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Black-hole thermodynamics: Entropy, information and beyond

SAURYA DAS
Department of Physics, The University of Lethbridge, 4401 University Drive, Lethbridge, Alberta, T1K 3M4, Canada
E-mail: saurya.das@uleth.ca

Abstract. We review some recent advances in black-hole thermodynamics including statistical mechanical origins of black-hole entropy and its leading order corrections from the view points of various quantum gravity theories. We then examine the problem of information loss and some possible approaches to its resolution. Finally, we study some proposed experiments which may be able to provide experimental signatures of black holes.

Keywords. Black-hole thermodynamics; quantum gravity.

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1. Introduction

Existence of black holes is one of the most intriguing predictions of general relativity. They are expected to have Bekenstein–Hawking entropy and radiate at their characteristic Hawking temperature [1]. Furthermore, these quantities should satisfy laws analogous to the laws of thermodynamics. For example, for a Reissner–Nordström (RN) black hole of mass M and charge Q in d-space-time dimensions with the metric:

$$\begin{aligned} \mathrm{d}s^2_{\mathrm{RN}} = &- \left(1 - \frac{16\pi G_d M}{(d-2)c^2\Omega_{d-2}r^{d-3}} + \frac{16\pi G_d Q^2}{(d-2)(d-3)c^4 r^{2(d-3)}}\right) c^2 \mathrm{d}t^2 \\ &+ \left(1 - \frac{16\pi G_d M}{(d-2)c^2\Omega_{d-2}r^{d-3}} + \frac{16\pi G_d Q^2}{(d-2)(d-3)c^4 r^{2(d-3)}}\right)^{-1} \mathrm{d}r^2 \\ &+ r^2 \mathrm{d}\Omega^2_{d-2}, \end{aligned} \tag{1}$$

the (outer) horizon radius, electrostatic potential at the horizon, Hawking temperature, and entropy are given by

$$r_+^{d-3} = \frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}} + \sqrt{\left(\frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}}\right)^2 - \frac{2G_d Q^2}{(d-2)(d-3)c^4}}, \tag{2}$$

$$\Phi = \sqrt{\frac{2(d-3)}{d-2}}\ \frac{Q}{r^{d-3}}, \tag{3}$$

$$T_{\rm H} = \frac{(d-3)\hbar c}{2\pi r_+^{d-2}} \sqrt{\left(\frac{8\pi G_d M}{(d-2)c^2\Omega_{d-2}}\right)^2 - \frac{2G_d Q^2}{(d-2)(d-3)c^4}}, \tag{4}$$

$$S_{\rm BH} = \frac{A_{\rm H}}{4\lambda_{\rm Pl}^2} = \frac{\Omega_{d-2} r_+^{d-2}}{4\lambda_{\rm Pl}^{d-2}}, \tag{5}$$

where $A_{\rm H}$ is the black-hole horizon area. These satisfy the zeroth, first, and second laws of 'black-hole thermodynamics'.

$$T_{\rm H} = \text{ constant over horizon}, \tag{6}$$

$$\mathrm{d}(Mc^2) = T_{\rm H}\mathrm{d}S_{\rm BH} + \Phi \mathrm{d}Q, \tag{7}$$

$$\Delta S_{\rm BH} \geq 0, \tag{8}$$

G_d and $\lambda_{\rm Pl}$ being the d-dimensional Newton's constant and Planck length respectively, and Ω_{d-2} the area of S^{d-2}.

One of the foremost problems in quantum gravity is to explain the origin of Bekenstein–Hawking entropy. In other words, to discover a set of fundamental degrees of freedom which give rise to a (large) degeneracy Ω, such that

$$S_{\rm BH} = \ln \Omega, \tag{9}$$

(where the Boltzmann constant has been set to unity). Various approaches to quantum gravity have attempted to answer this question, with different degrees of success. The related issue of Hawking radiation has also been examined in these approaches. In the following sections, we will review a few of these important approaches including loop quantum gravity and string theory. Associated with black holes is the so-called problem of 'information loss', which we examine in §5. Finally, we review some future experiments which could shed light on the nature of quantum gravity and test the correctness of some of the theories.

2. Statistical mechanical origins of entropy

2.1 *Horizon conformal field theory*

There have been several attempts to explain the origin of Bekenstein–Hawking entropy from an underlying conformal field theory (see e.g. [2,3]). Here, we review a recent approach which is applicable to a large class of black holes [3]. We start with the Einstein action in d-dimensions:

$$I = \frac{c^3}{16\pi G_d} \int R\sqrt{-g}\ \mathrm{d}^d x\ . \tag{10}$$

If one restricts oneself to the spherically symmetric sector with the metric

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + \phi(x)\mathrm{d}\Omega^2_{d-2}, \quad \mu,\nu = t, x, \tag{11}$$

where x is a radial coordinate and $\phi(x) = r(x)^2/\lambda^2_{\mathrm{Pl}}$, then the action (10) reduces to a two-dimensional dilaton gravity action of the form [4]

$$I = \int \mathcal{L}\sqrt{-\gamma}\ \mathrm{d}^2x = \frac{c^3}{2G_2}\int \left(\phi R_2 + \frac{1}{\lambda^2_{\mathrm{Pl}}}V[\phi]\right)\sqrt{-\gamma}\ \mathrm{d}^2x, \tag{12}$$

where the potential $V[\phi]$ depends on the matter content of the theory. Next, define a quantity analogous to the expansion of null congruences:

$$\Theta = \frac{1}{\phi}\ell^a\nabla_a\phi \equiv \frac{s}{\phi}\ , \tag{13}$$

where ℓ^a is the null normal. Black-hole horizons are characterised by vanishing Θ. Now, it can be shown that under the set of transformations

