

**IMPROVING THE SELF-EFFICACY OF MATH LEARNERS USING A DIRECT
AND FOCUSED APPROACH TO VOCABULARY CLARIFICATION**

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Abstract

Mathematics could be considered a language in itself, with numerous content-specific vocabulary and symbols. The language of mathematics must be explicitly taught through direct and focused instruction to promote the vocabulary development of students. The level of student self-efficacy in mathematics has been demonstrated to relate in a cyclical way to academic achievement; if one was to improve, so would the other. The purpose of this project was to explore connections between self-efficacy, vocabulary development, and student academic achievement. A self-efficacy questionnaire, *Math and Me*, was created that teachers could implement multiple times throughout a learning period. The results could be used to determine the growth of student self-efficacy after focused and direct instruction of vocabulary clarification. Use of the Collaborative Four-Square Frayer Model presented in this paper, in conjunction with the Vocabulary Instruction Implications for Teacher Practice guide, should increase student self-efficacy in math classes by providing resources to teachers that would aid in the instruction of mathematical vocabulary. To deepen student vocabulary knowledge, it would be essential for students to have experienced multiple and repeated opportunities for meaningful engagement with new mathematical vocabulary. Increases in student achievement have been linked separately to both increased vocabulary acquisition and increased levels of self-efficacy; the foundation of this project was built on the notion that a relationship must therefore exist between vocabulary knowledge and mathematical self-efficacy.

Preface

Mathematics was always my favorite subject in school. I loved the black and white nature of math; that there was always a right answer with multiple methods to find it. My experiences in high school steered me away from the study of mathematics. After completing a Bachelor's Degree in Chemistry and teaching science for three years, I eventually found myself back in the mathematics classroom. In the beginning, my lack of training as a math educator created challenges for myself and my students, but my love of learning mathematics created a positive learning environment that allowed my students to see success with the subject.

From the onset of my mathematics teaching career, I noticed the importance and significance of vocabulary development while learning math. I found students at all levels could complete mathematical procedures in straight-forward ways, but when a word problem covered the same concept, they were unable to apply that knowledge. Considerable time during assessments and work blocks was regularly spent clarifying word problems or instructions on given questions. Once I re-worded using vocabulary the students were familiar with, they could often complete the question with ease.

As I began teaching at the high school level, I noticed a lack of student requisite mathematical vocabulary knowledge. Most students would come to class not knowing what a reciprocal was (instead they would use the phrase "flipping" the fraction), how to properly say a number, or what opposites were. These were all essential vocabulary and mathematics skills grade nine students should have. Even when they were introduced to these words in earlier grades, most students would say that they were never expected to speak mathematically. My classroom guidelines always included this as a must; both

myself and my students were always required to use proper language, in oral conversation and while completing mathematical work. In my experience, this increased my students' levels of conceptual understanding of mathematics; there was less confusion on word problems and assessments. Most of the time, students could point out the vocabulary that was unclear, and this would facilitate instruction to the class. Once students would overcome this obstacle, they were often much more confident in completing their work and the academic conversations in my classes would improve.

My Masters Degree journey contributed significantly to my interest and concern with the development of mathematical vocabulary in students. My tacit knowledge led me to become intrigued with the notion of vocabulary development and the coursework I engaged in strengthened my curiosity. During my *Curriculum Studies and Classroom Practice* course, I found myself frequently recalling conversations with colleagues and interactions with students regarding mathematical vocabulary. The curriculum change I proposed for the Curriculum Change Project PowerPoint was an addition of mandatory vocabulary at every level in the Alberta Program of Studies for mathematics.

In my *Research as Assessment* course, I had the opportunity to conduct research on the construct of mathematical understanding. I found that vocabulary was rarely mentioned as an essential facet. Most academic research discussed the skills, dispositions, and knowledge required, and some mentioned creativity and real-life connections as essential. I was most intrigued with the irrelevant variances that were haphazardly addressed in the research. Such things as anxiety, time needed, readiness, reading skills, and relationships with teacher were regularly afterthoughts in the mention of mathematical understanding. Through conversations with other math teachers, along with

my tacit knowledge, I realized that these irrelevant variances may in fact be imperative to student mathematical understandings.

A lack of mathematical vocabulary may prevent students from developing mathematical skills, dispositions, and knowledge. In addition, how students feel about themselves as math learners could immensely impact their academic success in mathematics. It is these two facets of the mathematical construct, vocabulary development and self-efficacy that motivated the development of this project.

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Introduction

Although academic ability level and student understanding of the language of mathematics have not previously been explicitly linked, evidence in research has demonstrated that improved student mathematical self-efficacy could create gains in student academic achievement (Carpenter & Clayton, 2014; Md. Yunus & Wan Ali, 2008; Pajares, 1996; Pajares & Graham, 1999; Pinxten, Marsh, De Fraine, Van De Noortgate & Van Damme, 2014; Usher & Pajares, 2009). Carptener and Clayton (2014) stated that “a variety of choices including activity, effort, and resilience are influenced by self-efficacy beliefs, and all of these can impact learning” (p. 110). Students sometimes view mathematics as another language; one that would be unattainable for them to learn. Teachers, who clarify mathematical vocabulary through focused and direct methods such as the Collaborative Four-Square Frayer Model presented in this paper, could dispel the perception that mathematical vocabulary knowledge may be unreachable for students. By increasing student understanding and confidence with mathematical vocabulary, self-efficacy should improve, which could lead to more academic success with the subject. This project examined possible connections between student self-efficacy and the development of their mathematical vocabulary in current educational research. It provides teachers with suggestions for integration of mathematical vocabulary instruction into everyday practice.

Self-Efficacy

Definition

How students feel about themselves as math learners, known as their self-efficacy, may affect their willingness to experiment with questions or attempt new scenarios, and

their overall enjoyment of mathematics courses. Bandura (1997) describes the notion of self-efficacy as “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments” (p. 3). Carpenter and Clayton (2014) linked this definition to the education of children when they stated “self-efficacy is the belief students hold about their academic capabilities” (p. 110). In my experience, when students have negative feelings attached to mathematical learning, those feelings can be hard to change. By understanding more about self-efficacy and how it impacts student learning, teachers could increase the mathematical enjoyment in their classrooms by growing their students’ beliefs about their ability to learn math, which could lead to increases in student academic achievement.

Four Sources of Self-Efficacy

In my experience, the self-efficacy beliefs a student had regarding mathematics were often strongly developed by the time they reached the high school level. A student’s education and life experience shaped the way they viewed themselves and their abilities in math. Bandura (1997) hypothesised that human self-efficacy was established from four sources: mastery experiences, vicarious experiences, social persuasions, and emotional and psychological states (p. 79). These four categories have been used in various academic works and were therefore reasoned suitable for use when exploring the self-efficacy of students (Carpenter & Clayton, 2014; Usher & Pajares, 2009).

Mastery learning and social persuasion were thought important factors to increase student confidence in all fields of study. Mastery experiences, which Bandura (1997) stated as “the most influential source of efficacy information because they provide the most authentic evidence of whether one can muster whatever it takes to succeed” (p. 80),

and the influence of external supporters on students learning, called social persuasions, can be directly measured to determine student self-efficacy. Mastery experiences would be those where students have either been successful in learning or have failed in learning, which Usher and Pajares (2009) stated that “successful performance in a domain can have lasting effects on one’s self-efficacy” (p. 89). People who interact with math learners on a regular basis could immensely impact a child’s self-efficacy beliefs. Since teachers, parents, and peers could all have positive or negative effects on the self-efficacy of a student, encouraging and supportive messages may increase self-efficacy (Usher & Pajares, 2009). Usher and Pajares (2009) advocated contemplation of social persuasions when they stated:

Encouragement from parents, teachers and peers whom students trust can boost students’ confidence in their academic capabilities. Supportive messages can serve to bolster a student’s effort and self-confidence, particularly when accompanied by conditions and instruction that help bring about success. (p. 89)

Examining both the mastery experiences and social persuasions of a student’s mathematical experiences may give a comprehensive overview of their level of math self-efficacy.

