TEACHING MATHEMATICS WITH THE BRAIN IN MIND: LEARNING PURE MATHEMATICS WITH MEANING AND UNDERSTANDING

TANYA JOHNSON

B. Ed., University of New Brunswick, 1993
DAUS, University of New Brunswick, 1994

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I dedicate this work to

My Mom

When I thought I did not have the time and energy to finish my project; the strength and perseverance shown by my mom with her battle with cancer became my inspiration to keep going and to never give up in life.

Abstract

This culminating master's project applies data and information discovered in a content analysis of research documents to create a brain-based pure math teacher resource that will help teachers teach the pure mathematics 20 program with meaning and understanding. The resource includes a rationale, as well as explanations for the brain-based mathematics lesson framework. Teacher friendly daily lessons are laid out in a thematic unit for the algebraic equations, relations and functions section of the curriculum. The resource utilizes a brain-based approach to teaching and learning providing teachers with an easy to understand, practical, everyday guide that can easily be implemented into the pure math classroom. This resource is needed because students continue to feel inadequate and inferior in pure math classrooms across Alberta. Changes needed to resolve this disturbing situation include teachers themselves altering their teaching strategies to help minimize the existing problems in the pure math program, and this project contributes to the knowledge about improving best teaching practices. Research on the multiple intelligence theory reminds us of the different student learning styles and the fact that, more than one type of teaching strategy should be used to deliver the pure math program. Current research on the science of learning has brought to light some very interesting ideas of how a student's brain works and the applications of this work to classroom practice. As teachers, we can translate this information into classroom practice in order to help our students learn pure mathematics with meaning and understanding.

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Chapter One: The Beginnings

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge."

(National Council of Teachers of Mathematics, 2000, p. 20)

Introduction

Throughout chapter one, a solid foundation is created for the idea of teaching mathematics with the brain in mind. Key background information sets the stage for the compelling rationale for teaching pure mathematics with the brain in mind. Ideas associated with brain-based teaching are discussed. The reasoning behind how teaching styles should parallel learning styles is brought to the forefront in brain-based teaching so all students have the opportunity to learn pure mathematics with meaning and understanding. One tool in your toolbox approach to teaching will not meet the learning needs of all students. Justification for the use of various teaching strategies is reinforced throughout the chapter. The afore mentioned interrelated brain-based concepts give birth to a pure mathematics teacher resource that will help teachers teach the pure mathematics program with meaning and understanding.

Background: The Journey Begins

The Alberta Mathematics Program

The high school mathematics program has just undergone many curriculum changes over the past five years. Alberta Learning implemented a new mathematics program that included an applied mathematics stream and a pure mathematics stream. The high school mathematics programs were developed with the advice and support of many individuals and groups: teachers, school administrators, post-secondary educators, parent groups, representatives of professional organizations, such as the Alberta Teachers' Association, Mathematics Council Alberta Teachers' Association and the Alberta Chamber of Resources, and other members of the community. The programs were designed to help students develop skills to solve a wide range of problems both inside and outside of mathematics courses, and to keep pace with the latest information technology.

The Differences: Pure and Applied Mathematics

The main differences between applied mathematics and pure mathematics are the topics studied, and the approach to solving problems and developing understanding. Both programs help students develop the critical skill of using mathematics to find solutions to problems in real-life situations. Students planning to pursue programs requiring the study of mathematics at the post-secondary level need to be registered in the pure mathematics stream. Pure mathematics is a requirement for post-secondary level programs that possess a connection to mathematics. Pure mathematics courses emphasize the specialized language of algebra as the preferred method for learning mathematical concepts and for solving problems. Students learn about mathematical theories, find exact value solutions to equations, and use formal mathematical reasoning and models in problem solving. Therefore, all the time and money that was put into this new high school mathematics program will ensure its survival for a long time to come.

Rationale: Why Brain-Based Teaching?

Pure Mathematics: A Disturbing Situation

During the last four years of teaching pure mathematics, the following observations of students are a familiar teaching experience: blank stares, frightened looks, frustrated body language, and streaks of tears. Does every pure mathematics student encounter these symptoms of stress? Of course not, but each day I see students suffering through taxing encounters with the subject. As a teacher, I am disturbed by these student expressions of difficultly in learning. Therefore, I question the pure mathematics program and my mathematical teaching approach. Obviously, some students feel inadequate and inferior when subjected to the pure mathematics curriculum and I want to discover teaching strategies that will enhance their learning.

Mathematical Anxiety: Lasting Effects

Based on my own classroom practice of teaching the pure mathematics program over the last five years, students undergo a high level of mathematical anxiety each day. Students are fearful of what they are learning in class. Fear paralyzes both learning and teaching in the classroom. Caine & Caine (1990) claim recent research on how the brain learns supports the importance of classroom environments; threat and stress can inhibit learning. Students are being turned off mathematics. Some students report hating mathematics. Evidently, these intense emotions have lasting effects, because students are becoming complacent and satisfied with just passing grades. Some students are dropping out of the pure mathematics program. Furthermore, student whose dreams involve professions with a connection to mathematics are locking these imaginings into a black box and pursuing something else. Producing life-long mathematics learners has not been a realistic outcome of the program. Although, this scenario of failure does not occur in every pure mathematics class, I argue it happens more often than teachers would like to recognize. The situation is affecting the well-being and the future knowledge, careers, and skills of our students. Obviously, something needs to change. What is the solution to this problem? Should we change the curriculum or change our mathematical approach to teaching? or both?

Another Approach: Brain-Based Teaching

The question needs to be asked: what can be done to change this unsettling pattern of failure and dropout in pure mathematics? The same question of mathematical learning difficulties has been constantly etched in my mind since I started teaching the pure mathematics program. The solution to the question has been researched in current educational journals and books, spoken about by experts in the mathematical field, and experienced with fellow teachers. There are probably many solutions to enhancing mathematics learning; ranging from trying to motivate students to work harder to counselling more students to take the applied mathematics route. I am a teacher so I am interested in what teachers can do to help minimize the problems that exist in the pure mathematics program. One central solution lies in my research assumptions that teachers can change the teaching strategies they use to deliver the pure mathematics program to their students. Current research by E. Jensen (1995, 1997, 1998, 2000), P. Wolfe (1998, 1999, 2000, 2001), R. Sylwester (1990, 1994, 1995), and many others on the science of learning brought to light some very interesting ideas of how a student's brain works and the applications of this work to classroom practice. It is hoped that teachers can translate this information into classroom practice in order to enable our students to learn with meaning and understanding.

Starting Point: Brain-Based Teaching

Since teaching is complex and contextual, it is very hard to identify directly proportional correlated relationships between specific teaching practices and student success. There is no recipe that lists a set of ingredients and methods for good teaching for all teachers. Teaching is a highly personal act. Teaching is a process of human interaction involving the teacher and his or her students in particular contexts. As Parker Palmer (1998) remarks, "*The Courage to Teach*" is built on the premise that good teaching cannot be reduced to technique; good teaching comes from the identity and integrity of the teacher. Therefore, the problem situation with the pure mathematics program is more complex than a solution of altering teaching strategies based on brain-based research can support. Nevertheless, according to brain researchers, brain-based teaching is a starting point that uses the way students learn to formulate and structure daily classroom lessons. As well, teachers themselves can participate in this solution. Teachers can directly help their students learn the pure mathematics curriculum with understanding. Pure mathematics teachers themselves can learn and teach mathematics with meaning.

Caution: Teachers Need to Analyze the Brain-Based Approach

Other researchers like Bruer (1998) believe current brain research has little to offer educational practice. New results in neuroscience point to the brain's lifelong capacity to reshape itself in response to experience. Educators' great challenge is developing learning environments and practices to exploit the brain's lifelong plasticity. Some researchers feel brain science should return to the back burner. More research needs to be done on how our brains learn before we can apply this knowledge to learning in the classrooms. Even Jensen (2000), who has written numerous accounts in support of brain research in the classroom, warns although neuroscience has much to offer teaching and learning, teachers must remain cautious about applying lab research to classrooms.

Teachers need to verify the brain-based approaches to learning with their own depth of knowledge and experience about the way their students learn.

Teaching Styles Parallel Learning Styles: Mathematics for all Students

One Teaching Style: Who Loses?

How is the pure mathematics program currently delivered to students in the classroom? Of course, there is no one set way, although through my own experience, many teachers teach pure mathematics primarily from a direct approach. The direct approach includes the presentation and development of processes or algorithms for carrying out mathematical tasks. Craig Loewen (2001), a professor at the University of Lethbridge, explains that a typical lesson plan in a direct approach to teaching might involve the presentation of a process or algorithm followed by a series of teacher-led examples, and then by the assignment of a set of questions that directly apply the algorithm or process. Direct teaching encourages students to memorize what is being taught. Also, the mathematical content is being presented as a series of unrelated concepts. Since everything traditionally, was learned in isolation and no relationships were established amongst the mathematical content, our brains will simply forget what we have learned over a short period of time. Furthermore, in a class of thirty students there is more than one type of learning style. For example, Erualer (2003) reiterates the typical classroom usually includes 46% visual learners, 35% kinaesthetic learners, and 19% auditory learners. As well, Brandt (1990) contends because students are unique and will respond differently to a variety of instructional methods we need to respect the individual differences among us so all students are given an opportunity to learn in their own way. Therefore, if there is more than one student learning style in a classroom then should there not be more than one teaching style to ensure understanding by all the students. If only one teaching style is used to teach the pure mathematics program many students will lose out on learning with meaning and understanding.

Varied Teaching Styles: Who Wins?

In contrast, teaching pure mathematics with the brain in mind establishes rich connections amongst the mathematical processes being taught to the student. It is like giving each student a key to a door in their brain that will allow them to learn the pure mathematics curriculum with understanding. As educators, we recognize these different types of learning. Some of what we teach in pure mathematics requires students to engage in rote rehearsal learning. Examples are basic algebraic processes such as operations with polynomials, and solving equations. However, much of the pure mathematics curriculum cannot be memorized and be fully understood. For example, understanding an algorithm in pure mathematics does not include the method of rote rehearsal. Wolfe (2001) suggests for these types of learning, elaborative rehearsal strategies are much more effective. Elaborative rehearsal strategies encourage the learner to elaborate on the information in a manner that increases understanding and retention of that information. For example, some elaborative rehearsal strategies include real-life problem solving, making connections to previous learning, developing concepts by making connections to other ideas, visual representations and models, math projects, writing activities, scientific applications, mnemonic patterns, and motivated active learning. In many instances, these strategies make the information more meaningful and relevant to the learner. Elaborative rehearsal strategies connect meaning and emotion to what is being learned in order to enhance retention. If varied teaching styles are used to teach the pure mathematics program students will win; more students will learn mathematics with meaning and understanding. Both R. Brandt's (1990, 1993, 1994, 1997, 1999, 2000, and M. D'Arcangelo's (1998, 2000, 2001) research support these ideas.