$$\delta g_{ab} = \nabla_c(f\ell^c)g_{ab}, \tag{14}$$

$$\delta\phi = (\ell^c\nabla_c h + \kappa h), \tag{15}$$

where f is an arbitrary function, $\kappa(= 2\pi T_{\mathrm{H}}/\hbar)$ the surface gravity and $h = sf/\kappa$, the variation of the Lagrangian takes the following form:

$$\delta\mathcal{L} \sim \Theta, \tag{16}$$

which vanishes at the horizon. Transformations (14) and (15) thus constitute an asymptotic symmetry. It is generated by the Hamiltonian

$$L[f] = -\frac{c^3}{2G_2}\int_\Delta (2\ell^a\nabla_a s - \kappa s)\, f\sqrt{\gamma}\mathrm{d}^2x, \quad s = \ell^a\nabla_a\phi, \tag{17}$$

where Δ is an element of the horizon. The function f can be expressed in terms of the basis functions

$$f_n = \frac{\phi_+}{2\pi s}z^n, \quad z = \exp(2\pi i\phi/\phi_+) \ \text{ with } \ \{f_m, f_n\} = i(m-n)f_{m+n} \tag{18}$$

as

$$f = \sum c_n f_n. \tag{19}$$

Computation of Poisson brackets yields

$$\{L[f_m], L[f_n]\} = -\frac{24\pi s}{G_2\kappa}\ \frac{n^3}{12}\ \delta_{m+n,0}, \tag{20}$$

where the RHS can be identified with the central charge of the Virasoro algebra: $\mathcal{C} = -24\pi s/G_2\kappa$. The eigenvalue of L_0, Δ is given by

$$L[f_n] = -\frac{\kappa\phi_+^2}{4\pi G_2 s}\delta_{n0} \equiv \Delta\delta_{n0}. \tag{21}$$

The asymptotic density of states is then given by the Cardy formula, whose logarithm gives the microcanonical entropy

$$S = \ln\rho(\Delta) = 2\pi\sqrt{\frac{\mathcal{C}\Delta}{6}} = \frac{2\pi\phi_+}{G_2} = \frac{A_{\rm H}}{4\lambda_{\rm Pl}^2} = S_{\rm BH}. \tag{22}$$

This is observed to agree with the Bekenstein–Hawking entropy of the black hole. In other words, in this approach, the conformal field theoretic (CFT) degrees of freedom appear to be responsible for the black-hole entropy.

2.2 *Loop quantum gravity*

Next, we examine loop quantum gravity (LQG) [5]. Imposition of the null condition on Δ, as well as those of no radiation falling in and no rotation are equivalent to the condition:

$$\frac{A_{\rm H}}{2\pi\gamma}F_{ab}^{AB} + \Sigma_{ab}^{AB} = 0, \tag{23}$$

where F is the field strength corresponding to the $SL(2,C)$ connection A_{aA}^{B}, $\sigma_a^{AA'}$ is the soldering form for $SL(2,C)$ spinors, the metric $g_{ab} = \sigma_a^{AA'}\sigma_{bAA'}$, $\Sigma_{ab}^{AB} = 2\sigma_{[a}^{AA'}\sigma_{b]A'}^{B}$ $(a,b,\ldots(A,B,\ldots)$ are space-time (internal) indices) and pull-backs of F and Σ to the horizon two sphere are understood. The gravity action can be written in terms of these variables as

$$I = -\frac{i}{8\pi G}\int \mathrm{Tr}\,(\Sigma\wedge F) - \frac{i}{8\pi G_4}\frac{A_{\rm H}}{4\pi}\int_\Delta \mathrm{Tr}\left(A\wedge \mathrm{d}A + \frac{2}{3}A\wedge A\wedge A\right). \tag{24}$$

The second term is the appropriate boundary term on Δ, which is nothing but the Chern–Simons action. The quantum version of (23) can be written as

$$\left(I\otimes\frac{A_{\rm H}}{2\pi\gamma}F_{ab}\cdot r + \Sigma_{ab}^{AB}\cdot r\otimes I\right)\Psi_{\rm V}\otimes\Psi_{\rm S} = 0, \tag{25}$$

where r is an internal vector and $\Psi_{\rm V}$ and $\Psi_{\rm S}$ are volume and surface states, which are eigenvalues of the first and second terms of (25) respectively. It is the surface degrees of freedom which are responsible for entropy. These puncture Δ in a finite number of points n, at each of which there is an associated spin. Thus, the collection of surface states can be collectively written as

$$\mathcal{P} = \{(p_1, j_{p_1}), \ldots, (p_n, j_{p_n})\}, \tag{26}$$

where j_{p_i} is the spin labelling the puncture p_i.

The horizon area can be thought of as being built up of individual bits of area associated with the punctures, where a spin j_p contributes a quantum of $8\pi\gamma\lambda_{\rm Pl}^2\sqrt{j_p(j_p+1)}$, γ being the unknown Immirzi parameter. Thus

$$A_{\rm H} = 8\pi\gamma\lambda_{\rm Pl}^2 \sum_p \sqrt{j_p(j_p+1)} \tag{27}$$

(applications of LQG with a different regularisation, as well as in the context of black-hole coherent states, an equispaced area spectrum is predicted, although the area proportionality of entropy remains unchanged [6,7]). The dimensionality of the Hilbert space is a product of those for each spin, which is

$$\dim(\mathcal{H}) \sim \prod_{j_p\in\mathcal{P}} (2j_p+1) \approx 2^P, \tag{28}$$

where the last step follows from the fact that $j = 1/2$ dominates the spin configuration. The number of punctures P can be estimated from (27) under the same assumption, which when plugged into (28) gives us the microcanonical entropy:

$$\begin{aligned} S_{\rm BH} &= \ln\dim(\mathcal{H}) \\ &\approx P\ln 2 \\ &= \left(\frac{\gamma_0}{\gamma}\right)\frac{A_{\rm H}}{4\lambda_{\rm Pl}^2}, \quad \gamma_0 = \frac{\ln 2}{\pi\sqrt{3}}\ . \end{aligned} \tag{29}$$

We see that the area proportionality of entropy emerges naturally, although the prefactor is 1/4 only for $\gamma = \gamma_0$. It may be mentioned that the value of γ_0 does not depend on the specific black hole that one considers and is the same for dilatonic and charged black holes.