Learning, Achievement, and Self-Efficacy

Students confident in a subject area could be more willing to engage in new situations and concepts than their uncertain peers. Confident students may be more motivated to ask questions that would ensure a deeper understanding of the material. Studies found a positive correlation between an increase in mathematics self-efficacy and an increase in mathematical academic achievement (Carpenter & Clayton, 2014; Pajares,

1996; Pinxten et al., 2014). Pajares (1996) indicated that “academic performances are highly influenced and predicted by students’ perceptions of what they believe they can accomplish” (p. 325). Teachers that endeavour to improve the confidence level of students in their mathematics classes could notice such positive results as increases in student engagement, more positive classroom environments, and growth in student academic achievement levels.

Students come to mathematics classrooms with various levels of math self-efficacy and academic achievement. The cyclical nature found between self-efficacy and academic achievement indicated that strengthening one could strengthen the other: increasing self-efficacy could lead to an increase in achievement and an increase in achievement could lead to an increase in self-efficacy (Carpenter & Clayton, 2014; Pinxten et al., 2014). Pajares (1996) found “some self-efficacy researchers have suggested that teachers should pay as much attention to students’ perceptions of capability as to actual capability, for it is the perceptions that may more accurately predict students’ motivation and future academic choices” (p. 340). As these two facets of a student’s learning experience were found to be explicitly linked to one another, by focussing on one, self-efficacy, academic achievement could be positively impacted.

Academic achievement was found to be but one influence of self-efficacy: the amount of effort expended, the perseverance and resilience throughout a task, and the worry associated with mathematic coursework all contributed to self-efficacy beliefs (Alsawaie, Hussien, Alsartawi, Alghazo, & Tibi, 2012; Carpenter & Clayton, 2014; Md. Yunus & Wan Ali, 2008). Teachers must be aware that these factors influence students’ beliefs about themselves as learners. Pajares (1996) concluded that “students who lack

confidence in skills they possess are less likely to engage in tasks in which those skills are required, and they will more quickly give up in the face of difficulty” (p. 340).

Students with low levels of self-efficacy in mathematics may benefit if their teachers attempted to determine the reasons behind the associated behaviors.

Self-efficacy beliefs could strongly impact a student’s decision making regarding their future academic and career choices (Carpenter & Clayton, 2014; Pajares, 1996; Usher & Pajares, 2009). In my experience as a high school mathematics teacher, I have witnessed students choose courses beneath their academic capabilities because they either struggled in the previous course, didn’t connect with their former teacher, or felt like they had no chance of success at a higher level. Teachers should increase the confidence level of their students with focused and direct mathematical vocabulary development instruction, thereby increasing their levels of mathematical self-efficacy.

Evaluating Student Self-Efficacy

A questionnaire used before, during, and after a learning period could demonstrate a trend, positive or negative, of students’ mathematical self-efficacy beliefs. The self-efficacy questionnaire, *Math and Me (Appendix A and B)*, created for this project, utilized several components from the Sources of Self-Efficacy in Mathematics (SSEM) instrument created by Usher and Pajares (2009). This 6-point scale (where 1 = definitely false and 6 = definitely true), the categorization of questions based on Bandura’s (1997) sources of self-efficacy, and the questions aimed specifically at mastery experiences and social persuasions were duplicated exactly. Usher and Pajares (2009) stated mastery experiences “can be better obtained through self-report items that invite students to rate the degree to which they have experienced success rather than through concrete

indicators of past performance such as grades” (p. 90). Specific mathematical vocabulary questions were added to narrow the focus for students and to aid teachers in analyzing questionnaire results in order to pinpoint successes and failures of their direct and focused instruction on mathematical vocabulary. This questionnaire “focused on perceived rather than actual effort expenditure” (Pinxten et al., 2014, p. 156). Table 1 illustrates how the questions were categorized, whether as mastery experience or social persuasion components of self-efficacy or as mathematical vocabulary exclusively.

Table 1

Categorization of Questions within Self-Efficacy Questionnaire

Question	Categorization
1. I make excellent grades on math tests.	Mastery Experience
2. I have always been successful with math.	Mastery Experience
3. Even when I study very hard, I do poorly in math.	Mastery Experience
4. I got good grades in math on my last report card.	Mastery Experience
5. I do well on math assignments.	Mastery Experience
6. I do well even on the most difficult math assignments.	Mastery Experience
7. My math teachers have told me that I am good at learning math.	Social Persuasion
8. People have told me that I have a talent for math.	Social Persuasion
9. Adults in my family have told me what a good math student I am.	Social Persuasion
10. I have been praised for my ability in math.	Social Persuasion
11. Other students have told me that I’m good at learning math.	Social Persuasion
12. My classmates like to work with me in math because they think I’m good at it.	Social Persuasion
13. When completing a word problem in math, the language used is often confusing.	Mathematical Vocabulary
14. I understand most of what is said during math class.	Mathematical Vocabulary
15. I know that mathematics has its own language that can be figured out.	Mathematical Vocabulary

This project utilized parts of the SSEM to evaluate the effects of focused and direct vocabulary instruction on student mathematical self-efficacy. Carpenter and Clayton (2014) stated that “Usher and Pajares established the construct validity of the instrument and demonstrated its reliability across genders, ethnicity (African American and White only), and mathematical ability level (on level/above level)” (p. 112). As well, research on the Sources of Self-Efficacy in Mathematics indicated it to be an effective tool when measuring student levels of self-efficacy (Carpenter & Clayton, 2014), giving credibility to the use of these questions on the *Math and Me* questionnaire.

Modifications were made to the SSEM tool created by Usher and Pajares (2009) and used by Carpenter and Clayton (2014) during the creation of the self-efficacy questionnaire, *Math and Me*, presented in this paper. The questions Usher and Pajares (2009) included concerning vicarious experiences, and emotional and physiological states were not integrated into the *Math and Me* questionnaire. This decision was made on purpose as the questionnaire developed was to focus only on mathematical vocabulary - a limited subject matter. The number of questions was reduced to be mindful of the amount of class time needed to complete the survey. The 6-point scale used in both Usher and Pajares' (2009) and Carpenter and Clayton's (2014) questionnaires lacked detailed statements for each number on the scale. For this project, the same scoring criterion was used as in the SSEM, however, detailed statements were added for numbers two through four on the scale to clarify the distinction between each number.

The purpose of the self-efficacy questionnaire, *Math and Me*, would be to review students' self-efficacy as math learners continually. Usher and Pajares (2009) reflected on the use of SSEM:

The sources scale offered here may also provide middle school teachers with a quick assessment tool for understanding the antecedents of their students' self-efficacy beliefs. Such an understanding would certainly be useful to all who are interested in nurturing students' competence and confidence. (p. 100)

The questions would probe into a student's past experiences with mathematical learning, their successes and failures, and the reactions they have received from others in regards to their ability levels. Pajares and Graham's (1999) study found that "self-efficacy was the only significant motivation variable to predict performance both at beginning and end of

year” (p. 133). Having students complete the self-reflection questionnaire, *Math and Me*, multiple times throughout a learning process – beginning, middle, and end – could help teachers gauge student progress and improvement of math self-efficacy before, during, and after direct and focused instruction on mathematical vocabulary.

