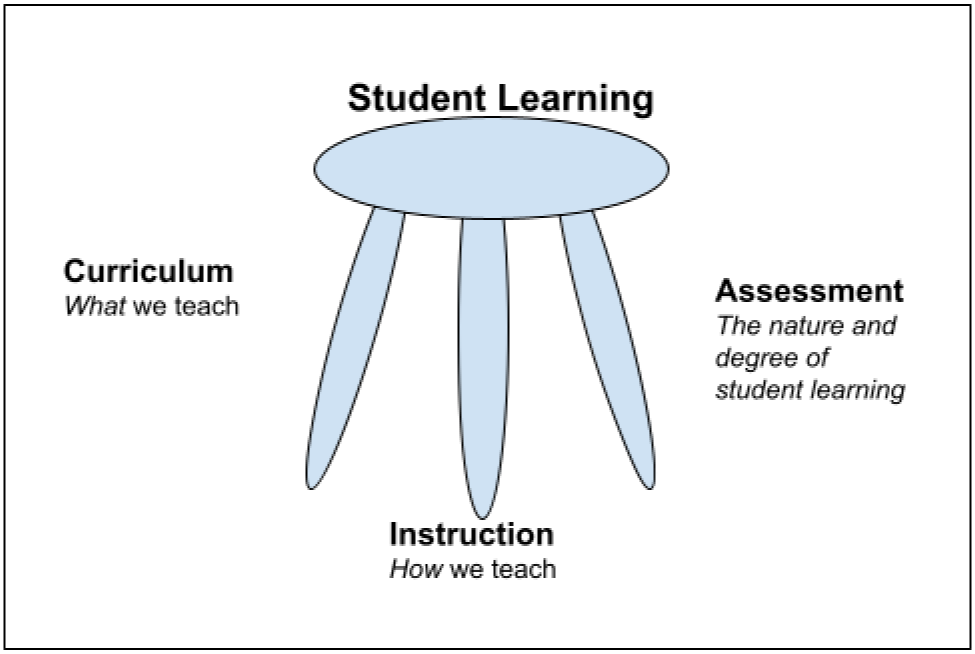

Figure 7: A Model of Curriculum, Instruction and Assessment (Gareis & Grant, 2015, p.4)

From Figure 7, Gareis and Grant (2015) further highlight the role assessment plays by stating how if "assessment is the means to discern student learning, then, in its absence, teaching becomes all about teachers and their decisions and *not* about the students and their learning" (p. 4-5). Without the assessment component being embedded into our teaching, there is no option but to allow the stool to fall, and that cannot be accepted. Yet, any old assessment is not always good enough to positively impact students at an individual level. As educators, we want to ensure the students are the central focus. Unfortunately, educators have not been known for their ability to design assessments to meet the needs of students in a consistent and effective way (Campbell, 2013). If we wish to have the students truly as the center of the assessment practice, we as educators must be willing to view assessment in a new light; one that does not focus on the traditional forms, but one grounded in the ongoing observation and conversation with our students. This is not to say that we should throw out the paper and pencil tests entirely; they certainly have their place. However, Dewey (1928), referenced by Heritage (2013), speaks of how “a much more highly skilled kind of observation … is needed to note the results of mechanically applied tests” (p. 182) and communicate those results back to the student with more than a simple grade. Teachers need to engage in continual observation of student learning by “paying close, firsthand attention to specific aspects of students’ developing understanding and skills as teaching and learning is taking place in real time” (Heritage, 2013, p. 179) and using such observations as feedback for students’ next steps.

Feedback is not a new concept when it comes to assessment, and it “is among the most critical influences on student learning” (Andrade, 2013, p. 25), but we are not always utilizing feedback in the best ways possible to impact students learning. To ensure that the feedback that we provide to students is centered on them as individuals and not as a blanket feedback for the whole group, opportunities for us to, literally, sit beside our students must be made available. This individualized feedback provides opportunity for metacognition to occur as students reflect on their work in a non-threatening, non-judgmental conversation which “can inform them about errors and misconceptions that need to be addressed” (Hattie et al., 2017). Andrade (2013), noted that “metacognition is associated with better learning and achievement” (p. 24) and if we can incorporate the feedback process as another opportunity for students to think about how they are completing their work, another supporting leg can be added to the student learning stool (Figure 8) to further support and stabilize the student learning process.