2.3 *String theory*

Next, we turn to string theory, one of the best explored approaches to quantum gravity. One of the first black holes explored within string theory was a four-dimensional charged black hole with a singular horizon [8]. However, the one for which Bekenstein–Hawking entropy is best explained in terms of string states is the five-dimensional extremal RN black hole. This will be reviewed in the next sub-section [9], following which anti-de Sitter–Schwarzschild black holes will also be examined in the context of AdS–CFT correspondence [10].

2.3.1 *Extremal charged black holes*: We start with the ten-dimensional low energy effective action of Type-II string theory in the strong coupling (large G_{10}, equivalently large string coupling g) limit:

$$I = \frac{c^3}{16\pi G_{10}}\int {\rm d}^{10}x\sqrt{-g_{10}}\left[R + \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}{\rm e}^{\phi}H_{(3)}^2\right], \tag{30}$$

where ϕ is the dilaton and $H_{(3)}$ is the RR–3-form field strength. Compactifying on a $T^4 \times S^1$ (note that originally the compact manifold $K3 \times S^1$ was considered), the above has a metric solution of the form

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\chi}\mathrm{d}x_i\mathrm{d}x^i + \mathrm{e}^{2\psi}(\mathrm{d}x_5 + A_\mu \mathrm{d}x^\mu)^2 + \mathrm{e}^{-2(4\chi+\psi)/3}\mathrm{d}s_5^2 \ . \tag{31}$$

In the above, χ and ψ are scalar fields, A_μ is a gauge field, the first and second terms represent metrics on T^4 and S^1 respectively, while $\mathrm{d}s_5^2$ is the five-dimensional extremal RN metric. The above configuration has a description in terms of D-branes and Kaluza–Klein (KK) momenta in the weak coupling (small G_{10}) limit. More precisely, the latter consists of $N_1 D$–1-branes (which couple to $H_{(3)}$), $N_5 D$–5-branes (which couple to $^\star H_{(3)}$) and N-units of KK momenta on S^1. Extremality condition for black holes translates to the condition of BPS saturation for these branes. In the case in which these three charges are equal, they are related to the black-hole charge Q and horizon radius r_+ as

$$N_1 = N_5 = N = \frac{Q}{\sqrt{c\hbar\lambda_{\mathrm{Pl}}}} = \left(\frac{r_+}{\lambda_{\mathrm{Pl}}}\right)^2 \ . \tag{32}$$

Now, since open strings begin and end on D-branes, there will be a total of $N_1 N_5$ oriented strings stretching between the various 1- and 5-branes (it can be shown that those that begin and end on the same brane do not contribute to entropy to leading order). Each such string has 4-bosonic and 4-fermionic degrees of freedom associated with it, corresponding to the four transverse directions of $T^4 \times R$ (i.e. total of $n_{\mathrm{B}} = n_{\mathrm{F}} = 4N_1N_5$). Each such degree of freedom has an energy of $N\hbar c/L$, where L is the length of S^1. Moreover L is taken to be much larger compared to the length dimensions of T^4. Then, the entropy of this one-dimensional gas of bosons and fermions is given by [9]

$$S = \sqrt{\frac{\pi(2n_{\mathrm{B}} + n_{\mathrm{F}})LE}{6\hbar c}} = 2\pi\sqrt{N_1N_5N} \ , \tag{33}$$

which using (32) yields

$$S = \frac{\Omega_3 r_+^3}{4\lambda_{\mathrm{Pl}}^3} = \frac{A_5}{4\lambda_{\mathrm{Pl}}^3} = S_{\mathrm{BH}}. \tag{34}$$

Thus we see that the entropy of the one-dimensional gas exactly reproduces the Bekenstein–Hawking entropy of the corresponding black hole. Furthermore, since the counting is done for BPS branes, supersymmetry ensures that it suffers no renormalisations and the result continues to hold even at strong coupling.

2.3.2 *Asymptotically anti-de Sitter black holes*: Next, we consider black holes in the context of the AdS/CFT correspondence. The metric of an AdS–Schwarzschild (AdS–SC) black hole in d-dimensions is given by

$$\begin{aligned}\mathrm{d}s_d^2 = &-\left(1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2}c^2 r^{d-3}} + \frac{r^2}{\ell^2}\right)\mathrm{d}t^2 \\ &+\left(1 - \frac{16\pi G_d M}{(d-2)\Omega_{d-2}c^2 r^{d-3}} + \frac{r^2}{\ell^2}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{d-2}^2\end{aligned} \tag{35}$$

whose entropy and Hawking temperature are

$$S_{\rm BH} = \frac{\Omega_{d-2} r_+^{d-3}}{4\lambda_{\rm Pl}^{d-2}} \approx c_1' T_{\rm H}^{d-2}, \quad \left[c_1' = \frac{\Omega_{d-2}}{4\lambda_{\rm Pl}^{d-2}} \left(\frac{4\pi\ell^2}{\hbar c(d-1)}\right)^{d-2} \right] \tag{36}$$

$$T_{\rm H} = \hbar c \frac{(d-1)r_+^2 + (d-3)\ell^2}{4\pi\ell^2 r_+} \approx \hbar c \frac{(d-1)r_+}{4\pi\ell^2}, \quad r_+ \gg \ell, \tag{37}$$

where the approximation $r_+ \gg \ell$ is known as the 'high-temperature limit'.