Instructions for Use of Self-Efficacy Questionnaire “Math and Me”

Teachers would follow these guidelines when giving the questionnaire to students:

- Either paper (Appendix A) or digital (Appendix B) versions could be used and would yield the same results.
- The importance of no right or wrong answer should be stressed; that this self-reporting tool would not influence students’ grades in the course.
- Read instructions aloud to students before they begin the questionnaire and allow time for clarifying questions from students (Carpenter & Clayton, 2014).
- Allow students enough time to finish at their own pace; have a quiet, simple activity for students to work on when completed.
- Compare individual student reports over time (beginning, middle, and end).
- Look at overall class trends - likely there will not be definite features that worked for all/didn’t work for any.
- Teachers should change their practice using the information provided by questionnaires to meet the needs of current students and improve their self-efficacy, otherwise the questionnaire would not be meaningful.

- Teachers could analyze the answers on a scale ranging from no confidence at all, to completely confident (Clutts, 2010).

The recommendations mentioned above should aid teachers in successfully implementing the *Math and Me* self-efficacy questionnaire in their classrooms and give concrete suggestions for how to evaluate the answers provided by their students.

Strategies to Improve Student Self-Efficacy

If students felt safe and appreciated in the classroom, they may be more willing to challenge themselves and persevere through seemingly complicated tasks. Teachers determined to raise student self-efficacy should incorporate frequent authentic praise of student effort and behaviors during learning episodes. Mueller and Dweck (1998) demonstrated the importance of praising for hard work rather than for intelligence to “lead children to value learning opportunities” (p. 48). Students should be given opportunities for mastery learning of mathematical vocabulary through repetition of key words, continual constructive feedback from peers and teachers, and interactive and engaging activities. These strategies could increase overall student well-being, as they were linked to improvements in self-efficacy found in Carpenter and Clayton (2014), Pinxten et al., (2013), and Usher and Pajares (2009). Teachers should provide ample opportunities for students to feel good about the effort they are putting in and the work it produces; it would be through these processes, that students’ self-efficacy could improve.

Vocabulary Instruction

Math as a Language

Learning mathematics is not only about numbers, equations, and their solutions; learning mathematics requires the introduction and comprehension of a new language –

the language of mathematics. Mathematics has a sizable number of content-specific words, as well numerous ordinary language words that are used with unique and vastly different mathematical meanings than in everyday context (DeVries, 2002; Dunston & Tyminski, 2013; Smith & Angotti, 2012). Teachers need to recognize that vocabulary development could be a key element of mathematical understanding that may be essential for student success.

Communication was included as a central aspect of teaching and learning mathematics in various programs of study (Alberta Education, 2014; National Council of Teachers of Mathematics, 2000). To demonstrate effective communication in mathematics, as in any foreign language, participants need to be explicitly taught content-specific vocabulary. Schell (1982) brought to light the enormity of this task in mathematics when she identified that math had “more concepts per word, per sentence, and per paragraph than any other area” (p. 544). Not much of the vocabulary in mathematics is used in everyday conversation. Therefore, students might not have experienced nor be comfortable with such language. Teachers could assist math learners through this struggle by focussing their instruction on clarifying misconceptions of ordinary vs. mathematical usage of words and by directly teaching content-specific vocabulary using a multitude of pedagogical strategies.

Vocabulary Instruction is Every Teacher’s Responsibility

Teaching vocabulary should no longer be viewed as only the English teacher’s responsibility. In a study completed by Cantrall, Burns, and Callaway (2009), it was noted that “21% of teachers expressed skepticism about content area teachers’ responsibility to teach literacy, and all were mathematics teachers” (p. 84). Math teachers

would be better equipped than other content area teachers to provide instruction on key terms that would allow students to advance successfully through their mathematics learning. Having a good grasp on relevant vocabulary could strengthen understanding of mathematical concepts, aid students in functioning better throughout the subject-matter, and assist with content-area reading (Adams & Pegg, 2012; Barton, 1997; Rekrut, 1996). Literature-based text and content-area vocabulary require the use of very different reading strategies when attempting to comprehend each type of text (Barton, 1997). Expecting English teachers to skillfully instruct on mathematical vocabulary would be like expecting the engineer who built a skyscraper to successfully manage the company that worked inside it. English teachers can be more adept on the fundamentals of language, but mathematics teachers are more familiar with the fundamentals of the language of mathematics and should oversee the instruction of it.

Benefits of Vocabulary Instruction

Studies have demonstrated positive correlations between student achievement and vocabulary instruction in mathematics classrooms (Barton, 1997; Dunston & Tyminski, 2013; Pierce & Fontaine, 2009). Many errors on assessments in mathematics have been linked to comprehension skills, reading problems, and lack of vocabulary knowledge (Barton, 1997; Blachowicz, Fisher, Ogle, & Watts-Taffe, 2006; Dunston & Tyminski, 2013). Dunston and Tyminski (2013) stated that “developing mathematics vocabulary knowledge allows adolescents to expand their abstract reasoning ability and move beyond operations to problem solving” (p. 40). Incorporating any means of vocabulary instruction has been shown to positively affect student learning in mathematics (Dunston & Tyminski, 2013; Petty, Harold, & Stoll, 1968). Consequently, teachers should be

provided with instructional guides and professional development opportunities to increase competence with this component of mathematics instruction.

Reading and Vocabulary Knowledge Connection

Vocabulary acquisition has been proven to be crucial to reading comprehension improvement for students (Graves, 1985; Stahl & Fairbanks, 1986). Word problems in math textbooks and assessments may contain real-life scenarios, bringing forward even more vocabulary that a student would have to sift through to decipher the meaning of the question. The mathematics teacher's role should include teaching content-specific vocabulary words that support student comprehension of word problems, for as Stahl and Fairbanks (1986) found, to understand any segment of language, the reader must understand most words used in the segment. Pre-teaching vocabulary may assist student reading comprehension when they encountered mathematical word problems (Graves, 1985), for if students understood all context-specific and math-specific vocabulary, they could move on to the meaning-making and computational steps of solving problems. Blachowicz et al. (2006) indicated that "vocabulary knowledge is a critical factor in the school success of English-language learners" (p. 526). However, despite the connection found between increased vocabulary knowledge and positive student achievement, this increase in knowledge would not necessarily translate to computational proficiency (Dunston & Tyminski, 2013). Teachers should recognize the limitations of their students' vocabulary knowledge, how those limitations could impact their academic achievement, and how to facilitate their mathematical language development.

Explicit instruction of vocabulary could create strategic mathematics readers. In my teaching experience, students have generally been proficient in the computational

aspect of a concept, but had difficulties once asked to apply those skills to a written scenario. By explicitly teaching vocabulary through classroom discussions and other instructional techniques illustrated in this project, teachers may demonstrate strategies students may apply when they approach a difficult math question. Stahl and Fairbanks (1986) suggested that “participation in a discussion, or even anticipation of being called upon to participate, might lead to more active processing of information about a word’s meaning, resulting in higher retention” (p. 77). During lessons, teachers focused on vocabulary development could guide students through solving word problems, focusing on the language of mathematics while including students in the discussions. Students should be given the opportunity to witness how their teacher thinks mathematically and reasons through problems; this pedagogical approach could lead to students developing reading comprehension strategies of their own.

Quality Vocabulary Instruction

Important aspects of quality vocabulary instruction include student engagement in the process, having ample opportunities for repetition, direct and explicit instruction, and connection making (Blachowicz et al., 2006; Greenwood, 2002; Harmon, Hedrick, & Wood, 2005; Rekrut, 1996; Smith & Angotti, 2012). Constructing definitions with students could lead to deeper processing of the vocabulary as students might recall the new terminology better if they had a meaningful encounter with it. Teachers should demonstrate their own strategies during class lectures and discussions in order for students to understand when vocabulary may cause confusion, what part of the vocabulary may be confusing, and how the teacher would resolve the issue. Blachowicz et al. (2006) described in detail that “effective vocabulary instruction requires a repertoire

of teaching activities and instructional strategies coupled with the teacher's ability to choose appropriately within this repertoire" (p. 528). Teachers should continually demonstrate their thinking processes throughout all mathematical instruction in order to facilitate student vocabulary acquisition while still engaging their audience.