Teaching Mathematics with the Brain in Mind

Personal Reflections

With eight years of teaching experience, I feel developing a learning environment with the brain in mind can be a successful approach to learning for both math teachers and students. When I reflect about what enabled me to learn mathematical concepts with understanding; I begin to realize these various brain strategies are what made the ideas meaningful for me. As I observe the way students learn, these brain related strategies make perfect sense of why it is easier for some students to approach learning in this manner. Researchers such as Jensen (1998) continually review recent research and theory on the brain and balance this information with tips and techniques for using the information in classrooms. In many instances, brain-compatible learning is explored with the teacher and student in mind.

Project Purpose

The purpose of my project is to develop a practical brain-based resource for pure mathematics 20 teachers. More specifically, I want to investigate teaching the high school pure mathematics 20 program with the student's brain in mind. I will develop a teaching unit that applies brain-based teaching methods that enable mathematics teachers to teach students pure mathematics with meaning and understanding. The teaching unit will be a pure mathematics resource for teachers with daily brain-based lesson plans that correlate with the objectives of the pure math 20 curriculum. The thematic unit in the teacher resource will entail the curriculum area of algebraic quadratic equations, relations and functions. The reason this unit of the pure math 20 curriculum was chosen is because a lot of the mathematical concepts in this section require abstract thinking about theoretical ideas. Many students find this part of the pure mathematics program to be challenging and frustrating as this area of curriculum is in the most urgent need of development. I believe that using brain-based elaborative rehearsal strategies to teach these mathematical concepts will enable students to learn mathematics with meaning and understanding. The daily lessons will be developed around a framework that approaches learning with the brain in mind.

Words: Contextual Meaning

The meaning of some key words in my purpose must be explained in terms of the context of my project. What does it mean when my project refers to teaching pure mathematics with the brain in mind? Teaching pure mathematics with the brain in mind is a teaching practice that incorporates research on how the brain learns and implements these brain-based strategies in daily classroom lessons to help students learn mathematics with meaning and understanding. Caine and Caine (1995) cite that brain-based teaching and learning uses a holistic approach stressing the importance of how the brain learns in order to produce meaningful learning.

The brain-based approach to learning incorporates elaborative rehearsal strategies. Wolfe (2001) describes how these strategies encourage learners to elaborate on the information being learned in a manner that increases understanding and retention of that information. In my project, these strategies make the information more meaningful and relevant to the learner. For example some elaborative rehearsal strategies involve making meaning using associations, analogies, metaphors, and visual representations.

Using these teaching strategies, students begin to learn mathematics with meaning and understanding. Loewen (2000) uses The National Council of Teachers of Mathematics (NCTM) to describe understanding mathematics in terms of connections. Connections can be made between mathematical ideas and other ideas, models, words, algorithms, and the world. Our learning of a concept is based on the connections we make cognitively with that concept and other related ideas. In this project, the more connections students make the more they understand about what they have learned. Furthermore, Hiebert and Carpenter (1992) define understanding as the way information is represented and structured. Historically, and still today researchers believe that a mathematical idea is understood if it is part of an internal network. Of course, the degree of understanding is linked to the number and strength of these connections.

Learning mathematics with meaning deals with students making their own individual meaning from a concept they are learning. The brain-based approach to learning mathematics with meaning uses strategies to help students find their own constructed meaning in a mathematical concept. Sousa (2001) states learning with meaning is a result of how students relate content to their past knowledge and experiences. Students break concepts down, compare concepts, and refer to concepts in a different way until they make sense or are meaningful to them.

Research Question

What elaborative rehearsal strategies are needed to develop a practical resource of daily lessons for pure math 20 teachers that includes a brain-based approach to teaching and learning that enables students to learn pure math with meaning and understanding?

Chapter Summary

The high school mathematics program has just undergone many curriculum changes over the past five years. Alberta Learning implemented a new mathematics program that included an applied mathematics stream and a pure mathematics stream. Based on my own classroom practice, students undergo a high level of mathematical anxiety each day in the pure mathematical stream. Students are fearful of what they are learning in class. I feel this is a disturbing situation. In order to minimize the problems that exist in the pure mathematics program teachers can change the teaching strategies they use to teach mathematics. Brain-based teaching is a starting point that uses the way students learn to formulate and structure daily classroom lessons. Nevertheless, teachers need to verify the brain-based approaches to learning with their own depth of knowledge and experience about the way their students learn. Many teachers teach pure mathematics primarily from a direct approach. In contrast, teaching pure mathematics with the brain in mind establishes rich connections amongst the mathematical processes being taught to the student. All of the aforementioned information will help to produce a brain-based pure mathematics teacher resource that will allow teachers to teach the pure mathematics program with meaning and understanding.

Chapter 2: Literature Review

"Students draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress."

(National Council of Teachers of Mathematics, 2000, p. 3)

Introduction

At the beginning of chapter two, an overview of educational researchers' thoughts on students' learning styles, multiple intelligence theory, brain-based learning, and learning pure mathematics with meaning and understanding are presented. The next section investigates what researchers think about facilitating learning through brain-based teaching. Researchers argue about the application of brain research to classroom practice. As well, ideas are explored on topics such as: how our brains learn; importance of teachers using previous learned knowledge; attention and learning; emotion and learning; movement and learning; rehearsal and learning; memory and learning; and elaboration and learning. Elaborative rehearsal strategies are offered for use in brain-based classrooms. Lastly, researchers such as Caine and Caine (1990, 1995, 1997, 1998), Jensen (1998), Erlauer (2003), Hardiman (2001), and King-Friedrichs (2001) explain their brain-based teaching and learning models.

Why Brain-Based Teaching: Different Learners

Based on five years of classroom teaching in pure mathematics, I believe students learn in different ways. Students possess their own unique ways of learning. Therefore, a pure mathematics-learning environment should be structured so each student's learning style is present. Imagine a mathematics classroom where all students are given an equal opportunity to learn. Learning mathematics with the brain in mind recognizes different student learning styles by presenting mathematical concepts through the varying manners in which the brain learns.

Students' Learning Styles

Learning styles is a popular topic in educational research. Many researchers believe students' learning styles should be an important consideration for daily lesson planning. Brandt (1990) wrote an article on learning styles after conversing with Pat Guild, senior author of ASCD's *Marching to Different Drummers*, and he advised teachers to identify key elements of the student's learning style and match your instruction and materials to those individual differences. As well, Brandt (1990) concludes students are different, students will respond differently to a variety of instructional methods, and we need to respect and honour the individual differences in our classrooms. According to Shields (1993) teachers catering to particular learning styles is just good use of common sense in the classroom. Teachers should validate students' personal approaches to learning by varying their instruction. The approach is high-quality lesson planning, you should reach all children through a variety of activities.

The Multiple Intelligence Theory

The multiple intelligence theory (MI) put forward by Gardner has been praised as one of the most important ideas to help with school improvement. After a conversation with Gardner, Checkley (1997) reviews what is involved in the multiple intelligences: linguistic intelligence, logical-mathematical intelligence, spatial intelligence, bodily kinaesthetic intelligence, musical intelligence, interpersonal intelligence, intrapersonal intelligence, and naturalist intelligence. Sweet (1998) believes allowing students to use their knowledge about how they learn best can increase their enthusiasm, raise their achievement levels, and foster growth in their other intelligences. Silver, Strong, Perini (1997) integrate Gardener's multiple intelligence theory and learning styles in order to better understand how students learn. Learning styles are concerned with the process of learning while the multiple intelligence theory looks at the content and products of learning. The multiple intelligence theory (MI) acknowledges the differences in the way each student's brain learns. MI supports teachers who use several teaching methods to teach one concept so all students have the opportunity to learn that concept.

Brain-Based Learning

Kalbfleisch and Tomlinson (1998) remind educators of three principles from brain research: emotional safety, appropriate challenge, and self-constructed meaning suggest that a one–size–fits–all approach to classroom teaching is ineffective for most students. Classrooms need to be responsive to students' varying readiness levels, varying interests, and varying learning profiles. Furthermore, Kalbfleisch and Tomlinson (1998) explain how teachers have a responsibility to present information in varied ways, for example, orally, visually, through demonstration, part to whole, and whole to part if they want to give all their students an opportunity to learn. Moreover, Given (2002) feels teachers can stimulate and facilitate learning in all children by addressing the need to know in multiple ways. By providing alternate ways of learning, students are at a liberty to gain new information through those learning styles most comfortable for them.

Overlaps: Learning Styles, MI, Brain-Based Learning

Multiple intelligences, learning styles, and brain-based education are theories with many overlaps. Chock-Eng and Guild (1998) state the application of the three theories have similar outcomes. They judge multiple intelligences, learning styles, and brain-based education bring an approach and attitude to teaching of focusing on how students learn and the unique qualities of each learner. Additionally, Chock-Eng and Guild (1998) deem the overlaps in these theories exist in the following ways: each theory is learner-centered and promotes diversity; as well teachers and students are reflective practitioners and decision makers. Therefore, the ideas associated with learning styles, and the multiple intelligence theory clearly portrays a rationale for brain-based education. Research documents the need for varied instruction in our classrooms in order to meet the different ways students' brains learn.

Learning Pure Mathematics with Meaning and Understanding

Teaching pure mathematics with the brain in mind is intended to emphasize learning mathematics with meaning and understanding. Caine and Caine (1990) support this idea by accentuating in their writings the objective of brain-based learning is to move from memorizing information to meaningful learning.

What does it mean to learn pure mathematics with meaning and understanding? Loewen and Sigurdson (2001) pronounce learning mathematics with meaning is based on the number of connections students can make between mathematical concepts. When students learn mathematics with meaning and understanding they trigger previous networks of knowledge in the brain and make links between these networks and new mathematical concepts (Loewen and Sigurdson, 2001). Furthermore, Carpenter and

Hiebert (1992) explain the number and the strength of the connections determine the degree of understanding.