First we examine whether the above thermodynamic properties can be modelled by a 'dual gas' consisting of a perfect fluid of bosons and fermions, residing in $\mathcal{D}$ space-time dimensions, in some appropriate boundary of the black-hole space-time (Note: here $\mathcal{D}$ is identical to the Δ of ref. [11].) We assume a general dispersion relation between the energy and momentum of the constituents of the gas, given by $\epsilon = \kappa p^\alpha$. It can be shown that the free energy and entropy of the gas in this case is related to its temperature as

$$F_{\rm gas} = -\frac{c_2' V_{\mathcal{D}-1}}{(\mathcal{D}-1)/\alpha + 1} T^{(\mathcal{D}-1)/\alpha+1}, \tag{38}$$

$$S_{\rm gas} = c_2' V_{\mathcal{D}-1} T^{(\mathcal{D}-1)/\alpha}, \tag{39}$$

where

$$c_2' = \frac{\Omega_{\mathcal{D}-2}\left((\mathcal{D}-1)/\alpha+1\right)\zeta\left(\frac{\mathcal{D}-1}{\alpha}+1\right)\Gamma\left(\frac{\mathcal{D}-1}{\alpha}+1\right)\left(n_{\rm B}+n_{\rm F}-\frac{n_{\rm F}}{2^{(\mathcal{D}-1)/\alpha}}\right)}{(\mathcal{D}-1)\kappa^{(\mathcal{D}-1)/\alpha}(2\pi\hbar)^{\mathcal{D}-1}}.$$

Further, if we assume that the gas is at a distance r_0, where it is in equilibrium with the Hawking radiation, then its temperature is related to the red-shifted Hawking temperature as

$$T = \frac{T_{\rm H}}{\sqrt{-g_{00}}} = \frac{\ell T_{\rm H}}{r_0}. \tag{40}$$

Plugging this into (39), we get

$$S_{\rm gas} = c_2' V_{\mathcal{D}-1} \left(\frac{\ell}{r_0}\right)^{(\mathcal{D}-1)/\alpha} T_{\rm H}^{(\mathcal{D}-1)/\alpha}. \tag{41}$$

Matching powers and coefficients of $T_{\rm H}$ in (36) and (41), we arrive at the following [11]:

$$\mathcal{D} = \alpha(d-2) + 1 \tag{42}$$

$$c_1' = c_2' \, \frac{(d-1)\Omega_{\mathcal{D}-1}\ell^{d-2} r_0^{(\alpha-1)(d-2)}}{(\mathcal{D}-1)/\alpha + 1}. \tag{43}$$

The above relations can be thought of as necessary conditions that any quantum theory of gravity must satisfy, if it wishes to describe AdS–SC black holes holographically. Note that only in the case $\alpha = 1$ (relativistic dispersion) is the usual holographic dimension ($\mathcal{D} = d-1$) recovered. Simultaneously, r_0 vanishes from (43) (i.e. the precise location of the dual gas becomes irrelevant), suggesting perhaps that $\alpha = 1$ is preferred. The full significance of $\mathcal{D} \neq d-1$ is general, including fractional $\mathcal{D}$ is yet to be understood.

With the above formalism at hand, let us test the validity of relations (42) and (43) in the light of $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence. The dual of the black hole in this case has been conjectured to be $\mathcal{N} = 4$, $SU(N)$ super Yang–Mills theory for large N. The number of bosonic/fermionic degrees of freedom and the relation between $\ell, \lambda_{\mathrm{Pl}}$ and N are in this case (here $\alpha = 1$) [10]:

$$N_{\mathrm{B}} = N_{\mathrm{F}} = 8N^2, \quad \left(\frac{\ell}{\lambda_{\mathrm{Pl}}}\right)^3 = \frac{2N^2}{\pi}. \tag{44}$$

Using the above, it is easy to show that [12]

$$c_1' = \frac{3}{4}c_2', \quad S_{\mathrm{BH}} = \frac{3}{4}S_{\mathrm{gas}}. \tag{45}$$

That is, the entropies of the quite different physical systems (black hole and gas) are almost identical! It has been conjectured that the discrepancy of the factor of 3/4 is due to the strong vs. weak coupling of the two systems, although a rigorous proof is lacking.

3. Leading order corrections to entropy

We now ask the following question: does the matching of equilibrium entropy of black hole and its dual (as in the case of AdS/CFT) guarantee matching of corrections to entropy due to thermodynamic fluctuations, which are always present for a thermodynamic system? In other words, we would like to explore whether the approximate agreement becomes better or worse when sub-leading terms are taken into account. To this end, we first compute the first-order corrections for an arbitrary thermodynamic system, using the canonical framework (for corrections due to fluctuations of geometry, see [13]). The partition function for such a system is

$$Z(\beta) = \int_0^\infty \rho(E)\ \mathrm{e}^{-\beta E}\mathrm{d}E, \tag{46}$$

where the density of states, $\rho(E)$, can be written as an inverse Laplace transform of the partition function

$$\rho(E) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} Z(\beta)\mathrm{e}^{\beta E}\mathrm{d}\beta = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{e}^{S(\beta)}\mathrm{d}\beta, \tag{47}$$

where we have used

$$S = \ln Z + \beta E \tag{48}$$

Close to the equilibrium temperature inverse $\beta = \beta_0$, one can expand the entropy function as

$$S(\beta) = S_0 + \frac{1}{2}(\beta - \beta_0)^2 S_0'' + \cdots,$$

where $S_0 := S(\beta_0)$ and $S_0'' := (\partial^2 S(\beta)/\partial\beta^2)_{\beta=\beta_0}$. Substituting the above in (47), we get

$$\rho(E) = \frac{\mathrm{e}^{S_0}}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{e}^{1/2(\beta-\beta_0)^2 S_0''}\,\mathrm{d}\beta \quad (\beta - \beta_0 = ix). \tag{49}$$