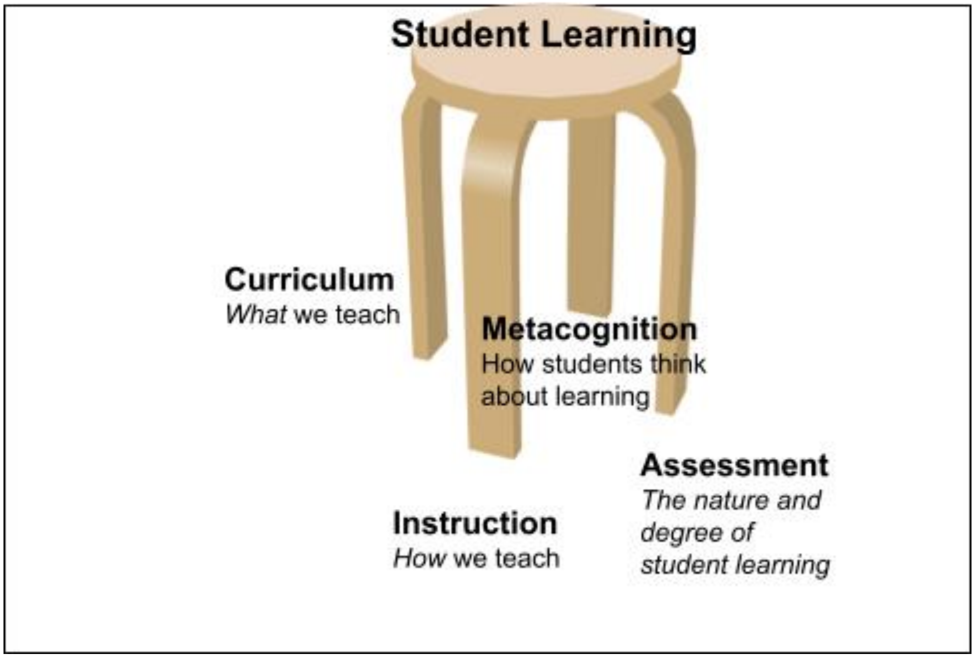

Figure 8: New Model of Curriculum, Instruction and Assessment with Metacognition (adapted from Gareis & Grant, 2015, p. 4)

After all, we as educators are the ones who set the environment that the students will engage in, and be influenced by those settings (Brookhart, 2013). If we are not providing them with realistic and scaffolded learning goals that lie within their zone of proximal development (Vygotsky, 1978), we are not providing a base for students to continue to grow from, and we reach a point where learning becomes stagnant (Andrade, 2103).

4.2 NOT SO STANDARDIZED

Boaler (2014) suggests that the perception of what doing mathematics is has been narrowed down to simply *performing math.* When students perform math, the belief is in order for them "to show they know math [they] can answer questions correctly — rather than to learn" (p. 469). As a result, the performance of how students achieve has been measured using scores they have received on paper-pencil, timed or standardized assessments and the essence of what assessment stands for (to sit beside) is often forgotten. It has been noted that when students undertake summative or standardized assessments, which are commonly multiple-choice, the aspect of variability is at risk of being ignored during the assessment (NCTM, 2014), and it has been found that grades alone are not suitable feedback as it does not lead to visible learning for the student (Hattie et al., 2017). A student's understanding is not reflected in their justification of an answer alone. They may understand the procedure that is required, but one single error may cost that student the ability to truly demonstrate what they know. Furthermore, the results of such assessments are not always utilized to inform and improve instruction for that student. Priority is often given to the reporting of an increase or decrease in scores, which is not a true reflection of the child's mathematical ability (Boaler, 2014; Campbell, 2013).

By embracing formative assessment, which is "the process of gathering evidence to inform instruction" (Hattie et al., 2017, p. 200), teachers have the task of paying attention to everything students do and use this information to inform their practice (McGatha & Bush, 2013). Some examples that reflect the ability to gather evidence for our use are: math journals, flexible groupings, pre-assessment quizzes, performance tasks, and questioning (McGatha & Bush, 2013) to name a few. Such a variety of tasks allows students to become the owners of their learning by not limiting them to a set way of solving and explaining a problem (Andrade, 2013; Hattie et al, 2017). If we utilize some of these assessment types, it becomes a form of regulation for students, versus a summative assessment which has the potential to "sort students and to convince many who earn lower grades that they cannot do mathematics, thus defeating the overall aim of ensuring mathematical success for all students" (NCTM, 2014, p. 95).

This regulation offered by formative assessment mimics the metacognitive skills that students need to advance their learning by assisting them to understand the process they use to set goals, implement the plan or strategy, and reflect to adapt their learning (Andrade, 2013).

One way these skills can be supported by the teacher is through the use of the Socratic method. This method "relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse" (NCTM, 2014, p. 35). These questions are designed with the purpose of exploring the level of student learning and creating points of conversations (or teachable moments) where students can embrace their metacognitive skills by interacting with those questions on an internal level (Hattie et al., 2017). These questions provide a foundational point where teachers can provide feedback on student learning, and give the student time to reflect and think critically on their work to establish where they are, and where they need to go (Gareis & Grant, 2015; Heritage, 2013; McGatha & Bush, 2013).