Instructional Techniques. Direct instruction and incidental learning should be used in tandem during direct and focused instructional approaches to vocabulary clarification (Dunston & Tyminski, 2013). Stahl and Fairbanks (1986) found that "the methods that did appear to produce the highest effects on comprehension and vocabulary measures were methods that included both definitional and contextual information about each to-be-learned word (or "mixed" methods)" (p. 101). This approach focuses beyond the dictionary definition of a word to when it should be used, in what contexts, what it could look like, relationships to other vocabulary. Both technical and symbolic connections should also be included in vocabulary instruction (Harmon, Hedrick, & Wood, 2005). If a situation were to occur during a lecture, discussion, or work block that would highlight the importance of vocabulary development, the teacher should take time to examine the situation with students. In addition, math learners should be actively involved in their vocabulary development. They could do this by describing what types of problems have caused them confusion in the past and by notifying their teacher when they encountered difficulties with a math problem because of the vocabulary used. By using real-life situations, with students at the center of instruction, students would be able to make connections to their learning processes and would see how the strategies could be used on an individual basis. When teachers instruct using both direct instruction and incidental learning of vocabulary development, students would understand vocabulary

not as a separate piece of math learning, but as an integrated and important facet of achievement and success.

Quality Vocabulary Instruction Takes Time. Time is valuable in most mathematics classrooms, therefore a focus on vocabulary instruction would need to be purposeful and worthwhile for student learning. Rekrut (1996) asserted “while direct instruction can teach students only a limited number of words each year, a rich teaching context can provide more information about words in less time than can the casual contact of reading” (p. 67). Since not all vocabulary could be explicitly taught within the time constraints of present-day math classrooms, teachers would need to decide which vocabulary was crucial for students’ success.

Teachers as Vocabulary Experts. Teachers should be familiar with important vocabulary associated with the mathematical concepts they teach and should be able to rank those words in order of importance for students learning. Mathematics, as noted previously, has its own language, and that language has “math specific words and ambiguous, multiple-meaning words with math denotations” (Pierce & Fontaine, 2009, p. 242). Teachers could use textbook resources and tacit knowledge to develop a list of common vocabulary that would be related to the concepts being taught. Beck, McKeown, McCaslin, and Burkes (1979) created a classification of “three levels of word knowledge: established, acquainted, and unknown” (p. 61). *Established words* would be those that students automatically recalled, *acquainted words* would be “recognized, but only after deliberate attention focused on it,” (Beck et al., 1979, p. 61) and *unknown words* would be those that students would have had no previous connection or association. Teachers could give pre-assessments on concept vocabulary then categorize words as either

established, acquainted, or unknown based on the results. This information could be used to plan vocabulary instruction for the students in their classrooms. By using a strategy such as this, teachers would not lose class time instructing on familiar and understood vocabulary.

Theoretical Framework

Learning is a social process. The direct and focused approach to vocabulary clarification presented in this paper followed the socio-constructivist lens; students and teachers would create the meaning of vocabulary together. Barron and Melnik (1973) indicated that “discussion of vocabulary exercises, in conjunction with procedures designed to enhance the understanding of the purposes and processes of engaging in the learning task, facilitates vocabulary learning” (p. 50). Having students participate in their own vocabulary acquisition by actively engaging them in classroom discussions could create stronger connections with words and develop deeper meanings. Vocabulary instruction focused on definitional knowledge only was proven ineffective (Monroe & Pendergrass, 1997). The use of a vocabulary instructional tool such as the Frayer Model (1969) could facilitate the engagement of students in the learning process and would provide more extensive knowledge on each vocabulary term.

The Frayer Model

Using the Frayer Model for vocabulary instruction has become commonplace in the teaching profession; the holistic paradigm of vocabulary development through the Frayer Model can be used at all levels of schooling. The model designed by Frayer, Fredrick, and Klausmeier (1969) was intended to “test the level of concept mastery” (p. 9). The schema developed (see Appendix C for example) to teach unknown vocabulary

included the following seven categories of information about a word that should be established:

(a) the names of the attributes which comprise the concept examples, and which are relevant and which irrelevant to the concept, (b) examples and non-examples of the attribute values, (c) the name of the concept, (d) concept examples and non-examples, (e) a definition of the concept, (f) the names of supraordinate, coordinate, and subordinate concepts, (g) principles entailing the concept, and (h) problems which may be solved by relating principles involving the concept.

(Frayer et al., 1969, p. 9)

The universality and comprehensiveness of the Frayer Model allowed it to be used in any subject matter to develop vocabulary knowledge; this feature made it a well-respected instructional technique almost fifty years later.

Research to Support Use of Frayer Model

The Frayer Model method of vocabulary development has been well researched and has been shown to enhance the vocabulary development of learners positively.

Monroe (1997) explained that “the human brain naturally organizes information into categories determined by past experience,” (p. 4), illustrating the reason for the success of graphic organizers that demonstrate conceptual relationships. The Frayer Model reveals relationships of similarity and difference between concepts, which has been shown to create deep connections and understandings that would be retained by students and retrieved for future learning experiences (Barton, 1997; Brunn, 2002; Gillis & MacDougall, 2007; Monroe, 1997). The Frayer Model has been used in a variety of

learning environments (educational institutions, professional development, etc.) and frequently can be used as the method to increase vocabulary acquisition.

Important Considerations While Using the Frayer Model

Although there have been well-researched benefits of the use of the Frayer Model in vocabulary instruction, one important hesitation must be considered. As Greenwood (2002) expressed, the Frayer Model was “the most time consuming and labor-intensive model” (p. 261). Teachers must be purposeful when selecting the vocabulary that would be developed using this model; the Frayer Model should be reserved for only the most challenging and conceptually hard to understand vocabulary (Greenwood, 2002). Teachers must limit the number of vocabulary words developed using the Frayer Model as issues of time management, loss of student interest, and the creation of an overwhelming amount of information could occur. Therefore, in order to be an effective vocabulary instructional tool, completing the Frayer Model with students should include both oral discussion and written information components (Monroe & Pendergrass, 1997). The benefits of this model have been well documented: the time taken to complete it was overshadowed by the positive retention of vocabulary knowledge student demonstrated after its use.

Adaptations and Implementation Techniques to the Frayer Model

The Frayer Model has been modified and implemented in various ways since its inception by Frayer, Frederick, and Klausmeier in 1969. Multi-modal implementations used to deliver this model of vocabulary instruction have included such low-tech methods as paper and pencil to such high-tech methods as phones, computers, and SmartBoards© (Fillippini, 2015). The use of technology could be engaging for students, as it would

permit the Frayer Model to be shared by classmates and then accessed from a variety of locations. Monroe and Pendergrass (1997) blended the Concept of Definition Model with the Frayer Model and coined the result the Integrated CD-Frayer Model (Appendix D). This graphic organizer was similar in shape to a web map; it incorporated category, example, and attribute labels from the Concept of Definition Model, and it included the non-examples label from the Frayer Model. Another modification to the Frayer Model was developed in the Four-Square Strategy presented by Brunn (2002). This model was more reminiscent of what educators typically associate the Frayer Model with today (Appendix E), with the page split into four quadrants and the concept label in the middle. Brunn's Four-Square Strategy (2002) took "the power of Frayer's model [that] lies in the visual imagery presented to the students" (p. 522) and changed the organization. This model encourages teacher autonomy over the decision of what information should be investigated in each of the four quadrants, but keeps the "underlying function [to] position several related terms, ideas, or concepts around one central element" (p. 524). No matter which particular model is used, graphic organizers have been well researched and recognized to help students develop deep understandings of difficult vocabulary concepts (Brunn, 2002; Gillis & MacDougall, 2007; Harmon, Hedrick, & Wood, 2005; Monroe, 1997; Monroe & Pendergrass, 1997). Whether the Integrated CD-Frayer Model, the Four-Square Strategy, low-tech or high-tech, all vocabulary instructional methods emphasized the central theme of the original Frayer Model: vocabulary instruction should be focused on discovering the connections between new words and their relationships to background knowledge.