Defining $\beta - \beta_0 = ix$ and performing a contour integration, we get

$$\rho(E) = \frac{\mathrm{e}^{S_0}}{\sqrt{2\pi S_0''}} \tag{50}$$

whose logarithm gives the corrected entropy, taking into account the thermal fluctuations

$$\mathcal{S} := \ln\rho(E) = S_0 - \frac{1}{2}\ln S_0'' + \text{ (smaller terms)}. \tag{51}$$

Next, using

$$E \equiv \langle E\rangle = -\left(\frac{\partial \ln Z}{\partial\beta}\right)_{\beta=\beta_0} = -\frac{1}{Z}\left(\frac{\partial Z}{\partial\beta}\right)_{\beta=\beta_0},$$
$$\langle E^2\rangle = \frac{1}{Z}\left(\frac{\partial^2 Z}{\partial\beta^2}\right)_{\beta=\beta_0} \tag{52}$$

and the definition of specific heat

$$C \equiv \left(\frac{\partial E}{\partial T}\right)_{T_0} = \frac{1}{T^2}\left[\frac{1}{Z}\left(\frac{\partial^2 Z}{\partial\beta^2}\right)_{\beta=\beta_0} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial\beta}\right)^2_{\beta=\beta_0}\right] = \frac{S_0''}{T^2} \tag{53}$$

it follows that

$$S_0'' = \langle E^2\rangle - \langle E\rangle^2 = CT^2. \tag{54}$$

This shows that it is indeed the fluctuation of the total energy which gives rise to corrections. Then from (51) it follows that [14]

$$\mathcal{S} = S_0 \; - \; \frac{1}{2}\ln(CT^2) + \cdots. \tag{55}$$

The above formula applies to all stable thermodynamic systems (for some refinements, see [15]). However, for black holes, we will make the substitutions $S_0 \to S_{\mathrm{BH}} = A/4\lambda_{\mathrm{Pl}}^{d-2}$ and $T \to T_{\mathrm{H}}$. For example, for a BTZ black hole,

$$C = T_{\mathrm{H}} = S_{\mathrm{BH}}, \tag{56}$$

implying

$$\mathcal{S} = S_{\rm BH} - \frac{3}{2}\ln S_{\rm BH}. \tag{57}$$

For AdS–SC black hole one uses (36) and (37) and the specific heat

$$\begin{aligned} C &= (d-2)\left[\frac{(d-1)r_+^2/\ell^2 + (d-3)}{(d-1)r_+^2/\ell^2 - (d-3)}\right] S_0 \\ &\approx (d-2)S_0[\ell \ll r_+], \end{aligned} \tag{58}$$

using which, we get

$$\begin{aligned} \mathcal{S} &= S_{\rm BH} - \frac{1}{2}\ln\Big(S_{\rm BH}S_{\rm BH}^{2/(d-2)}\Big) \\ &= S_{\rm BH} - \frac{d}{2(d-2)}\ln(S_{\rm BH})\,. \end{aligned} \tag{59}$$

For the dual gas on the other hand, from $C = {\rm d}(F_{\rm gas} + TS)/{\rm d}T$ and eq. (38), we get

$$\mathcal{S} = S_{\rm gas} - \frac{d}{2(d-2)}\ln S_{\rm gas} \quad + \quad \frac{1}{d-2}\ln\left[(n_{\rm B} + n_{\rm F})V_{\Delta-1}\right]. \tag{60}$$

Note that although the second term in the RHS of the above exactly matches the leading order correction term for the black hole in eq. (59), there is no such counterpart of the third term. In other words, leading order entropy matching does not guarantee the matching of corrections! This is in spite of using the same master formula (55) to compute corrections, since the free energies (and hence partition functions) of the two systems are in fact not equal. More work is required for a clearer understanding of this apparent discrepancy.

4. Hawking radiation

Another important feature of quantum black holes is Hawking radiation (HR). Both LQG and ST attempt to explain this phenomenon in terms of microscopic degrees of freedom that are relevant for each theory. In LQG, the picture of HR is as follows: the states that puncture the horizon two-sphere can jump from a higher to a lower spin state (lowering the horizon area), emitting a quantum of radiation in the process. A calculation of the corresponding radiation rate shows that this can indeed account for the qualitative behaviour of HR [16]. Using the so-called 'area canonical ensemble', it was shown that the intensity of radiation for bosons (in the s-wave sector) has the following form:

$$I(\omega){\rm d}\omega = \frac{\omega^3\sigma_{\rm abs}{\rm d}\omega}{{\rm e}^{\hbar\omega/T_{\rm H}} - 1}\,, \tag{61}$$

where $\sigma_{\rm abs}$ is the absorption cross-section of the black hole under consideration, which is also known as the greybody factor.