It is beneficial that the assessments being used are "interesting tasks and of obvious utility value… [as these types of tasks] give the best evidence that maximize both learning and future interest in the subject" (Brookhart, 2013, p. 46). As students work through the process, teachers can support the quality of the work by adopting an evidence-gathering strategy that aligns to the learning goals and constructs they are looking to achieve (Heritage, 2013). Heritage (2013) expands on this point by emphasizing that:

> teachers will need to employ strategies that tap into the individual knowledge that students manifest. Whatever strategies a teacher selects, they should account for the range of students [who] have the opportunity to show where they are in their learning and have the prospect of moving forward from their current status. (p. 185)

The challenging part of this process is managing the time to ensure that the feedback is within a point of time that can positively impact the student (Hattie et al., 2017). Teachers need to ensure that once information is collected, through questioning or other activities, and feedback is given, in whatever manner that is, that actions are put into place based on the evidence that has been observed (Heritage, 2013). Assessment in this manner cannot be a waiting game. It needs to be prompt, focused, and actionable by that student or its impact on affecting student learning are minimal at best (Hattie et al, 2017; Wiliam, 2013).

4.3 SELF-ASSESSMENT

Throughout this paper the topic of giving students control of their learning has come up repeatedly and is not a component of student mathematical development that is to be overlooked. In order to accomplish growth, the students, need to be entrusted to evaluate themselves during their process not only because it embraces the metacognitive skills that have be emphasized, but also allows them to look at their work with an objective view, compare their work to the determined goals, and reflect on where, what and how to get better (Brown & Harris, 2013). Essentially they are evaluating themselves and asking: Where am I going? What are my goals? How am I going to get there? What next? (Hattie et al., 2017). As educators we can assess a student until we are blue in the face and see little to no progress in the student's abilities for those efforts. However, Pintrich (2002) notes how "students who know their own strengths and weaknesses [have the ability to] adjust their own cognition and thinking to be more adaptive to diverse tasks and this facilitate learning" (p. 222). Self-assessment is not the replacement to other forms of assessment, merely another support to the complex and intertwined dynamic that is student learning related to mathematical number sense. It has been noted that a difference tends to exist between students' own grading of an assessment compared to that of their teacher or other outside evaluators (Brown & Harris, 2013. Although there may be a difference between a teacher's grade and a student when it comes to assessment of a specific task, the student needs to see that this difference exists in order to get an understanding of where they truly stand in relation to classroom expectations. “It is much more important to have accurate perceptions and judgements of one’s knowledge base and expertise than to have inflated and inaccurate self-knowledge” (Pintrich, 2002, p. 222) and work off the misconception that they are achieving when they are not. If the student doesn’t realize that there are issues or gaps in some aspect of their knowledge on a topic that needs to be addressed, it is unlikely that they will make any effort to rectify those areas. As educators our goal is not to inflate their self-esteem to the point of hindering growth but provide students with a realistic picture of where they are, and what needs to be done to move forward (Pintrich, 2002). One of the best ways we can provide this to our students is through the use of portfolios (traditional or online), which will be explored more in the next section as well as on the connecting website.

4.3.1 PORTFOLIOS

Portfolios grew out of the concerns regarding timed and standardized tests that were taking away from the student-centered curriculum and instruction. Belgrad (2013) draws attention to this when he addresses how students “have determined that their performance on standardized tests is more important than the what and the how of their learning experience and achievements” (p. 334). Portfolios provide assessment opportunities for educators that allow students to display a collection of their work that demonstrates achievement over time and ensures an active process for students to participate in (Belgrad, 2013). A study by Birgin and Baki (2007) claimed that “portfolio[s] give more reliable and dynamic data about students for teachers, parents and also [the] student [them]self” (p. 76). This form of assessment is empowering for students, but also for educators, as it puts the learner once again as the center of their education and the role of the teacher as a facilitator for that student. By adopting portfolios as a primary form of assessment, students get a two-prong approach in supporting their learning;

being provided with a voice in how they present their learning, and by having the ability to reflect on their work and identify errors and successes.

It has been identified that teaching with portfolios has positive impacts on students by providing an individualized focus and ability to scaffold metacognitive strategies aiding in the student's development (Belgrad, 2013; Bruand & DeLuca, 2018; Huang, et al., 2012). Along with that, portfolios also give learners a chance to bear witness to their own strengths and weaknesses as they go through their work (Ballard, 1992; Birgin & Baki, 2007).