The Collaborative Four-Square Frayer Model

The Collaborative Four-Square Frayer Model (CFSF) created for this project integrated pieces from the original Frayer Model, the Four-Square Strategy, the Integrated CD-Frayer Model, and as well considered the use of technology as integral in student vocabulary development. My school has transitioned to being a 1:1 school, where technology is available for every student at all times; this project was built upon the premise that technology would be accessible and available for students use. Google Docs© was selected as the software to display the CFSF Model (Appendix F) for the following reasons: students would be able to collaborate in real-time with each other on the same document; individual contributions would be recorded; it would allow for direct and indirect instruction; the creations could be individual, group or whole-class; and last, CFSF Models could be shared with all individuals in the classroom. *Attributes, examples, non-examples, and problems* were all included in the Collaborative Four-Square Frayer Model and originated from Frayer, Frederick, and Klausmeier's (1969) *Schema for Testing the Level of Concept Mastery*. Contribution from the Integrated CD-Frayer Model included the component of *concept category* to the CFSF Model. Finally, the visual organization of Brunn's (2002) Four Square-Strategy was utilized to structure the CFSF Model. Used to assist in the instruction of student vocabulary clarification, the Collaborative Four-Square Frayer Model included all significant components of other vocabulary instructional tools previously mentioned here.

Collaborative Four-Square Frayer Model Implementation Guide

The Collaborative Four-Square Frayer Model could be implemented in a variety of ways in the classroom; teachers would be encouraged to modify the process to suit their pedagogical requirements and the needs of their students.

Gradual Release of Ownership. In order to ensure accurate student usage of the CFSF Model, teachers should take an incremental approach when implementing it. Teachers should first lead the activity by demonstrating the completion of the attributes, examples, problem situation, and concept categorization for one specific word. Next, teachers may have students direct the CFSF Model vocabulary development process, but still be in control of its completion. This teaching technique could clarify any student misunderstandings on how to develop vocabulary in using this model. The end goal would be to have student-generated CFSF Models that either the whole class or individual groups had created. Monroe (1997) maintained that:

Student-constructed graphic organizers appear to be more beneficial than those constructed by teachers. Monroe and Readence suggested that when students construct their own graphic organizers, they participate actively and process ideas themselves. Further, student-constructed graphic organizers allow for teacher observation of level of understanding so that instruction interventions may be provided. (p. 5)

By gradually increasing the ownership and accountability for creation of the Collaborative Four-Square Frayer Model, teachers would maintain the integrity of the model and could ensure that it was being used to its highest potential.

Finding Vocabulary Information. Since research using technology would result in an infinite amount of information made available to students, teachers would need to work with them to accurately filter non-essential information when researching elements of new vocabulary during the completing of the Collaborative Four-Square Frayer Model. It is recommended that teachers have students use their textbooks only as a source of information when first introducing the use of CFSF Models. Moreover, the material provided in textbook resources would typically be at grade level and would have been filtered to include important information for concept development, so once students became proficient at obtaining accurate vocabulary information from the textbook, teachers could provide specific websites to use as additional resources. As well, allowing students to insert images from the Internet into their CFSF Model could create visual connections to the vocabulary being developed and therefore help with concept retention. Once students were proficient at collaboratively developing vocabulary with limited resources (textbooks and authorised websites), teachers could then allow the use of the Internet freely, but would need to teach strategies to aid students in filtering out unimportant information. By gradually introducing more resources, students would learn which resources best fit their individual learning styles when encountering an unknown mathematical word. Varying the resources students could use would allow for multiple entry points for individuals to be able to contribute in equally productive and beneficial ways to the development of vocabulary using the Collaborative Four-Square Frayer Model.

How Google Docs© Could Assist. The Collaborative Four-Square Frayer Model would be most valuable if students collectively worked together on the document at the

same time – the use of technology would aid in this process. If the CFSF Model was completed on paper in small groups, it would still help in the development of student vocabulary, but not to the level that using technology would. By having all students working on the same document all at once using Google Docs®, every student would be expected to contribute. Participation could be in the form of research and addition of new information, comments or questions regarding information others included, or the discussions that should take place when contradictory ideas were provided. Students not comfortable speaking in group settings may be more willing to add information through the Google Docs® program. Teachers would be able to see which students were contributing by using the revision history tool on Google Docs®, and individual conversations could then happen with students not engaging in the process. Teachers could place requirements on participation, such as needing at least one contribution per CFSF Model (dependent on classroom structure, size, environment, etc.). Through the use of technology, the Collaborative Four-Square Frayer Model would engage all students throughout the process of their vocabulary development.

Vocabulary Instruction Implications for Teacher Practice

The following is a guide for teachers undertaking the task of vocabulary development in their mathematics classrooms. Teachers should select the strategies presented that best fit their individual teaching styles and student needs. The information presented is not intended to be prescriptive, but rather a catalogue of ideas that could be used for vocabulary instruction.

Effective Practices

To strengthen the vocabulary development of their students, teachers must be willing to alter their instructional practices to position vocabulary instruction at the forefront. First, teachers should become well-versed in the content-specific vocabulary and the ordinary language that could be used in their mathematics courses. They could collaborate with colleagues, not only at grade level, but also with previous and subsequent grades as well. Compiling a variety of resources on a curriculum that included textbooks and the program of study would pinpoint important vocabulary for mathematical concepts they taught. Also, attending professional development workshops that provided content area specialists with literacy strategies could be very beneficial for math teachers. Barton (1997) expressed that “training teachers in strategic reading skills need not be a time-consuming staff development process” (p. 28). Many schools have access to literacy experts (which may be the English teacher) that could lead this in-service workshop to other content area teachers. Generally, mathematics educators already have the skills to build vocabulary development for their students; they would only need to discover what skills they used themselves and determine how to transfer those skills to their students. If teachers explored their personal strategies while clarifying vocabulary and then could articulate their thinking processes aloud, students could be given real-time first-hand experiences with vocabulary development during instruction. Cantrall, Burns, and Callaway (2009) found that “teachers often reported that once they had an opportunity to implement and assess the impact of a strategy in their own classrooms with their own students, their discomfort diminished” (p. 87). For those who may find mathematical vocabulary instruction uncomfortable, increasing their confidence

and awareness of the strategies they use to acquire new terminology could allow them to translate this knowledge and new-found confidence to their students.

The vocabulary needs of one class would not match the needs of the next. Likewise, vocabulary important for direct and focused instruction would change from year to year dependent on the students in the room. Therefore, it would be imperative that teachers get to know their students and base vocabulary instruction decisions on this information. Flanigan and Greenwood (2007) introduced four factors that should be considered when contemplating vocabulary instruction: (a) the students being taught, (b) the words chosen to teach, (c) the instructional purpose for each word, and (d) the strategies that would be used. No one-size-fits-all approach to vocabulary development has been found, so teachers would need to be fluid and flexible throughout learning processes to support individual classes and student needs.