In string theory on the other hand, D-brane interactions provide the necessary mechanism. Open strings on D-branes interact to form closed strings, which leaves the brane and propagates to asymptopia. A careful computation of the rate in this case reproduces the following HR rate for the $d = 5$ RN black hole including all prefactors [17,18]:

$$I(\omega)\mathrm{d}\omega = \frac{\omega^4 A_{\mathrm{H}}\mathrm{d}\omega}{\mathrm{e}^{\hbar\omega/T_{\mathrm{H}}} - 1} , \tag{62}$$

where it was shown that the five-dimensional greybody factor is precisely the area of the horizon A_{H}. The agreement appears to persist even in the high energy tail of the spectrum [19]. For HR from the AdS/CFT perspective, see [20].

Another explanation for HR which has been proposed does not depend on the specific underlying theory that one is considering and that radiating black hole behaves like an excited black hole [21,22]. This can be seen by starting from the assumption that the spectrum of the black-hole horizon is discrete and equispaced, and of the specific form

$$A_{\mathrm{H}} = \Omega_{d-2} r_+^{d-2} = 8\pi \left(n + \frac{1}{2} \right) \ell_{\mathrm{Pl}}^{d-2}. \tag{63}$$

Using (2) and (4) it can be easily shown from the above that the fundamental frequency of emission (for uncharged black holes) due to a transition from one area level to the next lower one, $\omega_0 \equiv \delta M c^2/\hbar$ $\left[\text{where } \delta M = \left(\frac{\partial A}{\partial r_+}\frac{\partial r_+}{\partial M}\right)^{-1} \delta A \right]$, is given by

$$\omega_0 = \frac{(d-3)c}{r_+} = \frac{4\pi T_{\mathrm{H}}}{\hbar}. \tag{64}$$

Comparing with Wien's law, which states that the frequency at which the radiation is brightest, $\omega_{\mathrm{Max}} \approx T_{\mathrm{H}}/\hbar$, we see that (64) implies $\omega_0 \approx \omega_{\mathrm{Max}}$. Thus, there will be only a few visible spectral lines in the range where the luminosity is significant. In other words, the emission lines are widely separated and lie under a Planckian envelope, which can have interesting experimental consequences [23].

5. Information loss from a black hole

Since a black hole emits uncorrelated thermal radiation, if it evaporates completely, then the large amount of information that went inside the black hole while it was being formed, is destroyed forever. This is the origin of the problem of information loss for a black hole, which can also be understood in the following way. Consider a Cauchy surface which intersects collapsing matter forming a black hole, as well as the subsequent Hawking radiation. Now, as the black hole evaporates, since the information that went inside the black hole cannot be transmitted along the Cauchy surface to a region containing the observer at $\mathcal{I}^+$, the information is lost. One resolution of this apparent paradox (since quantum mechanics at the fundamental level is expected to be unitary, which forbids such loss of information) is to assume that the information inside the black hole gets 'cloned' at the horizon such that copies of every bit that is inside are transmitted via Hawking radiation to the outside observer. Unfortunately, such cloning is forbidden by unitary quantum mechanics, as the following line of reasoning demonstrates. Let $|\Psi\rangle$ be an arbitrary state to be cloned to a 'target' state $|T\rangle$. One can think of the former as the page of a document and the latter as a blank sheet of paper in a quantum photocopying machine. Assume there is a unitary operator $\mathcal{U}$ which does the cloning, i.e. takes $|T\rangle \to |\Psi\rangle$:

$$\mathcal{U}|\Psi\rangle \otimes |T\rangle = |\Psi\rangle \otimes |\Psi\rangle, \quad \mathcal{U}^\dagger = \mathcal{U}^{-1}. \tag{65}$$

The machine should also be able to photocopy another arbitrary state $|\Phi\rangle$:

$$\mathcal{U}|\Phi\rangle \otimes |T\rangle = |\Phi\rangle \otimes |\Phi\rangle. \tag{66}$$

Taking the inner product of (65) and (66):

$$\langle\Psi| \otimes \langle T|\mathcal{U}^\dagger \, \mathcal{U}|T\rangle \otimes |\Phi\rangle = (\langle\Psi| \otimes \langle\Psi|) \cdot (|\Phi\rangle \otimes |\Phi\rangle) \tag{67}$$

$$\Rightarrow \quad \langle\Psi|\Phi\rangle = (\langle\Psi|\Phi\rangle)^2 \,. \tag{68}$$

The last equation implies

$$\langle\Psi|\Phi\rangle = 1 \;\; \text{or} \;\; 0 \tag{69}$$

meaning $|\Psi\rangle$ and $|\Phi\rangle$ cannot be an arbitrary quantum state. In other words, there is no quantum photocopier, and unitary cloning is impossible! If the above possibility is ruled out, are there other ways of resolving this paradox? There has been a host of other proposals, of which we mention a few here. For other proposals, see [24].

5.1 $\mathcal{U}$ *is not unitary*

An obvious way to circumvent the non-cloning theorem is to abandon the requirement of unitarity. If $\mathcal{U}^\dagger\mathcal{U} \neq \mathcal{I}$, then of course the above theorem does not hold and cloning is still possible [25]. Although viable, this proposal is at odds with quantum mechanics as we know it as well as most of the proposed theories of quantum gravity, including LQG and string theory.

5.2 *Planck-size remnant*

Another conjecture put forward by various authors is that the black hole does not radiate completely and that the evaporation stops when it reaches Planck size. This 'remnant' could then contain all the information that went into the black hole. While a remnant ground state is suggested in many cases, it is far from clear whether such a large information can be concentrated in such a small volume and more work needs to be done in this direction [22,26,27].