By witnessing one's strengths and weaknesses learners are not able to hide behind the veil that they didn't know and are forced to address their work critically through reflection and self-evaluation. This form of self-assessment is not only about getting to think about one's work with the goal of understanding oneself better but "judging, evaluating, and considering one's own academic work or abilities" (Harris, 2013, p. 369) in a way that makes sense to them. It provides students with a way of sharing what work they are proud of, and why.

Portfolios are a means for teachers and students to engage in different ways of thinking and reflection. It does so by emphasizing a "focus on how the ongoing process of student inquiry captures the cognitive abilities that underscore successful achievement and engage students themselves as participants" (Belgrad, 2013, p. 332). As a result, portfolios have the ability to provide insights of the assessment process by allowing students to learn during assessments and, in turn, be assessed during learning without the pressures of standardized testing looming overhead (Birgin & Baki, 2007). If educators are stuck in a curriculum that is driven by testing scores, they are not allowed the opportunity to engage, assess and explore some of the higher-level, 21st-century skills that are necessary for students as they move from school into the real-world (Belgrad, 2013; Huang et al., 2012). Figure 9 reflects that challenge as it compares portfolio assessment to standardized testing:

Portfolio Assessment	Standardized Testing
❖ occurs in the child's natural environment	❖ is an unnatural event
❖ provides an opportunity for student to demonstrate his/her strengths as well as weaknesses	❖ provides a summary of child's filatures on certain tasks
❖ gives hands-on information to the teacher on the spot	❖ provides little diagnostic information
❖ allows the child, parent, teacher, staff to evaluate the child's strengths and weakness	❖ provides ranking information
❖ is ongoing, proving multiple opportunities for observation and assessment	❖ is an one-time "snapshot" of a student's abilities on a particular task
❖ assesses realistic and meaningful daily tasks	❖ assesses artificial task, which may not be meaningful to the child
❖ invites the child to be reflective about his/her work and knowledge	❖ asks child to provide a singular desired response
❖ invites the parents to be reflective of child's work and knowledge	❖ provide parents with essentially meaningless and often frightening numerical data
❖ encourages teacher-student conferencing	❖ forces teacher-administration conferencing
❖ informs instruction and curriculum; places child at center of the educational process	❖ reinforces idea that the curriculum is the center of the educational process

Figure 9: Comparing to Portfolio Assessment with Standardized Testing (Birgin & Baki, 2007, pg. 83)

In order to “ensure that next generation, 21st-century knowledge, dispositions, and abilities … a holistic, systematic approach to collecting and reporting evidence of student achievement is needed” (Belgrad, 2013, p. 343). As educators we need to embrace this and ensure that the focus of a child’s learning is in their hands, not simply a quest for grades and scores.

CHAPTER 5: IMPLEMENTATION IN THE CLASSROOM

> Since it has become clear that metacognitive awareness and skills are a central part of many academic tasks, a critical question for educators is how do we foster the development of metacognition in students?
>
> -McCormick, 2003, p. 90

As stated by McCormick (2003) above, a challenge we face as educators is being able to develop metacognition in students without any formal training in doing so. This project, the development of my website (www.metacogmath.weebly.com), is a tool that I hope will allow educators to connect with resources to assist them in developing number sense in their classrooms through metacognitive activities. Scruggs et al. (1985), as referenced in Noushad (2008), "suggests that direct instruction in metacognitive strategies leads to increases in learning" (p. 16), however, direct instruction in metacognition may not be beneficial for students especially when strategies of problem solving are imposed rather than generated by the students themselves, their performance may be impaired. My website is a means to assist teachers in expanding their own understandings in the hope that it allows them to generate new experiences where students are allowed to "experience the need for problem solving strategies, induce their own, discuss them, and practice them to a degree that they become spontaneous and unconscious, [in doing so] their metacognition seems to improve" (Louca, 2003, p. 17). For this reason, the primary information that has been put onto the website involves activities, videos, and background information that are research supported and that emphasize the student being in charge of their learning. If we, as teachers, address the focus of metacognition in our classrooms the opportunities for increased learning and development of number sense are made available for students. We expect that students are coming into classrooms, with varying levels of understanding and ability to progress (McGarvey, 2018). By its very nature, metacognition is an individual progression which allows each student to focus on their own skills and understandings at their own pace. Due to this fact the links, videos, and write-ups of various programs, ideas, or recommendations have been provided to allow for teachers to select programs that best fit their classroom context, as we cannot blanket one approach and expect it to provide the same impact across various groups. The reason metacognitive activities were selected is due to Djudin (2017), who argued that metacognitive skills not only benefit the majority of one's class but affects the learning potential of all students at every learning level. As we are involved in metacognitive practices everyday whether we know it or not, and this lack of recognition of when we are using metacognition is another reason why a website like this is necessary, not only for our students but for educators as well. By having access to various resources all in one place, teachers can provide support to bring this practice into the classrooms on a daily basis and consciously emphasize its practice by the students. The challenge we face is that teachers, traditionally, are not trained in how to teach metacognition to students even though it commonly occurs in our classrooms without our knowledge. Just because educators are not formally trained does not mean that we are not able to effectively teach the necessary skills and through this website, the pressure of searching for programs that best reflect metacognitive skills can be limited. This in turn can provide a stepping stone into more frequent metacognitive skills being present in the classroom because research supported methods, support documentation, and professional learning communities are available for all who wish to access it.