Teaching for meaning and understanding can be achieved using alternate representations of mathematical concepts (Loewen and Sigurdson, 2001). Such other forms like visual drawings, context stories, hands-on math manipulatives, and real life problems increase the opportunity to stimulate prior mathematical knowledge networks in the brain. Varied teaching methods activate different parts of the brain and student's learning styles that enable students to construct knowledge through the connection of mathematical concepts to ensure learning with meaning and understanding.

Other educational researchers enlighten us with their ideas about learning with meaning and understanding in very similar ways. Carpenter and Lehrer (1999) point out learning math with meaning and understanding is important because when students acquire knowledge with understanding they can transfer that knowledge to learn new concepts and solve novel problems. They believe learning with meaning and understanding is developed through constructing relationships, extending and applying mathematical knowledge, reflecting about learning experiences, communicating what one knows, and making mathematical knowledge one's own (Carpenter and Lehrer, 1999).

Checkley (1997) discovers, while interviewing Howard Gardner, learning mathematics with understanding means students can take ideas they learn in school or anywhere and apply those appropriately in new situations. We know students understand something when they can represent the knowledge in more than one way. Kalbfleisch and Tomlinson (1998) agree and continue to comment how learners should spend more time constructing their knowledge so they can witness the relationship between the parts and the whole of a concept. Brandt (1994), while conversing with Magdalene Lampert, refers to understanding as sense making. Learning with understanding and meaning is merely sense making by traversing the same territory in lots of different directions. Also, Brandt (1994) reminds us how many more connections are made between mathematical concepts during daily review, especially if you can do it with other people and talk about different perspectives.

Teaching pure mathematics with meaning and understanding is an achievable outcome for brain-based learning. A variety of learning situations and brain stimulations will enable students to construct knowledge through the connections they make between previously learned and new mathematical concepts.

Facilitating Learning: Brain-Based Teaching

A brain-compatible mathematics classroom uses what research says about how students learn to improve teaching. Many of the findings in brain research validate effective teaching practices. Brandt and Wolfe (1998) suggest that educators, because of their experience and knowledge base, are in the best position to recognize what brain research is saying to supplement, explain, or confirm current practices. Brain-based teaching highlights certain aspects of learning such as connections to previous knowledge; attention and learning; emotion and learning; movement and learning; rehearsal and learning; memory and learning; elaboration and learning. These brain-based ideas are interwoven throughout the strategies teachers' use in the classroom. Brain strategies match how the brain learns best.

Applying Brain Research to Classroom Practice

Researchers do expect teachers to be cautious when interpreting and applying brain research to their teaching practices. Some researchers more than others warn teachers about the application of brain research. For instance, Bruer (1998) advises teachers not to link brain research in any meaningful way to educational practice because neuroscience does not know enough about brain development and neural function. Furthermore, Bruer (1997) admits neuroscience has discovered a great deal about the brain, but argues not nearly enough to guide educational practice. This project contributes to linking neuroscience research to educational knowledge. On the other hand, Brandt (1999) responds to Bruer's articles on brain research with his own argument. Brandt (1999) feels findings from neuroscience are yielding additional insights into the learning process if used in conjunction with other sources, and educators would be foolish to ignore this growing body of knowledge. Using neuroscience in the classroom is challenging because teachers cannot rely exclusively on brain research. Students are complex and unique; not just one strategy will work. Caine (2000) considers integrating brain research with other fields of research can provide teachers with a foundation for excellent teaching.

There is a spectrum of opinions for and against the application of brain research to educational practice. Nevertheless, most educational researchers portray a common theme when analyzing the use of brain research in the classroom. The purpose of neuroscience research is to learn how the brain functions; the purpose is not to tell teachers what they should be doing in the classroom to increase student understanding. Brandt (1999) believes if we use the findings about the physical brain with our knowledge about human behaviour and learning we have derived from psychology, social sciences, along with educational research and professional experience it can further illuminate our understanding of how students' learn best. As well, Caine and Caine (1998, 2000), and Jensen (1998, 2000) explain there is a vast body of knowledge of brain research out there but it is up to educators to carefully interpret what brain science means for classroom practice. Teachers possess an immense background of knowledge about teaching and learning. Teachers gain this knowledge from educational research, cognitive science, and classroom experience. Therefore, teachers are in the best position to analyze and interpret how brain research applies to classroom practice. Brandt (2000) challenges teachers to keep informed about current brain research so they can provide the best possible learning environment for their students.

How Our Brains Learn

Teaching strategies will help our brains learn concepts. But, what is actually happening in our brains when we gain knowledge. Hardiman (2001) explains learning occurs through the growth of neural connections stimulated by the passage of electrical current along nerve cells and enhanced by chemicals discharged into the synapse between neighbouring cells. In more general terms, Wolfe (1999) comments as the brain receives information from other cells connections are made between the cells. The more connections you make the more you will remember. Teachers need to strengthen these neural connections in their students. Wolfe (1999) reminds teachers neurons that fire together wire together. Thus, learning involves both the attainment of information and the ability to retrieve the information. The more teachers use various strategies to make connections in students' brains the more students will remember and understand what they have learned.

The Importance of Teachers Using Previous Learned Knowledge

One of the most challenging things to do as a teacher is to take rather meaningless information and make it meaningful for our students. In seconds of information entering our brains, the brain will decide if the information is meaningful. Wolfe (1999) suggests in order to make meaning out of new information our brains must have previous information to reactivate connections that were made before. The first time a neuron makes a connection in our brains a certain amount of energy will be needed (Wolfe, 1999). As further connections are made, less and less energy is needed. Eventually the connection is automatic and this produces a strong memory. Brandt (2000) agrees with the idea of using less brain energy when performing familiar functions than when learning new concepts. This is why the use of prior knowledge is so important when learning something new. If a teacher is able to link a new concept to something previously learned less brain energy will be needed to make the new concept a wired memory. In order for teachers to create meaning in their classrooms every encounter with something new requires the brain to fit the novel information into an existing connection of neurons (Wolfe, 1999). Therefore, learning new information must occur within the context of what the learner already knows.

Everyday our brains make the decision to keep or get rid of information. A major factor as to whether or not a brain hangs on to new information is if the brain recognizes a previous pattern. Westwater and Wolfe (2000) declare our brains grasp new data by searching through previously established neural networks to see whether it can find a place to fit the new information. Educational researchers, Hardiman (2001), Jensen (1998), King-Friedrichs (2001), Lowery (1998), Sousa (2001), Westwater and Wolfe (2000), have demonstrated that previous knowledge enhances the understanding of new information. Westwater and Wolfe (2000) assume if the brain can retrieve stored information that is similar to new information, the brain is more likely to make sense of the new information. Furthermore, Sousa (2001) recalls the importance of teachers helping students to transfer information from what the learner knows to other new concepts. As well, Lowery (1998) reminds teachers when teaching a new concept to your students make sure they can assimilate the concept into their prior constructions. Therefore, in order for our brains to keep new information teachers must be aware of the significance of previously learned knowledge.

Brain Research: Attention and Learning

What does brain research say about paying attention? Cho and Sylwester (1992-93) agree teachers should adapt their instructional strategies to meet students' attentional needs. There are some essential principles to remember when trying to catch students' attention. Erlauer (2003) contends the first ten minutes of a lesson is when students will learn the most so you do not want to review during this time. Teachers should hit the students with the new stuff and then later on in the lesson tie in the review from previous lessons to relate new concepts to background knowledge. Additionally, Erlauer (2003) has faith in the fact students need a break in concentration every 20 minutes in order to keep student attention. Teachers need to shift activities at least every 20 minutes to maintain the attention of their students. Nevertheless, teachers should still teach the same concept but in different ways to ensure understanding by all students. Jensen (1998) deems students can only hold their attention in short bursts; no longer than their age in minutes. Brain-based classrooms usually include fifteen minutes of teacher time used for key ideas, directions, lectures, reviews, or closings, and the rest of the time is used for student processing of the new information in their brains through various learning activities such as projects, discussions, group work, writing, etc (Jensen, 1998).

Furthermore, D'Arcangelo (2000) feels teachers can use emotion to direct attention, and that attention will lead to better learning. Teachers must use imaginative teaching strategies like games and competitions to increase students' attention on unemotional learning activities. For example, teachers should use fun energizing rituals for class openers, closings, and most of the repetitious classroom work (Jensen, 1998). Thus, repetitive sedentary activity is followed by an active, enjoyable activity. These strategies will help grab students' attention when teaching the abstract concepts in pure math.

Brain Research: Emotion and Learning

Learning is strongly influenced by emotion. Sousa (2001) states emotions are an integral part of learning, retention, and recall. The stronger an emotion connected to an experience the stronger the memory of that experience (Brandt and Wolfe, 1998). Furthermore, Jensen (2000) reveals powerful evidence that embedding intense emotion such as those that occur with celebrations, competitions, drama, in an activity may stimulate the release of adrenaline, which may more strongly encode the memory of the learning. But, if the emotion is too strong, then learning is decreased. Caine and Caine (1998) cite an important aspect in teaching for meaning includes a state of mind in the learner characterized by low threat and high challenge. When threat and fear are relatively absent, the brain appears to be able to engage in more complex processes. Emotionally stressful classroom environments are counterproductive because they can reduce students' ability to learn. In the classroom, stress might be released through drama, peer support, games, exercise, discussions and celebrations (Jensen, 1998). Good teachers consider the emotional climate in a classroom to be critical. Teachers should invest the first few minutes of every class to get the learner into a positive learning state.

Brain Research: Movement and Learning

Sousa's (2001) research told us that 46% of our students are visually preferred learners, 19% are auditorily preferred learners, and 35% are kinaesthetically preferred learners. Since usually one third of our classes are kinaesthetic learners, teachers need to make sure movement and learning are parallel processes in their classrooms. Ronis (1999) defines bodily-kinaesthetic learner as the ability to use one's whole body to express ideas and feelings, and capacity to use one's hands to produce or transform things. Brain Research confirms that physical activity, moving, stretching, walking, can actually enhance the learning process (Jensen, 2000).

Teachers should encourage their students to move more to learn more. We think well on our feet because more blood and oxygen go to our brains when we are standing and moving (Erlauer and Myrah, 1999). Movement increases heart rate and circulation, which often increases performance. Certain kinds of movements can stimulate the release of the body's natural motivators: adrenaline, and dopamine. Teachers should help students learn with movement. For instance, exercise while you are learning is one type of movement, while others include active lessons, stretching, drama, simulations, games, etc. As well, Erlauer (2003) comments on how changing locations can have an improved effect on student memory so it provides more vivid memory triggers. Active learning has significant advantages over sedentary learning. The advantages include learning in a way that is longer lasting, better remembered, more fun, and that reaches more kind of learners (Jensen, 2000).