Acknowledging Students Prior Vocabulary Knowledge

Teachers must understand the current knowledge students bring to the classroom when considering vocabulary development and instruction. Pre-assessment on vocabulary would demonstrate students' current levels of understanding and retention of words. Observation would be key to discovering the struggles or concerns students have had with vocabulary. Teachers focused on the vocabulary development of their students should watch for problem terminology and be willing to address concerns promptly (Devries, 2012). Some methods teachers could use to pre-assess would be Quizlet Live© quizzes, Kahoot© quizzes, and vocabulary matching exercises. All pre-assessments would include preparation work on behalf of the teacher that would take a significant

amount of time; teachers would be encouraged to take it one step at a time, as in one unit at a time, to move their vocabulary instruction practice forward.

The first step to pre-assessment would be to generate a list of prior vocabulary words with which students should be familiar. The process could entail the teacher conferring with the preceding grade teacher to generate a list of important vocabulary previously taught, collaboration with grade-level teachers, use of tacit knowledge of student struggles, and investigation of earlier grade textbook resources and curriculum documents. Once the vocabulary list was created, the teacher could then decide which pre-assessment method to use.

After the pre-assessment, teachers could analyze the results by categorizing each vocabulary term as unknown, acquainted, or established words (Beck et al., 1967). *Established words* would not require the teacher to take any further action. *Acquainted words* should be attended to through direct and explicit instruction to resolve any confusion. For any words that were indicated as *unknown*, teachers could use the Collaborative Four-Square Frayer Model to develop that vocabulary. If students were struggling with vocabulary in the pre-assessment, it would be likely that they would have further difficulties with new concepts, as mathematics builds upon itself. By following this process of pre-assessment, teachers would be addressing the needs of their current students and beginning new topics with a base level of understanding for all students in their classes.

Selecting Important Math Vocabulary for New Concepts

To begin new mathematical instruction, teachers should pre-assess students' vocabulary knowledge as discussed previously. Of equal importance, teachers consider in

advance the vocabulary students will be encountering at the current grade level.

Vocabulary important for student conceptual understandings of new mathematical outcomes should be selected prior to instruction (Monroe & Pendergrass, 1997; Smith & Angotti, 2012). Vocabulary selected should be words that would challenge students in their understanding of mathematical concepts. As with pre-assessment of vocabulary, teachers should use their tacit knowledge, in conjunction with collaboration with same-grade colleagues and curriculum resources (including the program of study and mathematical textbooks), to determine commonly unknown vocabulary words for students. Teachers would then decide how to instruct on the important vocabulary, whether using the direct approach of the Collaborative Four-Square Frayer Model or another vocabulary instructional strategies. A limited amount of vocabulary should be selected for focused instruction to allow for time for a deeper understanding of the words (Blachowicz et al., 2006; Smith & Angotti, 2012). By carefully considering essential vocabulary, teachers could streamline the focus of their instruction and better prepare students for experiences with the important words.

Potential Instructional Techniques

A variety of pedagogy techniques could be used when incorporating vocabulary instruction into mathematics classrooms; teachers would need to be introduced to these techniques, and then select the approaches that would work best for both themselves and their students. Directly teaching the differences between ordinary language and the mathematical usage of words, in addition to instruction on the synonyms in math (such as distribute, simplify and expand), would be some examples of approaches that could be included in math teachers' pedagogical repertoire.

Structural Analysis. A major component of vocabulary development is the understanding of the structural analysis of words (prefixes, roots, etc.) - this is also true of math vocabulary. This base knowledge could aid students when contemplating an unfamiliar word in all content areas. If math teachers were to become more familiar with the structure of words, they would be able to pass this information on to their students. Trigonometry would be an example of a word where such a breakdown would be possible, as trigonometry “comes from the Greek words *trigonon* (“triangle”) and *metron* (“to measure”)” (The Editors of Encyclopædia Britannica, Maor, & Barnard, 2016). Students could transfer word structure information to new vocabulary situations with comparable roots and prefixes, and could then begin to interpret unfamiliar words themselves.

Multiple Meanings. Much research on mathematical vocabulary has outlined the difficulty with ordinary language words that have different meanings in mathematical concepts (Devries, 2002; Harmon, Hedrick, & Wood, 2005). Students should be made aware of these words beforehand as they could often be the culprits of confusion. Teachers should locate these words before instruction by examining resources students would be using. Also, if a student was to inquire about the use of an ordinary language word while completing a problem, the teacher could address both meanings and clarify the use of the vocabulary in mathematics. Providing students with multiple opportunities to see and obtain clarification on words with multiple meanings could start the development of vocabulary knowledge, which could then be drawn upon while completing future mathematical problems.

Teacher Awareness of Own Vocabulary. Teachers should be careful to always use the appropriate mathematical vocabulary and not to simplify the language; students need to understand that some words in math are complex and cannot be simplified. In my experience, one common example of teachers' changing vocabulary to make it simpler for students was in using the word *flip* instead of *reciprocal*, when switching the numerator and the denominator. Using the inaccurate term *flip* can cause confusion as this term is synonymous with reflection in mathematics. The notion of *reciprocals* becomes important in higher level mathematics and therefore should be taught correctly from the start. Towers and Hunter (2010) stated that "by occasionally recording and listening to the talk in their own classrooms, then, teachers may recognize similar trends in their own practices and be motivated to pay special attention to language use in their classrooms" (p. 37). Likely, teachers are not intentionally using incorrect vocabulary, so drawing attention to these types of errors could only improve the mathematical instruction for students.

Classroom Environment. Environmental factors could determine the level of student success during vocabulary instruction in mathematics. First and foremost, participation should be expected from all students (Towers & Hunter, 2010); if students were to be passive observers rather than active participants in the vocabulary development process, they would likely not gain the requisite skills needed to be successful when approaching unfamiliar and difficult terminology. Considerable classroom discussion of mathematical vocabulary should occur, for as Pierce and Fontaine (2009) asserted, the discussion must be "rich and lively...to encourage deep processing of the word's meaning" (p. 239). Multiple opportunities should take place for

students to experience new vocabulary in diverse contexts (such as discussion, instruction, word problems, etc.). It would be the role of the teacher to lead discussions and make certain that vocabulary development was at the forefront of the learning so that students would understand the value and importance in building their vocabulary knowledge. When encountering an unfamiliar word, teachers should break down the vocabulary out loud in order to disclose the method they used to determine meaning. Lastly, speaking mathematically, for both students and teachers themselves, should be viewed as an essential skill (Barton, 1997; Dunston & Tyminski, 2013); by always using proper terminology, teachers would be demonstrating accurate mathematical language as it should be written. These small modifications to a classroom environment should deepen the vocabulary development process for students, as it would be perceived as the utmost importance in strengthening mathematical understandings.

Assessing Vocabulary Development

Student participation in vocabulary development tasks such as scholarly classroom discussions and the Collaborative Four-Square Frayer Model should deepen the understanding of mathematical words. Teachers would need to determine whether the process of vocabulary instruction they used was working or not. Teachers could both informally and formally assess the vocabulary development of their students throughout the semester.

Informal Assessments. Informal assessments could include observation of students during work blocks, CFSF Model creation, and classroom instruction. Teachers would need to be constantly mindful of vocabulary use and development in their classrooms and take every opportunity to listen for student confusion or clarify mishaps

with mathematical words. Listening to student conversations during group work and contributions to classroom discussions would provide teachers with evidence of vocabulary development; if students were consistently using proper terminology and speaking mathematically, then the vocabulary instruction could be determined successful.