5.1 STRUCTURING A METACOGNITIVE CLASSROOM

As my research in metacognition continues, it is apparent that any curricula, especially in mathematics, needs to include instruction on metacognition as a means to improve not only the abilities of students but the quality and amount of instruction that occurs (Carr, 2010). We can no longer accept the method of teacher-direct instruction as a way to meet the varying needs of the students in our classrooms and help them simply get to an answer. “It’s more about going beyond the answer and helping them think about their thinking. How did they think of that? What did you notice that helped you figure it out? What’s another way to solve that?” (Sharapan, 2015, p. 1). These skills are not going to magically appear for students, but they are skills that can be taught and built upon at any level of education and ability, but the structure of the classroom is essential in order to assist in this venture. Due to this necessity, my website has taken some of the more research supported activities that demonstrate the ability to develop the skills necessary to metacognition and put them in one place for teachers to access at their own discretion and as reference. It is by no means a step by step guide, but an initial point where teachers can draw from as they work to increase their own competency in this area and begin to incorporate more of the skills into their classroom. It is but a tip of the iceberg, but in order for us as educators to begin to implement metacognitive skills, we need to know what is available, and how these programs could potentially work for us.

When designing this website, the time factor of teachers was kept in mind to prevent the perspective of starting over from scratch, as for educators that is an unrealistic request. However, the various programs, tips and videos on this website can provide access to resources to implement subtle changes to how the classroom is planned and the types of activities that are imparted on the students by teachers. Shilo and Kramarski (2018) emphasized how the use of mathematical-metacognitive discourse is a means to address the lacking consistent metacognitive practices that exist in classrooms. This approach “demands a change in teacher’s pedagogical role in class, [by] emphasizing student-centered pedagogy” (p. 4) which should focus on students becoming autonomous in the construction of their understanding. Again, this does not mean that it is not happening in the classroom, but, through the website, the goal is to bring the resources forward so they are at teachers' fingertips and ensure that we are keeping these goals at the forefront of our minds when it comes to planning lessons. In order to assist in accomplishing this, teachers need to model the metacognitive process personally and use various techniques such as questioning, think-alouds, rephrasing/repeating and more to mirror students' ideas back to them to provide them with a sense of what is required (Carr, 2010; Djudin, 2017; Kramarski, 2004; Louca, 2003; Shilo & Kramarski, 2018).

Before anything can successfully be implemented teachers first need to know what those techniques are and understand how to use them. An example of the effectiveness metacognition can have in a classroom can be seen in Whitney Elementary School as reported in NCTM's *Principles to Action* (2014):

> After some conversation, the teachers realized that although they had all been saying "Check your work," they had not actually taught their students *how* to check their work. To address this situation, the teachers jointly planned a "How to Check your Work" activity, and they decided that it would incorporate students' suggestions about how to check work. The outcome would be a "Check Your Work" routine to help students perform this task regularly (p. 98).

Unless educators have the knowledge, opportunity, and resources of what activities allow for students to embrace their learning, we run the risk of continuously teaching classrooms in the traditional sense where we are the keepers of knowledge and passing it onto students (Parrish, 2011).

Through this project, my goal was to provide a means to pull teachers away from the front of the classroom and through implementation of metacognitive activities provide opportunity to embark on a role of the facilitator. Pashler et al. (2007), referenced by NCTM (2014), explained that by providing students with opportunities to practice using concepts and skills over extended periods of time, and combining feedback regarding their performance, helped students create a deeper level of understanding. The attention paid to this practice allowed students to retain, reflect, generalize, and transfer knowledge and skills. The skills that students need cannot be assumed to just develop as they mature, the opportunity to witness and, in turn, experience what they witnessed allows for creation of their own connections. The most effective way we can influence the metacognitive skills in our students is to create a culture that promotes these skills on a daily basis. There is so much information out there for educators to sift through, this website, allowing educators to pinpoint programs that appeal to them and provides reference on how they can implement it within their own context.