Brain Research: Rehearsal and Learning

Sousa (2001) exclaims rehearsal is essential for retention and learning. Replicated studies have demonstrated that cells that fire together wire together. King-Friedrichs (2001) claims neuronal circuits that are continually activated together become stronger and they require less energy to activate as remembering becomes more automatic. Teachers must build into the learning context rehearsal activities that will allow students to recall concepts when needed. Sometimes, the terms rehearsal and practice are used as synonyms. Lowery (1998) argues practice and rehearsal do not mean the same thing. Practice is an activity that does the same thing over and over to improve a performance (Lowery, 1998). Rehearsal is an activity that takes place when people do something again in a similar but not identical way to reinforce what they have learned while adding something new (Lowery, 1998). Rehearsal activities are usually simulations where students draw the concept, write the concept, act out the concept, and solve a problem with the concept. Using rehearsal as a learning strategy increases the likelihood the knowledge will be transferable and useful in a variety of ways. Rehearsal activities are vital to teaching pure mathematics with meaning and understanding. Students must apply and transfer concepts learned in one unit to other units.

Brain Research: Memory and Learning

Memory and learning must be parallel processes to ensure learning with meaning and understanding. A new concept is usually related to prior knowledge or experience to be understood. The information is then practiced or manipulated, and used and applied numerous times before it becomes ingrained in the brain's long-term memory (Erluaer, 2003). Student's memories are not stored as one memory; students must reconstruct the memory. Wolfe (1999) indicates the human brain can only work with seven (plus or minus two) bits of information at one time. Therefore, student's recollections are mostly at the mercy of their elaborative rehearsal strategies. These strategies encourage learners to elaborate on the information being learned in a manner that increases understanding and retention of that information. Brandt (1999) contends only those aspects of experience that are targets of elaborative encoding processes have a high likelihood of being remembered.

Both rehearsal and reflection time is essential for long-term memory. Students need conceptual understanding of how a new skill works. During rehearsal sessions, students should be reflecting on the new concept and adapting or reshaping it. This deeper examination of the new concept, as well as the distinct ways of rehearsing through reshaping will promote useful application of that knowledge in the future (Erluaer, 2003). Through these processes, learning pure mathematics with meaning and understanding can be achieved.

Brain Research: Elaboration and Learning

The brain-based approach to learning incorporates elaborative rehearsal strategies. Wolfe (2001) describes how these strategies encourage learners to elaborate on the information being learned in a manner that increases understanding and retention of that information. These strategies make the information more meaningful and relevant to the learner. Jensen (1998) contends teachers need elaboration activities to strengthen their original contact with students because a synaptic connection is often temporary Additionally, King-Friedrichs (2001) explains students' recollections are largely at the mercy of their elaborations. Learning experiences should be elaborate and as deep as possible using a variety of learning experiences to help students deepen their understanding.

Brain Research: Elaborative Rehearsal Strategies

The possibilities for elaborating the curriculum and making learning meaningful to students are endless. These examples are but a few of the many brain-congruent activities available to teachers. If the content is rigorous and relevant, debates, storytelling, art, music with rhymes and rhythms, writing and illustrating books, constructing models, video making, visuals, active review, simulations, projects, hands-on activities, interactive note taking, drama, games, mnemonics, and graphic organizers can dramatically enhance student understanding (Jensen, 1997). Furthermore, chunking is an elaborative rehearsal strategy that uses association so more than seven bits of information can be remembered at one time. Westwater and Wolfe (2000) deem understanding that we create new neural networks through experience gives us a second avenue for making the curriculum meaningful. Since our strongest neural networks are formed from actual experience, we should involve students in solving authentic problems in their school or community. As well, analogies, metaphors, and similes are excellent ways to help the brain find links between new data and previously stored knowledge.

Journal writing to help students explore what they do or do not understand, and help them apply new concepts to their own experience. Mett (1989) discusses how writing in mathematics can be used at an intermediate time to make a transition, allow absorption of a new idea, or to personalize a theory by creating examples and applications. All of these brain-based learning and teaching strategies can be used to teach pure mathematics with meaning and understanding.

Brain-Based Teaching and Learning Models

The following depictions are an overview of brain-based teaching and learning models present in educational research. Although there are some differences in their work, many of their thoughts about brain-based teaching and learning models are similar.

Researchers: Caine and Caine (1990, 1995, 1997, 1998)

Educators who become aware of recent research on how the brain learns will gain exciting ideas about conditions and environments that can optimize learning. Caine and Caine (1990, 1995, 1997. 1998) offer the following brain principles as a general theoretical foundation for brain-based learning. These principles are simple and neurologically sound. Applied to education, they help us to reconceptualize teaching by taking us out of traditional frames of reference and guiding us in defining and selecting appropriate programs and methodologies. All conclusions reached are derived from and rest on the work of many researchers and research fields, including neurosciences, cognitive science, psychology, social sciences, and education. The brain principles are as follows: the brain is a complex adaptive system: body, mind and brain are one dynamic unity; the brain is a social brain; the search for meaning is innate; the search for meaning occurs through patterning; emotions are critical to patterning; the brain/mind processes parts and wholes simultaneously; learning involves both focused attention and peripheral perception; learning always involves conscious and unconscious processes; we have at least two ways of organizing memory: a spatial memory system and a set of systems for rote learning; learning is developmental; complex learning is enhanced by challenge and inhibited by threat; each brain is uniquely organized. These brain/mind-learning principles should be implemented into a climate of: relaxed alertness, meaningful experience, and active processing.

Researcher: Jensen (1998)

Many times we must reteach things because we do not teach in ways that match how our students' brains learn. Jensen (1998) mentions five brain-based principles to follow in order to understand the brain's learning process: neural history, context, acquisition, elaboration, and encoding. Neural history influences how students learn based on prior learning, character, environment, peers, and life experience. Learning context involves the emotional climate of the classroom. Unexpressed emotions can inhibit learning. Therefore, teachers should encourage activities, such as drama, discussions, and celebrations so students have an outlet for emotional expression in classrooms. As well, teachers need to recognize the majority of the acquisition of knowledge by students comes to them indirectly. Students should spend time processing what they have learned through projects, discussions, group work, partner work, writing, design, and feedback. Direct teaching should be limited to fifteen minutes per class. Jensen (1998) exclaims students who do the talking and the doing do the learning. Elaboration activities are needed so students can strengthen their neural connections with the new material being learned for deeper understanding. Finally, teachers need to permanently encode a classes learning in students' brains.

Researcher: Erlauer (2003)

Erlauer (2003) developed brain-based principles from what she knows about learning in order to improve teaching. The brain-based principles include the importance of the following in a classroom: emotion, movement, relevant content, interesting instructional strategies, value of student choice, time, and collaboration. Emotion deals with how students use emotions to remember their learning. Movement helps students learn in several ways. Even changing locations can have an improved effect on student memory as it provides more vivid memory triggers. Relevant content is significant to the learning process because the brain remembers information that is meaningful and linked to prior knowledge or experience. Teachers are challenged to make what students are learning interesting and be able to demonstrate how that information is relevant to them. Providing interesting instructional strategies is important to the learning process, as it is a method to gain student attention, and to meet different student learning styles. The value of student choice on learning activities allows students to work with different multiple intelligences to ensure understanding for all students. Time is essential for rehearsal and reflection activities for long-term memory. Lastly, collaboration in learning is a key teaching strategy as the brain learns socially.

Researcher: Hardiman (2001)

Hardiman (2001) links how the mind works during learning and current brain research to produce best practices for teaching all students. Also, Hardiman (2001) explains how the use of this model has resulted in exciting learning experiences for students as well as increased scores on state assessments. The model includes acquiring and integrating knowledge, extending and refining knowledge, using knowledge meaningfully, and habits of mind. Learning requires both the acquisition of information and the ability to retrieve and reconstruct that information whenever necessary. Extending and refining knowledge requires the brain use multiple and complex systems of retrieval and integration. Using knowledge meaningfully includes activities that require students to make decisions, investigate, conduct experiments, and solve real world problems. To end with, habits of mind involve mental habits that enable students to facilitate their own learning. For example, monitoring one's own thinking, goal setting, maintaining one's own standards of evaluation, self-regulating, and applying one's unique learning style to future learning situations.

Researcher: King-Friedrichs (2001)

King-Friedrichs (2001) introduces brain-based techniques for improving memory and learning. The brain-based techniques include: using emotion, connecting to prior knowledge, making sense, elaborating on key concepts, and incorporating rehearsal activities. When students are emotionally engaged with learning, certain neurotransmitters in the brain signal to the hippocampus, a vital brain structure involved with memory, to stamp this event with extra vividness. Teachers should connect new learning to prior knowledge because it is easier for the brain to reactivate neural connections than it is to learn something with no previous understanding. Students should make sense of new concepts by using writing to challenge students to organize and articulate what they have just learned. Teachers need to construct activities that allow students to elaborate on key concepts using a variety of learning experiences to help students deepen their understanding of the new material. Rehearsal activities are important to learning. Neural connections that are continually activated together become stronger and require less energy to activate as remembering becomes more automatic. Teachers must use rehearsal activities that will likely be present when students need to remember the new material.

Chapter Summary

The chapter examined the ideas, thoughts, and beliefs of many educational researchers on topics connected to brain-based teaching and learning. Students' learning styles and the multiple intelligence theory lay out the rationale for brain-based teaching and learning. One of the outcomes of the application of brain research to classroom practice is learning pure mathematics with meaning and understanding. A brain-compatible mathematics classroom uses what research says about how students learn to improve teaching. Nevertheless, researchers do expect teachers to be cautious when interpreting and applying brain research to their teaching practices. Educational researchers recommend teachers should be aware of certain aspects of brain-based teaching and learning such as being cognitive of how a student's brain learns; connecting previous learned knowledge to new concepts; gaining attention; using emotion and movement to drive learning; and stressing rehearsal, memory, and elaboration activities to motivate learning.

Chapter 3: Brain-Based Teacher Resource

"Students are resourceful problem solvers working alone or in groups productively and reflectively, with the skilled guidance of their teachers. Students communicate their ideas orally and in writing. Students value mathematics and engage actively in learning it."