Formal Assessments. The same methods introduced in the pre-assessment discussion could be used as formal assessments in the form of word sorts, Quizlet Live© quizzes, Kahoot© quizzes, and vocabulary matching exercises. Word sorts and vocabulary matching exercises would encompass a list of vocabulary definitions, terms, and examples that students would match; these methods would clearly demonstrate which vocabulary students were continuing to struggle with at the end of a learning period. Using the essential list of grade-level vocabulary developed before instruction, teachers could create Quizlet Live© and Kahoot© quizzes, testing student knowledge in an engaging manner.

Teachers could also complete more official formal assessments by either video recording lessons or having a colleague observe. Video recording lessons at the beginning, middle, and end of a learning period would demonstrate the vocabulary development of students after focused and direct instruction. Inviting an observer into the classroom could be beneficial as their interpretation could bring new insight. In either of these formal assessment approaches, teachers would be encouraged to set parameters of what they wanted to observe – specific word usage, mathematical vs. non-mathematical vocabulary used, etc.

Limitations and Implications for Further Consideration

The approaches described in this project for effective vocabulary instruction were based on research conducted by others. Research does not explicitly demonstrate how vocabulary acquisition affects mathematical achievement; much of the insights given are casual relationships and not definitive relationships. Though the individual facets of vocabulary instruction were well researched and documented by others, research has not been conducted with how they have been compiled in this project.

Most questions on the self-efficacy questionnaire, *Math and Me*, were duplicated from Usher and Pajare's (2009) *Sources of Self-Efficacy in Mathematics* survey, which demonstrated construct validity and was deemed valuable by other academics to determine whether students' self-efficacy had grown or declined (Carpenter & Clayton, 2014). Questions used in this tool would likely not be suitable across all grade levels, as the *Math and Me* questionnaire was created for a middle school math class, however, could be modified to meet the needs of both younger and older students by changing the language used.

A variety of vocabulary instruction methods exist; the Collaborative Four-Square Frayer Model presented in this project being one of many. Teachers should research and investigate methods that would best fit their teaching pedagogies and the needs of the students in their classes.

Teachers would need to be willing to change their practice and make vocabulary a priority in all classroom interactions to see improvement in student vocabulary knowledge. Vocabulary clarification would be a time-consuming practice both in preparatory work and instructional time; teachers must be willing to engage fully in the

process. Educators choosing to use the *Math and Me* self-efficacy questionnaire, Collaborative Four-Square Frayer Model, or to follow the suggestions provided in the Vocabulary Instruction Implications for Teacher Practice guide presented in this paper, should modify any materials as presented to meet the needs of their classrooms.

Professional development must be provided and sufficient time must be given for teachers to become experts in the vocabulary instruction component of mathematical pedagogy (Cantrell, Burns, & Callaway, 2009; Harmon, Hedrick, & Wood; 2005). Introducing pedagogical approaches to vocabulary instruction during teacher education programs would guarantee that vocabulary development would be considered essential to student successes by new educators in the profession (Blachowicz et al., 2006). Improving teacher knowledge and practice regarding the effective development of student vocabulary knowledge should be considered essential for mathematics teaching.

Conclusion

Research linking self-efficacy with academic achievement was a major motivator for this project. The goal of increasing students' mathematical vocabulary knowledge was to make them feel better about themselves as math learners, so that they would be more willing to take risks and be motivated to learn more deeply, consequently increasing their self-efficacy, and in turn, possibly increasing their mathematical academic achievement.

Insufficient student mathematical vocabulary knowledge has been demonstrated to be a concern. Reading comprehension has been linked to student achievement in mathematics; a lack of understanding of vocabulary could be detrimental to student success. Student self-efficacy has been shown to connect to student achievement in a cyclical way; when a student sees more success, they feel better about themselves, and

when they feel better about themselves, they see more success (Carpenter & Clayton, 2014; Pinxten et al., 2014). Since both vocabulary acquisition and self-efficacy have been linked to student achievement, the premise of this project was that there should be a relationship between these two factors as well. If students were given focused and clear instruction on mathematical vocabulary clarification, confronting challenging word problems should be less complicated. This would make the task of solving mathematics problems more straightforward. If students had strategies for approaching difficult problems and had a deeper base knowledge of mathematics, it would reason that they would feel better about themselves as math learners, simultaneously increasing their level of self-efficacy.

As the intention of this project was to clarify mathematical vocabulary, opportunities for students to develop mastery experiences were developed through the Collaborative Four-Square Frayer Model and the Vocabulary Instruction Implications for Teacher Practice guide. The *Math and Me* self-efficacy questionnaire should allow teachers to investigate whether student self-efficacy changed for better, worse, or not at all. By implementing the Collaborative Four-Square Frayer Model to develop vocabulary and by using pre/post vocabulary tests, students should receive continual feedback from not only their teachers but their peers as well.

All resources provided in this project promoted the direct and focused development of student mathematical vocabulary clarification. Petty, Herold, and Stoll (1968) stated that “some teaching effort causes students to learn vocabulary more successfully than does no teaching effort, that any attention to vocabulary development is better than none” (p. 85). Math teachers who focus their instruction on clarifying

vocabulary for students and on deepening their students' knowledge base, using the Collaborative Four-Square Frayer Model introduced or any other pedagogical approach, should see progress in student self-efficacy, mathematical achievement, or both.

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Appendices

Appendix A

“Math and Me” Self-Efficacy Questionnaire - Paper Version

Math and Me

Name: _____

Please fill out the following survey completely and honestly.

This information will be used by your teacher to understand more about you as a math learner.

		<i>Definitely False</i>	<i>Mostly False</i>	<i>More False than True</i>	<i>More True than False</i>	<i>Mostly True</i>	<i>Definitely True</i>
	Question	1	2	3	4	5	6
1	I make excellent grades on math tests.						
2	I have always been successful with math.						
3	Even when I study very hard, I do poorly in math.						
4	I got good grades in math on my last report card.						
5	I do well on math assignments.						
6	I do well even on the most difficult math assignments.						
7	My math teachers have told me that I am good at learning math.						
8	People have told me that I have a talent for math.						
9	Adults in my family have told me what a good math student I am.						
10	I have been praised for my ability in math.						
11	Other students have told me that I'm good at learning math.						
12	My classmates like to work with me in math because they think I'm good at it.						
13	When completing a word problem in math, the language used is often confusing.						
14	I understand most of what is said during math class.						
15	I know that mathematics has its own language that can be figured out.						

Appendix B

“Math and Me” Self-Efficacy Questionnaire - Google Forms Version

Math and Me

Please fill out the following survey completely and honestly.
This information will be used by your teacher to understand more about you as a math learner.

* Required

Email address *

Your email

*

		1 - Definitely False	2 - Mostly False	3 - More False, than True	4 - More True, than False	5 - Mostly True	6 - Definitely True
#1 - I make excellent grades on math tests.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#2 - I have always been successful with math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#3 - Even when I study very hard, I do poorly in math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#4 - I got good grades in math on my last report card.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#5 - I do well on math assignments.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#6 - I do well even on the most difficult math assignments.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#7 - My math teachers have told me that I am good at learning math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#8 - People have told me that I have a talent for math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#9 - Adults in my family have told me what a good math student I am.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#10 - I have been praised for my ability in math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#11 - Other students have told me that I'm good at learning math.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#12 - My classmates like to work with me in math because they think I'm good at it.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#13 - When completing a word problem in math, the language used is often confusing.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#14 - I understand most of what is said during math class.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
#15 - I know that mathematics has its own language that can be figured out.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Send me a copy of my response.

SUBMIT

Appendix C

Fruyer, Frederick, and Klausmeier's 1969 Schema for Testing the Level of Concept

Mastery

The information on which the quadrilateral items were based is as follows:

Attributes: closed figure, plane figure, simple figure, 4 sides (4 angles), relative length of sides, relative size of angles, parallelness of sides, size of figure, orientation of figure.