Students must have the opportunity to gain metacognitive knowledge, but linked with metacognitive experiences in unison, this is a key component on why this website was created. It is of benefit to both parties, teacher and student, because we cannot simply show students the skills and expect that they will master them; there must be time for experimentation and immersion into the practice of the activity. If one aspect is promoted more than the other, instead of at an equilibrium, there is no guarantee that metacognitive skills will be developed and/or maintained even if the skills have been shown (Bonnett et al., 2017; Carr, 2010; Ford et al., 1998; Mahdavi, 2014). We must “provide learners with both knowledge of cognitive processes as well as strategies and together with experience or practice in deploying ... metacognitive strategies and self-evaluation of outcomes of their learning” (Mahdavi, 2014, p. 533).

5.2 TEACHER/STUDENT METACOGNITIVE ACTIVITIES

As previously mentioned, teachers are not always trained in how to support the development of metacognitive skills in a classroom setting. The classroom is a dynamic place where metacognitive skills can be implemented at various points, through an array of activities that are controlled by the teacher. Through the website, educators are able to explore some of the most promising activities that can be utilized in their mathematics classroom, though not solely limited to math, to begin to establish the desired metacognitive skills. The elements included on the website are meant to accomplish two main goals: 1) to change traditional classroom structure and 2) to have students take a more active role in their learning.

One way to begin to promote metacognitive discourse in one's classroom is by breaking away from traditional classroom structures of teachers as the keepers of knowledge whose sole purpose is to pass that knowledge on (Parrish, 2011). How does one do that if they have not been formally trained to do so? My project provides connections to various activities that not only provide metacognitive skills, but training for the teachers on how they can implement such a task within their classroom. By having access to the resources, teachers can begin to step back into a guidance role by adopting some of the presented activities on the website that provide students with the opportunity to construct mathematical meaning by ... the use of self-metacognitive questioning" (Kramarski, 2004, p. 596). Self-questioning, using activities such as: think-alouds, reciprocal teaching, and cooperative teaching found on the website, students are able to partake in what Shilo and Kramarski (2018) call *metacognitive talk*.

This talk helps students stimulate their thinking by providing freedom for conversations between peers that allows them to verbalize their mathematical processes. Students, with peers or the teacher, can explain strategies, reasoning, justifications and in turn continuously look for more effective strategies by drawing on the understandings of those around them (NCTM, 2014;

Parrish, 2011). Figure 10, also found on the website, demonstrates a self-question model that provides samples that can be modelled by the teacher or implemented by students at any level.