(National Council of Teachers of Mathematics, 2000, p. 3)

Introduction

This chapter applies brain-based learning and teaching strategies to classroom practice in order to produce a practical brain-based resource for pure mathematics teachers. The following section demonstrates how to teach pure mathematics with the brain in mind. The resource includes the algebraic unit of study that investigates quadratic functions and equations. The reason this unit was chosen is because a lot of the mathematical concepts in this section require abstract thinking about theoretical ideas. Therefore, many students find this part of the mathematics program to be challenging and frustrating. The resource uses brain-based teaching, learning, and elaborative rehearsal strategies to teach these algebraic mathematical concepts that enable students to learn mathematics with meaning and understanding.

The Underlying Learning Principles for the Resource

Recognize Learning Styles

The uniqueness present in all human beings is reason enough why one approach to classroom teaching is ineffective for most students. Classrooms need to be responsive to students' varying readiness levels, varying interests, and varying learning profiles. Teachers should feel a responsibility to present information in varied ways, for example, orally, visually, through demonstration, movement, and hands-on activities if they want to give all their students an opportunity to learn. Therefore, teachers can stimulate and facilitate learning in all children by addressing the need to know in multiple ways. By providing alternate ways of learning, students are at a liberty to gain new information through those learning styles most comfortable for them.

Learning Pure Mathematics with Meaning and Understanding

Teaching pure mathematics with the brain in mind is intended to emphasize learning mathematics with meaning and understanding. Learning mathematics with meaning is based on the number of connections students can make between mathematical concepts. When students learn mathematics with meaning and understanding they trigger previous networks of knowledge in the brain and make links between these networks and new mathematical concepts. Teaching for meaning and understanding can be achieved using alternate representations of mathematical concepts. Such other forms like visual drawings, context stories, hands-on math manipulatives, and real life problems increase the opportunity to stimulate prior mathematical knowledge networks in the brain. Varied teaching methods activate different parts of the brain and student's learning styles that enable students to construct knowledge through the connection of mathematical concepts to ensure learning with meaning and understanding.

Brain-Based Teaching and Learning

A brain-compatible mathematics classroom uses what research says about how students learn to improve teaching. Many of the findings in brain research validate effective teaching practices. Brain-based teaching highlights certain aspects of learning such as connections to previous knowledge, attention, emotion, movement, rehearsal, memory, and elaboration. These brain-based ideas are interwoven throughout the strategies teachers' use in the classroom. Brain strategies match how the brain learns best.

Framework: Brain-Based Daily Lessons

The following brain-based teaching and learning strategies will form the framework for each daily math lesson in the resource. These ideas will be implemented into daily learning activities. Every part of the daily lesson is built around the brain-based framework. From the attention and goal activities to the rehearsal, and debriefing activities; all activities reflect brain-based teaching and learning principles.

Brain-Based Strategy #1: Previous Learned Knowledge

The brain uses less energy when performing familiar functions than when learning new concepts. This is why the use of prior knowledge is so important when learning something new. If a teacher is able to link a new concept to something previously learned less brain energy will be needed to make the new concept a wired memory. In order for teachers to create meaning in their classrooms every encounter with something new requires the brain to fit the novel information into an existing connection of neurons. Therefore, learning new information must occur within the context of what the learner already knows.

Brain-Based Strategy #2: Attention and Learning

There are some essential principles to remember when trying to catch students' attention. The first ten minutes of a lesson is when students will learn the most so you do not want to review during this time. Teachers should hit the students with the new stuff and then later on in the lesson tie in the review from previous lessons to relate new concepts to background knowledge. Additionally, students need a break in concentration every 20 minutes. Teachers need to shift activities at least every 20 minutes to maintain the attention of their students. Nevertheless, teachers should still teach the same concept but in different ways to ensure understanding by all students. Brain-based lessons consistently use the idea that students can only hold their attention in short bursts; no longer than their age in minutes. Brain-based classrooms usually include fifteen minutes of teacher time used for key ideas, directions, lectures, reviews, or closings, and the rest of the time is used for student processing of the new information in their brains through various learning activities such as projects, discussions, group work, and writing. As well, teachers should use imaginative teaching strategies like games and competitions to increase students' attention on unemotional learning activities. For example, teachers should use fun energizing rituals for most of the repetitious seatwork in math. These strategies will help grab students' attention and increase their motivation to do the important reinforcing concept work.

Brain-Based Strategy #3: Emotion and Learning

Learning is strongly influenced by emotion. The stronger an emotion connected to an experience the stronger the memory of that experience. Embedding intense emotion such as those that occur with celebrations, competitions, drama, in an activity may stimulate the release of adrenaline, which may more strongly encode the memory of the learning. But, if the emotion is too strong, then learning is decreased. Emotionally stressful classroom environments are counterproductive because they can reduce students' ability to learn. In the classroom, stress might be released through drama, cooperative learning, games, exercise, discussions and celebrations. As well, the learning activity Think Pair Share is a brain-based strategy that considers the relationship between emotion and learning. Bennett and Rolheiser (2001) believe when students have time to think and then share with a partner before sharing with the entire class, they are more likely to feel secure and experience success.

Brain-Based Strategy #4: Movement and Learning

Teachers should encourage their students to move more to learn more. We think well on our feet because more blood and oxygen go to our brains when we are standing and moving. Certain kinds of movements can stimulate the release of the body's natural motivators: adrenaline, and dopamine. Teachers should help students learn with movement. For instance, exercise while you are learning is one type of movement, while others include active lessons, stretching, drama, simulations, games, etc. Active learning has significant advantages over sedentary learning. The advantages include learning in a way that is longer lasting, better remembered, more fun, and that reaches more kind of learners.

Brain-Based Strategy #5: Rehearsal and Learning

Rehearsal is essential for retention and learning. Rehearsal is an activity that takes place when people do something again in a similar but not identical way to reinforce what they have learned while adding something new. Teachers must build into the learning context rehearsal activities that will allow students to recall concepts when needed. Rehearsal activities are usually simulations where students draw the concept, write the concept, act out the concept, and solve a problem with the concept. Using rehearsal as a learning strategy increases the likelihood the knowledge will be transferable and useful in a variety of ways.

Brain-Based Strategy #6: Memory and Learning

Memory and learning must be parallel processes to ensure learning with meaning and understanding. A new concept is usually related to prior knowledge or experience to be understood. The information is then practiced or manipulated, and used and applied numerous times before it becomes ingrained in the brain's long-term memory. Student's recollections are mostly at the mercy of their elaborative rehearsal strategies. These strategies encourage learners to elaborate on the information being learned in a manner that increases understanding and retention of that information. Both rehearsal and reflection time is essential for long-term memory. Students need conceptual understanding of how a new skill works. During rehearsal sessions, students should be reflecting on the new concept and adapting or reshaping it. This deeper examination of the new concept, as well as the distinct ways of rehearsing through reshaping will promote useful application of that knowledge in the future.

Brain-Based Strategy #7: Elaboration and Learning

The brain-based approach to learning incorporates elaborative rehearsal strategies. These strategies encourage learners to elaborate on the information being learned in a manner that increases understanding and retention of that information. These strategies make the information more meaningful and relevant to the learner. Learning experiences should be elaborate and as deep as possible using a variety of learning experiences to help students deepen their understanding. Some elaborative rehearsal strategies are debates, storytelling, art, music with rhymes and rhythms, writing and illustrating books, constructing models, video making, visuals, active review, simulations, projects, hands-on activities, interactive note taking, drama, games, mnemonics, and graphic organizers can dramatically enhance student understanding. Furthermore, chunking is an elaborative rehearsal strategy that uses association so more than seven bits of information can be remembered at one time. Since our strongest neural networks are formed from actual experience, we should involve students in solving authentic problems in their school or community. As well, analogies, metaphors, and similes are excellent ways to help the brain find links between new data and previously stored knowledge. Journal writing to help students explore what they do or do not understand, and help them apply new concepts to their own experience.

Brain-Based Strategy #8: Collaboration and Learning

The brain is a social brain. Intelligence is enhanced by social situations. The brain needs to be in situations where it is allowed to experience talk, teamwork, debate, and opinions. Cooperative learning activities support collaborative encounters. Therefore, activities like Think Pair Share where students are given time to think and then move on to sharing their ideas with someone else uses our social brain to develop a deeper understanding of what they are learning.

Brain-Based Lesson Plans

Each lesson in the teacher resource incorporates these brain strategies so students are able to learn pure mathematics with meaning and understanding. A pure mathematics 20 quadratic equation and function project is presented next. All lessons have a project work section. Students will work on this project daily throughout the unit. The daily project work enables students to apply their mathematical concepts to a real life situation. Daily brain-based lessons follow the project.

Mathematics Project: Quadratic Equations and Functions

Pure Mathematics 20 Quadratic Equation and Function Project

(Addison-Wesley, 2000, p. 42-47)

The Task:

In this project, you will design a golf video game. Your group will include three students. You will work with graphs of quadratic functions, characteristics of quadratic functions, and equations of quadratic functions. You will use your graphing calculator to find the maximum and x-intercepts of the graph of a quadratic function.

Background:

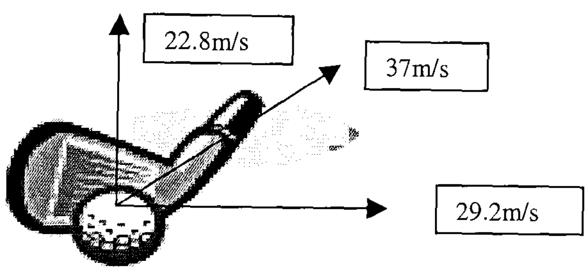
When designers and programmers develop video games they must make movement realistic. Whether it is in the flight of an object, the action of a ski jumper, or the movement of a golf ball, players of the game expect realism.

Many video games involve projectiles such as footballs, golf balls, or ski jumpers. A projectile is any object that is thrown, shot or propelled in some way into the air. Most projectiles are propelled at an angle, and move horizontally as well as vertically. The curved path, or trajectory, of a projectile is affected by the initial speed and inclination of a shot or kick, by gravity, and by air resistance. Gravity pulls everything on Earth toward the centre of Earth. Air resistance slows the projectile. Video game programmers use formulas that have been determined by engineers to model the flights of projectiles so their games seem real.

Getting Started:

When a golfer hits the golf ball squarely at the bottom of the swing, with the shaft perpendicular to the ground, the ball is propelled into the air in a direction perpendicular to the face of the club. For a 7 iron, the clubface is set at an angle of 38° with respect to the shaft of the club. The initial speed of the ball can be determined from stroboscopic photographs.

The speed of an object moving at an angle to the vertical can be resolved into its horizontal and vertical components, provided we know the angle of travel. Horizontal and vertical speeds are determined as the sides of a right triangle by the use of trigonometric ratios.