5.3 *Black-hole complementarity*

Another interesting proposal takes into account the prediction that even if Hawking radiation carries some information, an outside observer has to wait for at least half the life-time of a black hole to get just one bit of this information [28]. In other words, bulk information appears at very late times, when the black hole is almost Planck sized. To check whether this information obtained is indeed authentic, the observer can jump into the black hole, only to find that semi-classical physics has broken down, rendering its predictions null and void. Observers outside and inside the black hole are thus mutually exclusive and complementary to each other, and the above proposal is known as black-hole complementarity [29]. Although this attempts to avoid the information puzzle altogether, at best it seems to be an effective theory, a more precise microscopic picture being clearly warranted.

5.4 *Unique BH final state*

Recently, another proposal has been put forward, where it is conjectured that all information is transmitted to outgoing Hawking radiation by a mechanism similar to quantum teleportation, and that the final state of the black hole is in fact unique [30]. The entropy of the final state is thus $\ln 1 = 0$ and no information is lost. However, criticisms of this proposal include the observation that entangling interactions between the collapsing body and outgoing Hawking radiation spoil this unitarity even if in a weak sense [31].

6. Observations and experiments

The theoretical prediction of black holes is almost as old as the theory of general relativity itself [32]. Their fascinating properties have been extensively studied by all quantum gravity theories. However, actual observations of black holes involve enormous technical difficulties. Here we describe some recent advances in this direction. For other potential signatures of quantum gravitational effects, see [33–37].

6.1 *Astrophysical black holes*

There has been important progress in the detection of astrophysical black holes, and we draw the reader's attention to a review article [38] which summarises compelling experimental evidences for candidate black holes X-ray binaries and in galactic nucleii. Although these are massive, having negligible Hawking temperature, one cannot help but speculate whether any indirect evidence for black-hole thermodynamics can be extracted by these or future observations.

6.2 *Brane-world black holes*

There have been two recent proposals known as ADD and RS (collectively known as brane world scenarios) which attempt to solve the gauge hierarchy problem and which start with the assumption that our observed four-dimensional universe is embedded in a d-dimensional world (the 'brane') [39,40]. In the ADD scenario, which assumes that the unobserved part is a T^{d-4}, with each circle being of length L, the Planck scales in lower and higher dimensions are related as

$$M_{\mathrm{Pl}(d)}{}^{d-2} = \frac{\hbar^{d-3}}{c^{d-5}G_d} = \frac{\hbar^{d-3}}{c^{d-5}V_{d-4}G_4} = \left(\frac{\hbar}{cL}\right)^{d-4} M_{\mathrm{Pl}(4)}^2. \tag{70}$$

From the above it follows that even though the observed Planck energy is $M_{\mathrm{Pl}(4)}c^2 \approx 10^{19}$ GeV, for $d \geq 4$ and $L \approx 1$ mm, the fundamental Planck energy $M_{\mathrm{Pl}(d)}c^2$ could be as low as 1 TeV. In other words, the hierarchy problem does not exist in the full space-time. Now, energies of the order of TeV are expected to be produced within a few years in the Large Hadron Collider being built at CERN. The Schwarzschild radii corresponding to these energies also depend on the space-time dimension under consideration, and are related by

$$r_{+(d)} = \left(\frac{G_d M}{c^2}\right)^{1/(d-3)} = \left(\frac{V_{d-4}G_4 M}{c^2}\right)^{1/(d-3)} = \left(V_{d-4}r_{+(4)}\right)^{1/(d-3)}. \tag{71}$$

Once again, using $G_d = \hbar^{1/(d-3)}c^{5-d}/M_{\mathrm{Pl}}^{1/(d-2)}$ it can be seen that although $r_{+(4)} \approx 10^{-29}$ Fm is far beyond the realm of any realistic experiments, $r_{+(d)} \approx 10^{-4}$ Fm is not. In other words, the impact parameters can be adjusted to the above value such that the colliding protons are within each other's gravitational radii and thus form a black hole on colliding. These black holes will then Hawking radiate, measurements of whose signatures would confirm the existence of higher dimensions as well as that of black holes.

As shown in [41] however, the above set of inferences ought to be accompanied by caution. It is widely believed that near the Planck scale, the Heisenberg uncertainty principle (HUP) undergoes modifications and is replaced by a more refined version known as the generalised uncertainty principle (GUP) [27]:

$$\Delta x \geq \frac{\hbar}{\Delta p} + (A\lambda_{\mathrm{Pl}})^2 \frac{\Delta p}{\hbar}, \tag{72}$$

$$\Delta p = \frac{\hbar \Delta x}{2(A\lambda_{\mathrm{Pl}})^2} \left[1 - \sqrt{1 - \frac{4(A\lambda_{\mathrm{Pl}})^2}{\Delta x^2}}\right]. \tag{73}$$

The constant A is of order unity, but its precise value is theory dependent. HUP is recovered in the $\Delta x/\lambda_{\mathrm{Pl}} \gg 1$ limit. Now, a Hawking particle before it is ejected from the horizon has an approximate uncertainty of $\Delta x \approx 2r_+$. From eq. (73), the corresponding uncertainty of momentum (which being the only energy scale, is identified with the Hawking temperature)

$$T_{\rm H} = m^{1/(d-3)} \left[1 - \sqrt{1 - \frac{4A^2}{m^{2/(d-3)}}} \right] M_{{\rm Pl}(d)} \tag{74}$$

$$\approx \left[\frac{1}{m^{1/(d-3)}} + \frac{1}{m^{3/(d-3)}} + \cdots \right] M_{{\rm Pl}(d)}, \tag{75}$$

where we have used $r_+ \approx m^{1/(d-3)}$ and $m \equiv M/M_{{\rm Pl}(d)}$. Note that up to overall dimensionless factors, the first term on the right represents the usual Hawking temperature, while the second term gives leading order corrections to it. The latter being positive, the black hole will radiate faster on the brane according to the Stefan–Boltzmann law [42]:

$$\frac{{\rm d}m}{{\rm d}t} \propto ({\rm Area}) \times T_{\rm H}^4. \tag{76}$$

It can be seen from the exact expression in eq. (74) however, that the expression inside the square root becomes imaginary and hence the radiation stops at

$$M_{\rm Min} = (4A^2)^{(d-3)/2} M_{\rm Pl}. \tag{77}$$

Two conclusions follow: first, if the collider energy is below this threshold, black holes will not form, even if brane-world scenarios are correct. Second, if the energy is above this threshold, black holes will form and radiate, but there will be enormous amounts of missing energies corresponding to $M_{\rm Min}$. Detection of such missing energies would be a strong signature of brane worlds and black hole remnants.