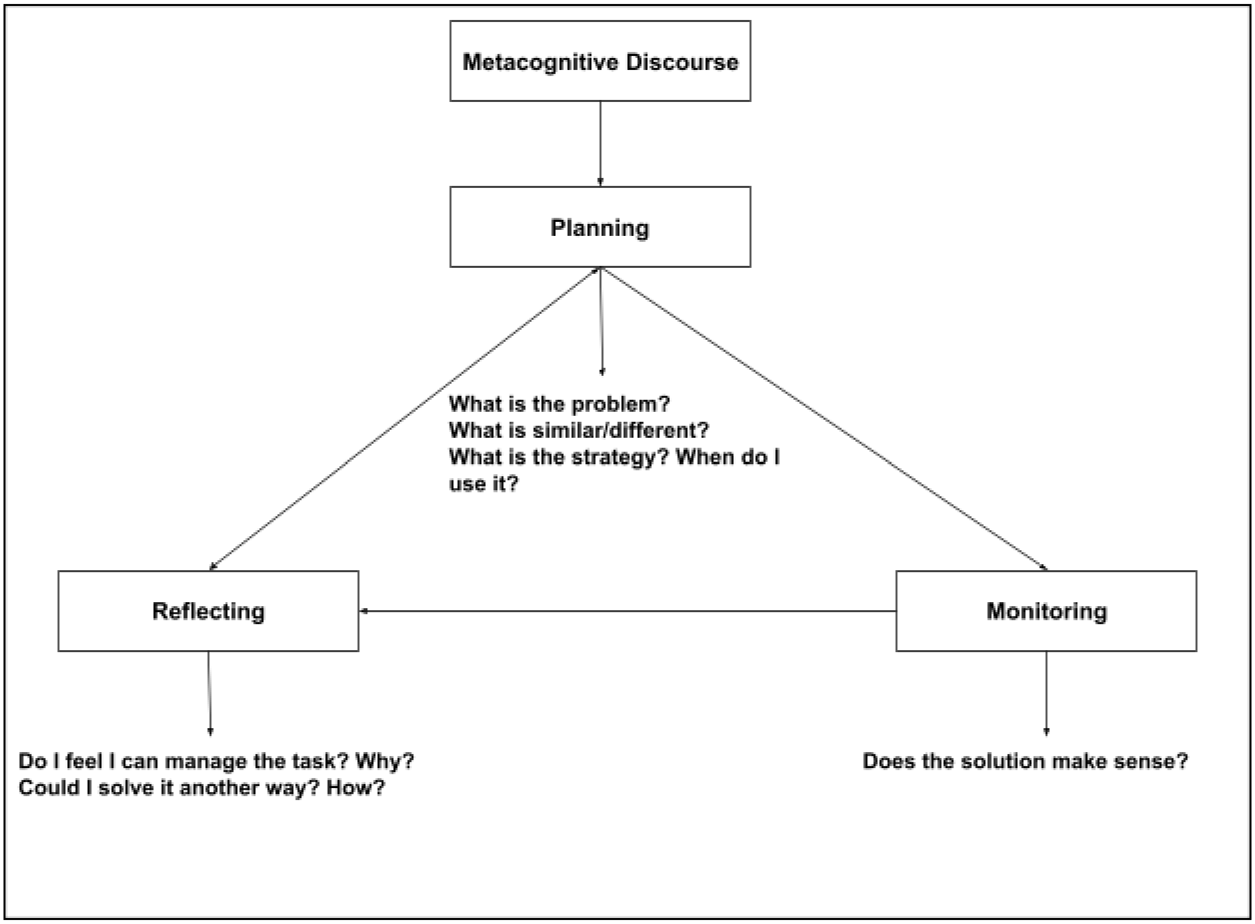

Figure 10: Generic Self-questions to Promote Mathematical-metacognitive Discourse (Shilo & Kramarski, 2018, p. 4)

Each of these components of metacognition is engaged in an interactive process "which is not of a linear nature, moving from preparation and planning to evaluation" (Mahdavi, 2014, p. 533) in a cyclical process allowing for constant revision and understanding (Ebdon et al., 2003). The process is never truly complete, but through greater competency, more opportunities should become available for students as teachers become more familiar with how they can implement metacognitive skills and activities.

Students need to take an active role in their mathematical learning if the goal is improvement and understanding. My project is designed as a means to support teachers in implementing foundational activities that, once practiced at the class level, will aid in building student competency with how to control one's learning. By utilizing the website activities such as: think-alouds, cooperative/paired problem solving, goal sketchers, thinking journals, role playing and reciprocal teaching (Carr, 2010; Louca, 2003), teachers are "promoting more complex thinking during discussions" (Carr, 2010, p. 184) and students have the benefit of being supported from multiple points as they delve into mathematics. These activities, which are grounded in metacognitive development, have the potential to help students move towards "independent thinking, more permanent knowledge, motivation for learning and higher achievement" (Louca, 2003, p. 26).

Having searched for activities to use in my own classroom I have noticed that there are a plethora of tasks at our disposal, unfortunately, filtering through them all to find ones that are going to be effective is a tedious task to say the least. The activities that are being selected need to be purposeful, if not targeted to a specific aspect that needs addressing, such as the Whitney Elementary School example of "Check Your Work". Access to my website at any time aids educators in their metacognitive venture by relocating some of the relevant metacognitive tasks in one common location. Ford et al. (1998) stated that "learning situations must facilitate the effective use of metacognitive or self-regulatory skills. A critical issue in the development and effective use of metacognitive skills is the opportunity for individuals to engage in" (p. 220) an environment where they can participate in activities that challenge them to connect knowledge (Schoenfeld, 1992). With the sheer variety and mass of activities available on the internet we cannot afford to select programs that are not building these skills.

The importance of metacognitive skills, as highlighted by previous sections, is one that needs to be addressed on a daily basis and continue to be addressed as they move beyond our walls. As educators incorporate some of the metacognitive activities from the website, the potential for students to develop a variety of new strategies that expand their thinking presents itself. The daily interaction with metacognitive skills forces students to engage with mathematics, or other subject areas, in ways they are not accustomed to and force new thinking pathways to emerge. This opportunity to work through new problems using new methods of thought forces students to explore different strategies and methods in contrast to students using teacher-taught procedures, which requires little connected knowledge, and hinders them to only one method.

5.3 ROLE OF THE TEACHER

The role of a teacher, even if they have not been trained in how to implement metacognitive practices, is one of guidance and facilitation (Parrish, 2011). This is the underlying theme behind the website as it allows activities to be implemented that alter the traditional role of the teacher. Educators are not needed as the omniscient leader in the room, who has all the answers or can show students the best methods, but as an observer who

> present[s] questions and select[s] tasks that challenge the students' thinking, listen carefully to ideas, know when to provide information, clarify issues, model a solution strategy, and know when to let students take the time to cope with difficulties and share ideas in the class. (Shilo & Kramarski, 2018, p. 4)

Simply put, one must put the learning of the students back into students' hands and provide them with the ability to adjust their learning to suit their own needs. Every student comes with their own set of strengths and weaknesses. To employ a method that blankets all of them not only takes away their individuality, but the opportunity for students to structure their own learning as well (Ford et al., 1998).