An engineer used photographs of a professional golfer to determine the average initial speeds for a variety of clubs. She then calculated the corresponding vertical and horizontal speeds. Her results are shown in the table below.

Club	Club Angle	Initial Speed M/s	Horizontal Speed m/s	Vertical Speed m/s
Driver	9.5°	86	84.8	14.2
3 wood	12°	71	69.4	14.8
3 iron	20°	51	47.9	17.4

5 iron	29°	42	36.7	20.4
7 iron	38°	37	29.2	22.8
9 iron	47°	34	23.2	24.9
Pitching Wedge	56°	32	17.9	26.5
Sand Wedge	65°	31	13.1	28.1

Quadratic Equation:

She found that, for the 7iron, the equation of the trajectory of the ball could be approximated by this quadratic equation:

$$Y = -4.9(x/29.2)^2 + 22.8\,(x/29.2)$$

$$Y = -4.9(x/\text{Hor. Speed})^2 + \text{Ver. Speed}\,(x/\text{Hor. Speed})$$

Hor. is short for Horizontal

Ver. is short for Vertical

Where y metres is the height, and x metres is the horizontal distance down the fairway.

Student Questions:

1. The constant – 4.9 in the equation is the engineer's way of accounting for the effect of gravity. Why is the number negative?

2. Where does the horizontal speed of the 7 iron appear in the formula? Where is the vertical speed?

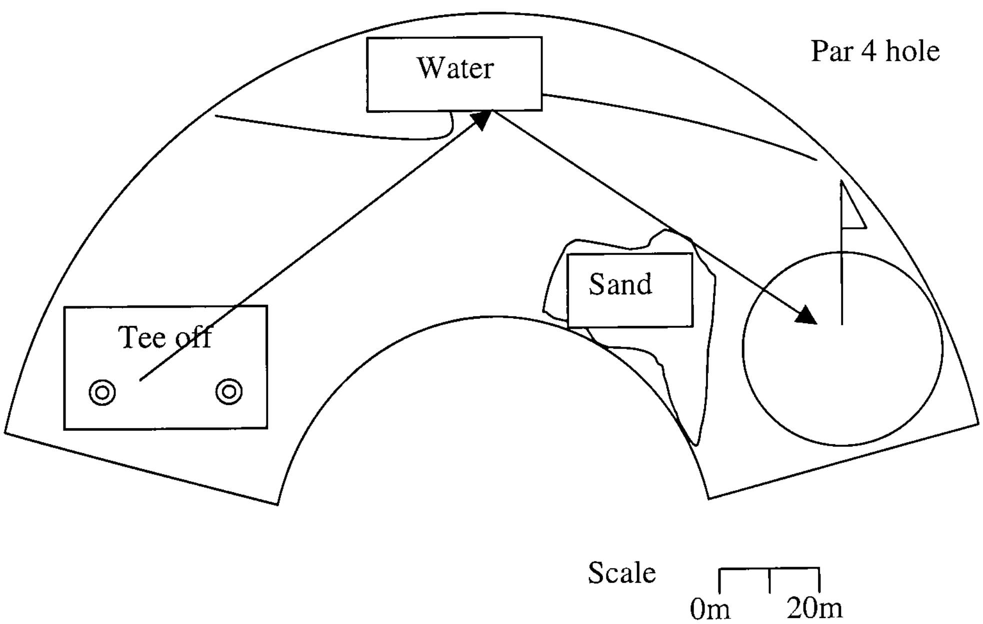
3. Use your graphing calculator to graph the trajectory of a golf ball hit into the air by a 7 iron.

4. Use the CALC feature of the graphing calculator to determine the maximum height of the ball, to the nearest metre.

5. Determine the maximum horizontal distance the ball travels while in the air. Why may this not be the total horizontal distance travelled by the ball?

6. Change the constants in the equation to those appropriate for a driver. Graph the trajectory this model predicts for a golf ball struck by a driver. Record the maximum height and distances.

7. Consider the layout of the par-4 golf hole on a golf course shown below. Assume a two-putt for the hole. One possible line of approach is shown. Calculate the distance for each shot. Would this line of approach be suitable for the club selection: driver, 7 iron, putter? Explain. Suggest other possible approaches.



Project Requirements and Presentation:

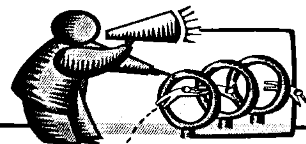
To design video games successfully, the programmers and course designers must work together to produce a course design that is interesting and fun to play as a video game. The programmers must use formulas that imitate the movement of a golf ball hit by different clubs and at different speeds. The course designers must make a course that is interesting to play, given the clubs available and how they are hit. Courses that are too easy are boring and those that are too hard aren't much fun either.

1. Make a table containing the flight data for each club. The flight data includes how far and how high the ball will travel under the conditions described in the table for each club. Using the quadratic equation given, graph the quadratic function for each club on your graphing calculator. Find how far and how high the ball will travel for each club using the zero and maximum functions on the graphing calculator.
2. Design three holes of a golf course that would be suitable for inclusion in a video game. Use the flight paths of a golf ball struck by each club in the table to set the distances and the shapes of each hole. To add interest to the course, include items such as ponds, sand traps, bunkers, creeks, and overhanging trees or make the green an interesting shape. Your golf course design could be presented in one of the following ways:
   - Through the use of computer programs
   - Through the use of a video camera
   - Through a dramatized play or skit
   - Through a golf brochure

- Through the construction of a hands-on model
- Through a child's storybook
- Create your own

3. Write a creative golf story with characters playing the three holes of your golf course. This should be read when your three-hole golf course design is presented. Mathematical data should be included in the story. An explanation of why certain clubs are picked for certain golf shots depending on their x and y graph distances should be included. As well, the mathematical information in the flight table for each club should provide support to your story.

Lesson One



Attention: Use various computer games that possess flights of balls in the form of a quadratic function and show these to students. Mention to students that computer programmers and engineers use quadratic equations and functions to depict the flight of an athletic ball in the computer games that you are showing them.

GOAL: To investigate quadratic functions and equations.

Rehearsal/Elaboration Activities:

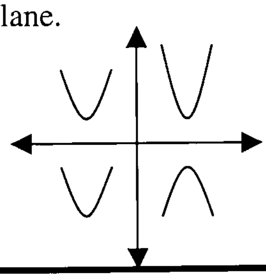
1. KWL Activity: On a sheet of paper, sketch a KWL chart. Explain or review the meaning of each letter of KWL (what we Know, what we Want to know, and what we Learned). Divide the students into groups of 2, 3 or 4. Introduce the topic of quadratic functions and equations. Ask each group to brainstorm what they already know about the topic. Record the ideas into the first column of the chart. When the K column is full, repeat the brainstorming process for the W column. Allow thinking time between responses as necessary. Display these KWL charts in the classroom during this unit. When the unit of study comes to an end and you are reviewing the material, one review activity would be to repeat the brainstorming process for the L column. This will provide an opportunity for students to organize and write down what they feel they have learned throughout the unit. Allow time for groups to discuss their ideas, as well get groups to present to the whole class.

What we Know	What we Want to know	What we Learned

<u>2. Data Set Inductive Thinking Activity</u>: (resource attached) Put students into a group of 2 or 3. The students should analyze the data sheet individually. Each student in the group is to make their own hypothesis of what a quadratic equation and function are. Then each member of the group should share their hypothesis while other members listen. As a group, students should decide on a hypothesis and use this educated guess to guide their group through the tester examples and the creation task. Small groups will share their decisions with the larger class group. As a class, conclusions are made about quadratic equations and functions.

<u>Problem Solving:</u>

Create quadratic equations of quadratic functions whose graph will fit into each quadrant of a Cartesian Plane.



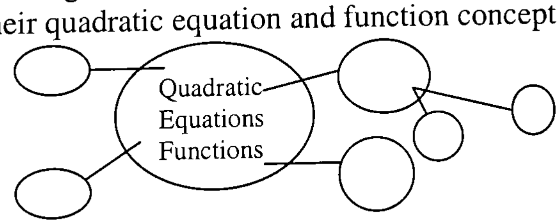
<u>Project Work:</u>

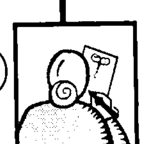
Video Game Design

Introduce project. Read the background information. Design project groups. In groups, students are to find one video game that they think would have used quadratic equations and functions in the design of the game. Students would share this information with other groups.

<u>Reflect – Pair – Share – Write</u>

Students should design a **concept map** that shows everything they know about quadratic equations and functions. Give students time to individually reflect on the task. Then students are to find a partner and share their knowledge with each other. Together they are to design their quadratic equation and function concept map.

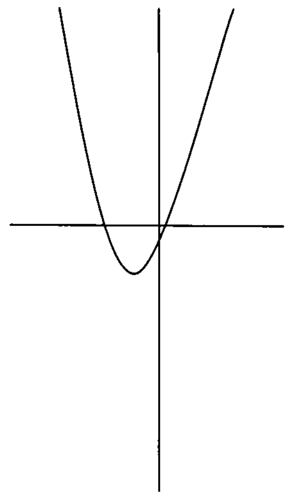


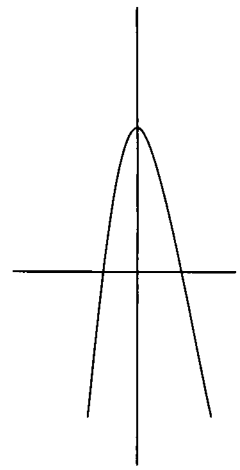


Data Set: Quadratic Equations and Functions

The following are examples of quadratic equations and functions:

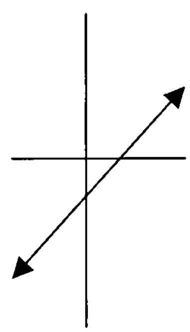
$y = x^2 + 3x - 4$ $\qquad$ $y = -2x^2 - x + 5$

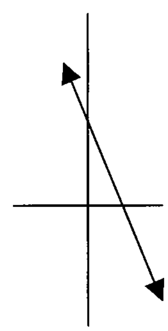




The following are not examples of quadratic equations and functions:

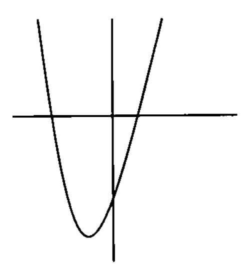
$y = x + 3$ $\qquad$ $y = 2x - 6$ $\qquad$ $y = x^3 - 5x + 2$





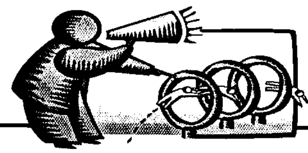
Tester: Which of the following are examples of quadratic equations and functions:

$y = x^3 + x - 7$ $\qquad$ $y = x^2 + 3x - 3$ $\qquad$ $y = x - 2$



Can your group create a quadratic equation and function.