6.3 *Analog black holes*

Another interesting arena where phenomena analogous to black hole thermodynamics can be potentially tested is in the context of condensed matter systems, which under suitable circumstances imitate black-hole horizons [43,44]. Consider the Navier–Stokes and continuity equations for an inviscid and irrotational fluid [44]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0, \tag{78}$$

$$\rho \left(\frac{\partial v}{\partial t} + (v \cdot \nabla) v \right) = -\nabla p, \tag{79}$$

$$\nabla \times v = 0 \Rightarrow v = \nabla \psi. \tag{80}$$

Perturbation around equilibrium to $\mathcal{O}(\epsilon)$ is of the form

$$\rho = \rho_0 + \epsilon \rho_1, \quad p = p_0 + \epsilon p_1, \quad \psi = \psi_0 + \epsilon \psi_1, \quad \vec{v} = \vec{v}_0 + \epsilon \vec{v}_1 \tag{81}$$

and combining the first-order equation into a single second-order equation yields

$$\frac{\partial}{\partial t}\left(c^{-2}\rho_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right)$$
$$=\nabla\cdot\left(\rho_0\nabla\psi_1-c^{-2}\rho_0 v_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right), \tag{82}$$

where $c^2\equiv\partial p/\partial\rho$ is the speed of sound in the fluid. Defining a 4×4 matrix

$$g^{\mu\nu}=\frac{1}{\rho_0 c}\left(\begin{array}{c|c} -1 & -v_0^j \\ \hline -v_0^i & (c^2\delta^{ij}-v_0^i v_0^j)\end{array}\right)$$

with inverse

$$g_{\mu\nu}=\frac{\rho_0}{c}\left(\begin{array}{c|c} -(c^2-v_0^2) & -v_0^j \\ \hline -v_0^i & \delta^{ij}\end{array}\right)$$

eq. (82) can be written as

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi_1\right)=0, \tag{83}$$

which is just the equation for a scalar field in a curved background. In this case, the scalar field represents phonons. The analogous infinitesimal line element is

$$\mathrm{d}s^2=\frac{\rho_0}{c}\left[-(c^2-v_0^2)\,\mathrm{d}t^2+\mathrm{d}\vec{r}^2-2\vec{v}\cdot\mathrm{d}\vec{r}\,\mathrm{d}t\right]. \tag{84}$$

Now, choose the following velocity and density profiles:

$$v_0=\sqrt{\frac{2G_4M}{r}},\quad \rho_0=kr^{-3/2}, \tag{85}$$

where G_4M is a constant and a new time coordinate:

$$t'=t+\left[\frac{4G_4M}{c^3}\arctan\left(\sqrt{\frac{2G_4M}{c^2r}}\right)-2\sqrt{\frac{2G_4Mr}{c^4}}\right]. \tag{86}$$

Then, from (84):

$$\mathrm{d}s^2=\frac{k}{c}\,r^{-3/2}\Bigg[-c^2\left(1-\frac{2G_4M}{c^2r}\right)\mathrm{d}t'^2$$
$$+\left(1-\frac{2G_4M}{c^2r}\right)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega_2^2\Bigg] \tag{87}$$

which is conformal to the Schwarzschild geometry. The corresponding Hawking temperature (which is conformally invariant), for typical fluid parameters is $T_{\mathrm{H}}=\hbar c^3/8\pi G_4M\approx 10^{-4}$ K. Similarly, it has been shown that observable super-radiance should result from these black-hole analogs [45,46]. It is hoped that one would be able to build suitable condensed matter systems in future which will demonstrate at least some of the above effects.

7. Conclusions

In this article, we have reviewed various approaches that try to explain the microscopic origin of black-hole thermodynamics. These include near-horizon conformal field theory, loop quantum gravity, and string theory. While the first two are able to address realistic Schwarzschild black holes in four dimensions, string theory primarily deals with extremal RN-type black holes. However, the agreements of microscopic and macroscopic results pertaining to entropy and Hawking radiation are exact and more spectacular in the case of the latter. On the other hand, whereas CFT and LQG pin-point the location of the degrees of freedom in curved space-time, that give rise to this entropy, in string theory it is unclear as to what the strong coupling counterparts of the D-brane degrees of freedom are. Moreover, most of the string theoretic results pertain to five space-time dimensions. Thus, the results of CFT, LQG and string theory appear to be complementary to each other. Although the approaches are diverse, since they all attempt to address similar problems, it is hoped that continuing research in all the fields will someday tell us the exact relationship between the degrees of freedom in each approach.

We have also studied the problem of information loss for black holes and some attempts at its resolution. Here too, the resolution is far from complete.

Finally, we have examined a few experimental scenarios which could test the existence of black holes in our universe as well as imitate black-hole thermodynamics in the laboratory. We hope that many of the unanswered questions will be satisfactorily addressed in the near future.

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