Instilling metacognitive strategies, such as the ones provided in this project, is one way that we as teachers can undertake the task of understanding and respecting that each student will develop at their own rate. It will be a tricky and challenging road ahead on both sides, and, since “metacognitive strategies are already in teachers’ repertoires” (Louca, 2003, p. 23) it simply requires teachers becoming aware of what strategies to use and when they can model such strategies and be purposeful in ensuring these are part of their daily plans (Carr, 2010; Djudin, 2014). Growth in this area will be subtle, especially at the start, and as facilitators we are looking for small measures of success by our students as well as in ourselves. Afterall, the “goals of mathematical instruction depend on one’s conceptualization of what mathematics is, and what it means to understand mathematics” (Schoenfeld, 1992, p. 334) and as teachers we need to understand that these goals will differ between each of our students.

By generating a platform where teachers can draw on ideas about how to bring metacognitive strategies into the class new possibilities open up on how teachers can support their students in this journey. Once teachers embrace metacognitive opportunities, they can begin to recognize the abilities each student has relating to metacognitive thinking. By identifying students who recognize their own thinking teachers are able to create plans to continue developing metacognitive skills and apply opportunities to support those students in their journey. These are steps in the right direction and need to be consistently addressed without avail. Faltering away from metacognition in one’s classroom and returning to traditional methods is

> just like giving a sick person a useless placebo injection, simply providing learners with answers [or algorithms] may enable them to resolve the immediate learning problem…[but] it is just a partial remedy that causes definitely as many problems as it solves. (Mahdavi, 2014, p. 533).

This is a pathway we need to avoid for the sake of our students' continued growth in mathematical understanding.

CHAPTER 6: CONCLUSION

> As far as I am concerned, this is essence of education: to facilitate a person's learning, to help that person become more intune with his or her own resources so he or she can use whatever is offered more fully
>
> -- Fred Rogers (Sharapan, 2015, p. 2)

Each child is unique in their learning and trying to find their place in the classroom to a point where they can develop the skills and understand which strategy is more appropriate as they work to build their own capacity (Pintrich, 2002). Metacognitive thinking is a practice that needs to be embedded into the culture of the classroom as well as the school as a whole in order to ensure this is being reinforced consistently year after year. As a solution Brownell (1945) suggests that those who wish to reach true understanding need to "acquire a true conception of the nature of meaning" (p. 482) and "they need to re-learn [mathematics]" (p. 482). While learning from strong teachers who have well established and structured programs are an aspect that cannot go overlooked, it is fair to say, based on the research provided, that it is more important for students to have experiences that coincide with metacognitively rich programs for students to draw their own meaning from. No matter how good a teacher or program is, it cannot be expected that they will be able to teach our students everything they need to know. So, if it is understanding that we strive for, it is fair to argue that the strongest mathematicians should be leading classrooms in the youngest grades. These are educators who have shown passion, dedication, and ability to develop the culture of mathematics and number sense. With people such as this at the helm, with our most malleable students, we could develop not only the minds of the students they interact with but spark a passion to dig deeper (Turkel & Newman, 1988).

From this project, ample research has been shown on the benefits of metacognition's role in the development of number sense skills, and "[i]t becomes increasingly evident that mathematics curricula need to include metacognitive instruction as a means of improving the quality and speed of learning" (Carr, 2010, p. 176). However, in order for students to demonstrate capabilities we require in any area, i.e number sense, they must go beyond what our teachers and programs have to offer and develop the metacognitive traits that will empower them to regulate their own learning and make the necessary steps into understanding in the classroom and eventually beyond (Mahdavi, 2014). This project's goal was not only to demonstrate the connection metacognition had to number sense, but to also provide a bridge for the teachers to connect their classrooms to the vast amounts of knowledge that is available to them. This allows teachers to expand their own understandings and move away from our current narrow view of mathematics; which not only deprives students from new and rich experiences, but prevents true understanding (Boaler, 2013; Schoenfeld, 1992; Turkel & Newman, 1988). Metacognition in mathematics should not be the only discipline where this skill is practiced; there is sufficient research to support the benefits of metacognition and how it offers the opportunity to impact all students at any level in any subject (Djuin, 2017). In order to support that development of number sense in our students the "practice in applying the metacognitive strategies should be executed by teachers of all content areas (subjects) since the primary years" (Djudin, 2017, p. 124). The sooner we begin this practice the greater the impact we can have on the students not only in mathematics but in learning and understanding as a whole.

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