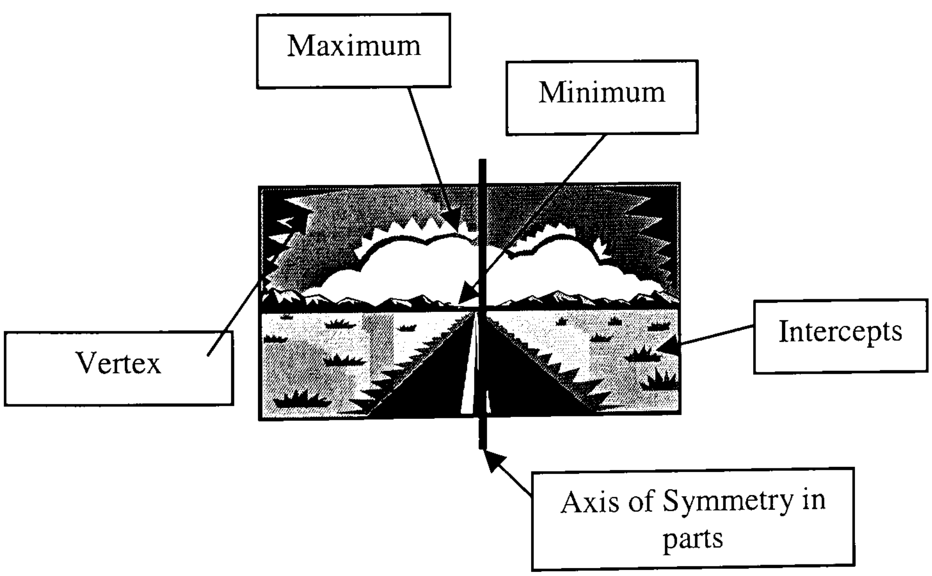
Lesson Two



Attention:

Analogy

In pairs, using a landscape picture, human body picture, or another type of picture, get students to identify on the picture the various characteristics of the graph of a quadratic function. The students should identify the characteristics in a very general manner so they can review their prior knowledge on the characteristics; yet gain an understanding of the main idea of the concept. Give students a few minutes to share in pairs, then one pair compare and share with another pair close to them. After a few minutes, students should share as a large group. Later in the lesson, the mathematical specifics of the characteristics will be added

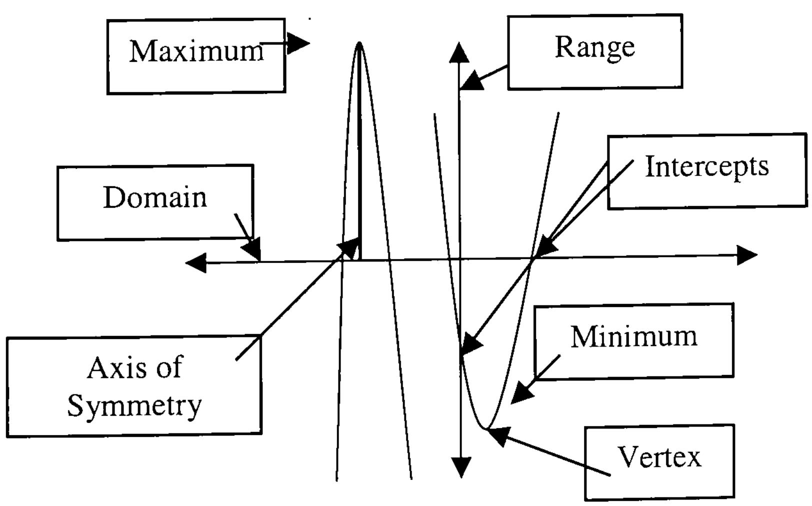


GOAL: Determine the following characteristics of the graph of a quadratic function:

a) maximum or minimum graph
b) axis of symmetry
c) vertex
d) intercepts
e) domain and range

Rehearsal/Elaboration Activities:

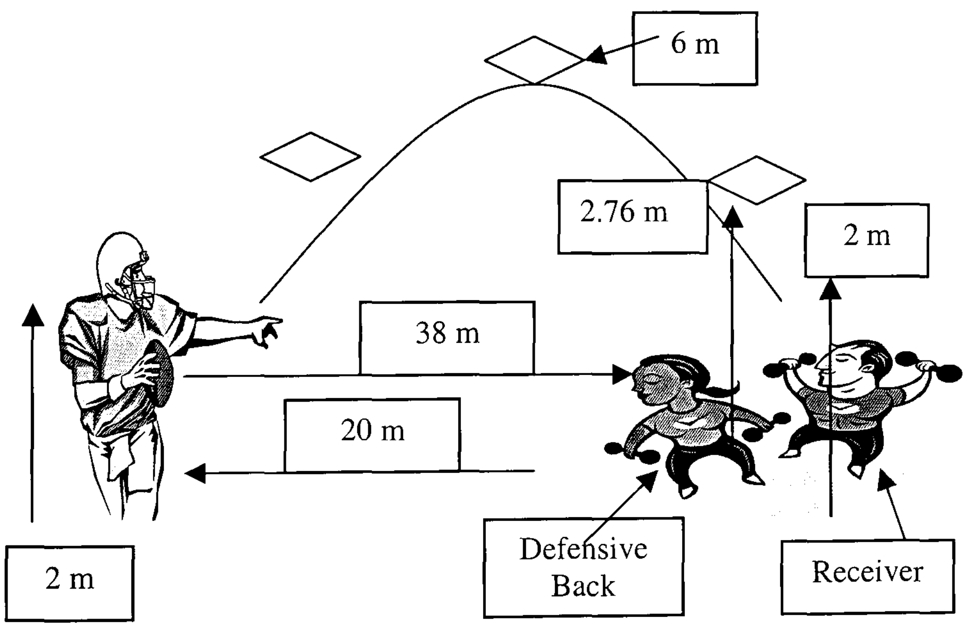
1. Think Tank Activity: Students work in groups of four, and put their four desks into a square formation. Teacher gives students a quadratic function equation to enter into their calculators to graph. Teacher should model the steps involved by using their graphing calculator and LCD projector screen. Students now have a visual diagram of the graph of a quadratic function. Students should use the other students in their think tank to transfer their general knowledge of the characteristics of the graph of a quadratic function and try to add in the mathematical specifics needed to identify exact characteristics. The teacher will go around to the groups giving hints and information where needed. Depending on the students understanding, a group of students could present to the class some characteristics on the LCD projector. Teachers need to be prepared to model the solutions depending on the knowledge of the class.



Problem Solving:
(see attached resource)
Football Problem
This problem is an in context application of a real life situation that reinforces the characteristics of the graph of a quadratic function. Students should work in pairs and make a large visual solution on poster paper. Put finished products up in class and teacher and students walk about giving feedback to groups.

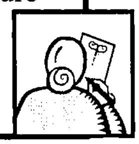
Project Work:
Video Game Design

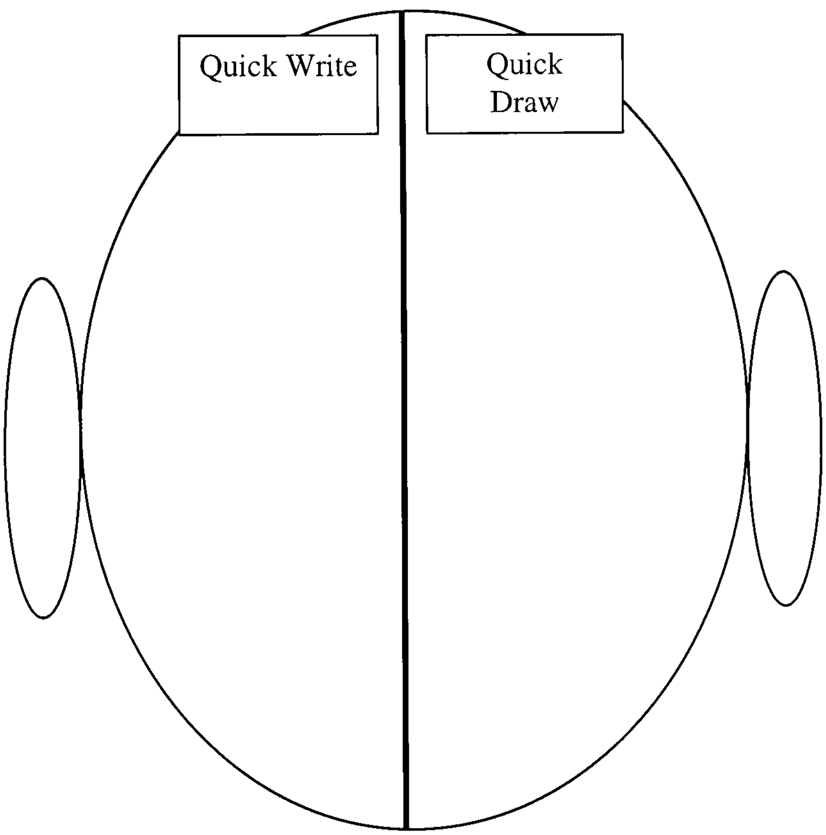
In project groups, with teacher guidance, students answer getting started questions.



Reflect – Pair – Share – Write

Teacher will give students a quadratic function equation. Students should write under the **Quick Write** heading the steps involved to find the characteristics of the graph of a quadratic function on their calculator. Next, students should draw under the **Quick Draw** heading the graph of the quadratic function and label the exact characteristics. The following brain organizer should be used to structure students' solutions.





Football Problem

A football quarterback passed the ball to a receiver 40m downfield. The path of the ball can be described by the function: $h(d) = -0.01(d - 20)^2 + 6$, where $h(d)$ is the height of the ball in metres, and d is the horizontal distance of the ball from the quarterback in metres.

a) What is the maximum height of the ball?

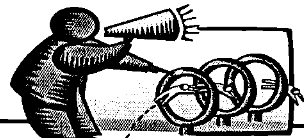
b) What is the horizontal distance of the ball from the quarterback at its maximum height?

c) What is the height of the ball when it was thrown and when it was caught?

d) If a defensive back was 2 m in front of the receiver, how far ahead was the defensive back from the quarterback?

e) How high would the defensive back have needed to reach to knock down the pass?