Attribute Value Examples: (e.g., closed figures)



Attribute Value Non-Examples: (e.g., non-closed figures)



Concept Label: quadrilateral

Concept Examples:



Concept Non-Examples:



Relevant Attribute Values: closed figure, plane figure, simple figure, 4 sides (4 angles).

Irrelevant Attributes: relative length of sides, relative size of angles, parallelness of sides, size of figure, orientation of figure.

Concept Definition: A plane closed figure with 4 sides.

Supraordinate Concept: polygon.

Coordinate Concepts: triangle, pentagon, hexagon.

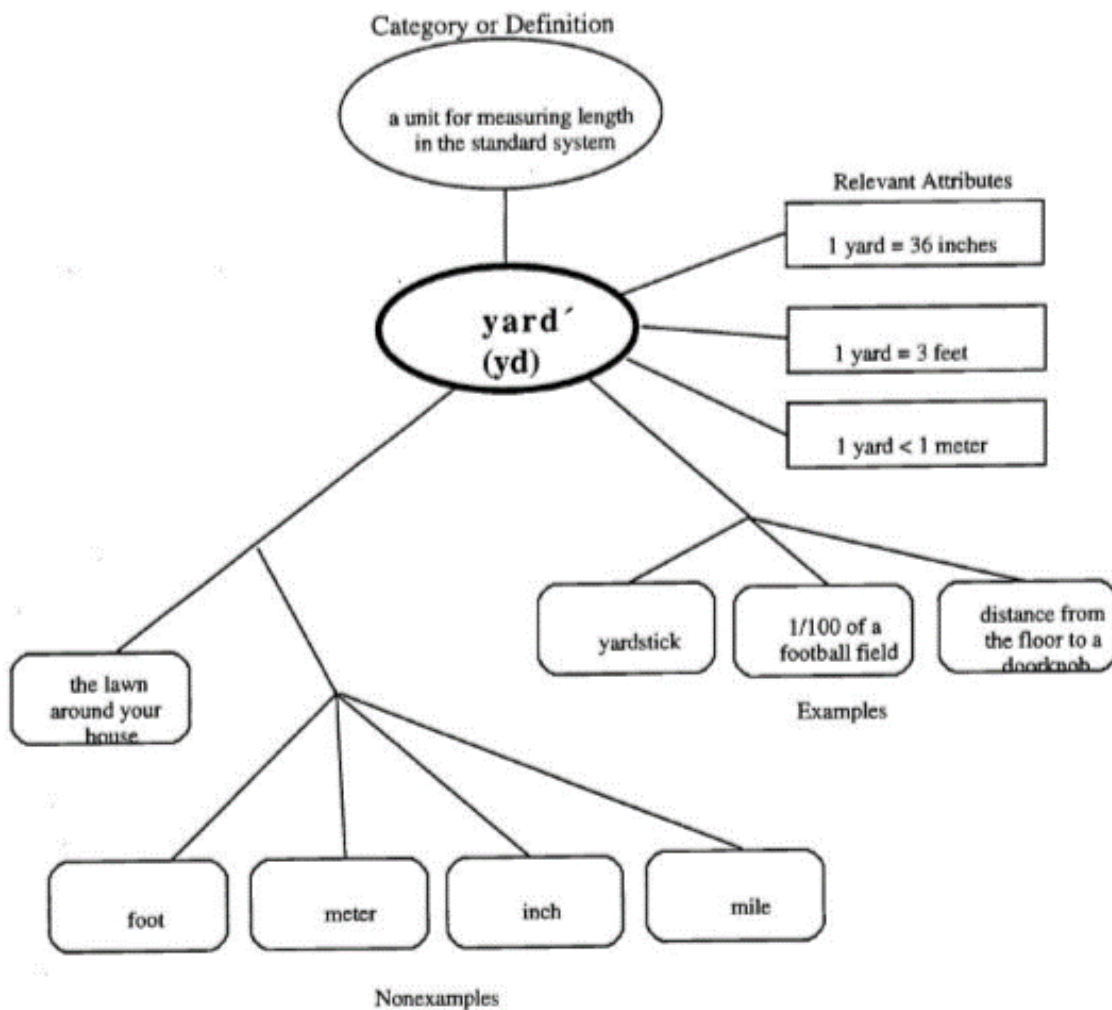
Subordinate Concepts: trapezoid, kite, parallelogram, rectangle, rhombus, square.

Principle: The perimeter is the distance around a quadrilateral.

Problem Situations: Find the perimeter of a given quadrilateral.

Appendix D

Monroe and Pendergrass' 1997 Integrated CD-Frayer Model



Appendix E

Brunn's 2002 Four-Square Strategy

This particular example was for the phonetic element "ch" and Brunn encouraged teachers to choose their own categories depending on the concept selected and student needs.

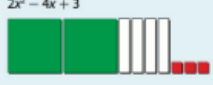
Target phonetic element

ch

Appendix F

Collaborative Four-Square Frayer Model

WORD: POLYNOMIAL

<p>Attributes:</p> <ul style="list-style-type: none"> - terms connected by addition and/or subtraction signs - expression (no equal sign) - can have one or more terms - can include variables, coefficients, constants, exponents - prefix 'poly' means many - some have specific names, but all considered polynomials Monomial - one term Binomial - two terms Trinomial - three terms - can be modelled with tiles $\underbrace{3x^2}_{\text{term 1}} - \underbrace{5x}_{\text{term 2}} + \underbrace{4}_{\text{term 3}}$ <p>(https://en.wikipedia.org/wiki/Polynomial)</p>	<p>Examples:</p> <table border="1"> <thead> <tr> <th>Name</th> <th>Number of Terms</th> <th>Example</th> </tr> </thead> <tbody> <tr> <td>monomial</td> <td>1</td> <td>$6x^2$</td> </tr> <tr> <td>binomial</td> <td>2</td> <td>$3x^2 - 5$</td> </tr> <tr> <td>trinomial</td> <td>3</td> <td>$-u^2 + 5uv + 1$</td> </tr> <tr> <td>polynomial</td> <td>more than 3</td> <td>$2s^2 - t^2 + st + 7t - 4$</td> </tr> </tbody> </table> <p>Mc Askill, B. (2009). <i>MathLinks 9</i>. Toronto: McGraw-Hill Ryerson.</p> <p>$2x^2 - 4x + 3$</p>  <p>Mc Askill, B. (2009). <i>MathLinks 9</i>. Toronto: McGraw-Hill Ryerson.</p> <p>Non-Examples:</p> <p><i>Biology</i>: a species name containing more than two terms. (http://www.dictionary.com/browse/polynomial?s=t)</p>	Name	Number of Terms	Example	monomial	1	$6x^2$	binomial	2	$3x^2 - 5$	trinomial	3	$-u^2 + 5uv + 1$	polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$
Name	Number of Terms	Example														
monomial	1	$6x^2$														
binomial	2	$3x^2 - 5$														
trinomial	3	$-u^2 + 5uv + 1$														
polynomial	more than 3	$2s^2 - t^2 + st + 7t - 4$														
<p>Problem Situation:</p> <p>23. Marion gives French lessons in the evening. She charges \$20 for adults and \$15 for children. The expression $20a + 15c$ represents her earnings.</p> <ol style="list-style-type: none"> What do the variables a and c represent? How much does Marion make if she gives lessons to four adults and nine children? Show your work. Write a new expression for Marion's earnings if she charges \$3 more for adults and \$2 more for children. <p>Mc Askill, B. (2009). <i>MathLinks 9</i>. Toronto: McGraw-Hill Ryerson.</p>	<p>Concept Category:</p> <p>Algebraic expression</p>															