Lesson Three



Attention: Introduce Mnemonic to use with students to help them remember the general steps to the complete the square process.

$$\mathbf{y = a\,(x - p)^2 + q}$$

BCC AS FSV

Bullies Cause Crying As Friends Supply Virtue

B = Brackets
C = Coefficient
C = Constant
A/S = Add or Subtract
F = Factor
S = Simplify
V = Vertex

$$\mathbf{y = 2x^2 + 12x + 1}$$

Brackets	$y = (2x^2 + 12x \quad\ ) + 1$
Coefficient	$y = 2(x^2 + 6x \quad\ ) + 1$
Constant	$y = 2(x^2 + 6x \qquad\ ) + 1$
	$(1/2 \text{ x } 6)^2 = 9$
	$y = 2(x^2 + 6x + 9) + 1$
Add/Subtract	$y = 2(x^2 + 6x + 9) + 1 - 18$
	$+ 18$
Factor	$y = 2(x + 3)^2 + 1 - 18$
Simplify	$y = 2(x + 3)^2 - 17$
Vertex	$(-3, -17)$

GOAL: Connect algebraic and graphical transformations of quadratic functions, using complete the square as required. Make sure students realize the complete the square form of a quadratic function allows for easier identification of the characteristics of a quadratic function from the equation form instead of the graph, especially the vertex.

Rehearsal/Elaboration Activities:

1. Math Graffiti Activity: The teacher should spend time reviewing students' prior knowledge on a few skills involved in the complete the square process. One of the biggest problems students have with complete the square is previously learned skills like factoring with a GCF, factoring a perfect square trinomial, and adding and subtracting fractions, not with the complete the square process itself. I find if students are given time to rehearse these skills a lot of the student error and frustration is alleviated.

Ex. Factoring with a GCF $y = \dfrac{2x^2 + 4x + 8}{2}$

$y = 2(x^2 + 2x + 4)$

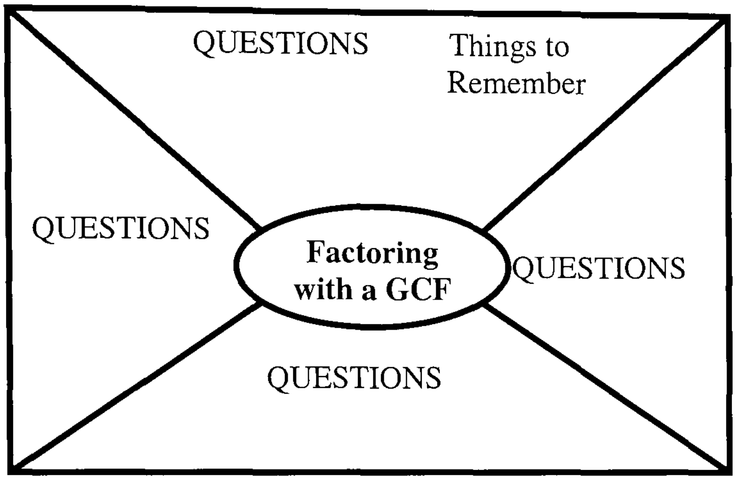
Ex. Factoring a Perfect Square Trinomial $y = x^2 + 10x + 25$

$y = (x + 5)^2$

Ex. Add or Subtract Fractions (common denominator) $\frac{2}{3} + \frac{1}{8}$

The Graffiti activity is a creative brainstorming process that involves collecting the wisdom of all or most of the students in the class. It starts by placing students in groups of four and providing a large piece of paper for each group. Each piece of paper has a topic in the middle. The topics for our math graffiti activity will be factoring with a GCF, factoring a perfect square trinomial, adding fractions, and subtracting fractions. Each sheet of paper has many math questions on it related to the topic. The students get 30 seconds to think and then 90 seconds to individually solve as many questions as they can related to that topic. Next, they get 30 seconds to write down one key thing to remember when solving those types of questions. They then stop, stand up, and go, as a group, to a different piece of paper. When they return to their original group they now have the collective wisdom of everyone in the class.

Graffiti Math Poster



2. Teacher Modeling: Teacher would model the complete the square process for students. Time should be given for students to do examples with teacher.

3. Game "Word Scramble" Activity: I would use this active game to motivate students to do rehearsal work on the complete the square process. There would be various large numbered index cards around my room with complete the square questions on them. In groups of 2, students will move around the classroom and try to find the answer to the question on the index cards. Students will verify their answer after each index card with the teacher. The teacher would have a sheet with the numbered index cards listed out with their answers. If the students got the question right they would be given a letter or letters depending on the complexity of the word scramble. Students must completely answer all index cards correctly and receive all letters before they can unscramble the word or words. The group that unscrambles the word first is the winner. I think you will find this game a highly motivating game for students.

[redacted]ex Card
[redacted]mplete the
[redacted] form:

$y = x^2 + 6$

#2 Index Card
Put into complete the
square form:

$y = x^2 + 10x + 28$

#3 Index Card
Put into complete the square form:

$y = x^2 - 5x + 2$

#4 Index Card
Put into complete the square form:

$y = 3x^2 - 12x + 17$

Teacher Solution List:

#1	$y = (x + 3)^2 - 9$	S, A
#2	$y = (x + 5)^2 + 3$	E, D
#3	$y = (x - 5/2)^2 - 17/4$	R, R
#4	$y = 3(x - 2)^2 + 5$	U, C

Word Scramble Solution: **CRUSADER**

Problem Solving:

Using the Mnemonic, write out the steps you would use to change a quadratic function into complete the square form. Make up your own quadratic function and use your steps to change it into complete the square form.

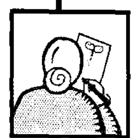
Project Work:
Video Game Design

Make a table containing the flight data for each club. The flight data includes how far and how high the ball will travel under the conditions described in the table for each club.

Reflect – Pair – Share – Write

Reciprocal Teaching

Pair students and allow each person in the pair to pick a letter A or B. Then give the students the question:

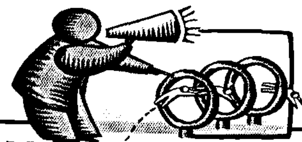


$$y = -2x^2 - 4x + 1$$

Ask the "A" partners to teach the "B" partners how to change that quadratic function into complete the square form. The "A" and "B" partners can switch their roles throughout the teaching process if one of them needs help teaching.

Next, ask the "B" partners to teach the "A" partners how to graph that quadratic function and then identify the characteristics of the graph. (maximum or minimum graph, axis of symmetry, vertex, intercepts, domain and range) The "A" and "B" partners can switch their roles throughout the teaching process if one of them needs help teaching.

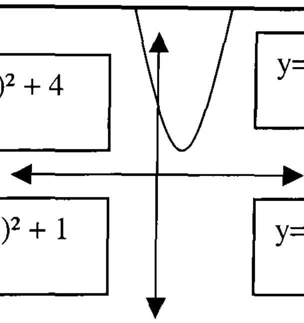
Lesson Four



Attention: Mix and Match
Teacher displays a large quadratic function graph in the classroom. Hand pairs of students an envelope with 4 small index cards in it. Students will be asked to analyze the large graph in front of them and then open their envelopes so they can look at the index cards. The small index cards have one completed square form quadratic equation on them. The pairs try to pick the equation that best matches the large quadratic function graph. Small groups defend their choices to the rest of the class. As a class, pick the equation that best matches the graph.

$y=(x-3)^2 + 4$

$y=(x-1)^2 + 1$



$y=(x+1)^2 + 1$

$y=(x-3)^2 - 1$

GOAL: Determine the following characteristics from a quadratic equation in complete the square form:

a) maximum or minimum graph
b) axis of symmetry
c) vertex
d) horizontal and vertical translations
e) domain and range
f) narrow or wide

Rehearsal/Elaboration Activities:

1. Inquiry Activity: Individually, students will be given the following sheet.

Completed Square Equation	**Graph**
1) $y = (x - 2)^2 + 5$	
2) $y = (x + 3)^2 - 4$	

Students will be asked to enter the completed square form equation into their calculators and display the graph. Students will then sketch the graph onto their paper and then label the following characteristics on the graph: maximum or minimum graph, axis of symmetry, vertex, horizontal and vertical translations, domain and range. Students will compare the characteristics of the graph to the completed square form equation. Using this comparison, students should be able to identify these characteristics in the equation. Students will be asked to pair and share. The teacher must be prepared to help some groups. Later, come together as a whole class to discuss solutions.

$$y = + \text{ or } - a\,(x - p)^2 + q$$

(p, q)	vertex
x = p	axis of symmetry
p	horizontal translation, domain
q	vertical translation, range
a	narrow or wide
sign	(+ or -) maximum or minimum

2. Station Activity: This is a rehearsal motivation activity. Put students into a group of 4. Around the room there will be posted stations numbered 1 to 6 or depending on the size of the class. The groups of students will start at any station. The station will give a completed square form equation and students will be asked to identify the characteristics using only the equation. The answers will be on the back of the station paper. Once the group feels confident in their answers they can flip the page over and check their answers. If they get all answers right they will get one point. Students should be given about 90 seconds at each station. Once time is up the teacher will give instructions to students to rotate to next station. The group with the most points after all stations are complete is the winner.

Problem Solving:

Crusader sweatshirts are sold to students for $20 each, and 300 students are willing to buy them at that price. For every $5 increase in price, there are 30 fewer students willing to buy the sweatshirt. What is the maximum revenue? Solve using the complete the square process.

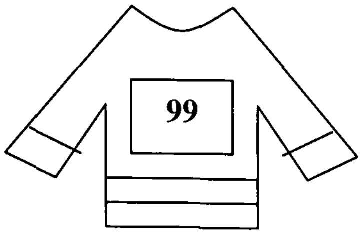
Project Work:

Video Game Design

Design three holes of a golf course that would be suitable for inclusion in a video game. Your golf course design could be presented in one of the following ways:

- Through the use of computer programs
- Through the use of a video camera
- Through a dramatized play or skit
- Through a golf brochure
- Through the construction of a hands-on model
- Through a child's storybook
- Create your own

Sweatshirt $20

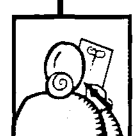


Let x be the number of $5 increases
Revenue = Cost x Number
$= (20 + 5x)(300 - 30x)$
$= -150x^2 + 900x + 6000$
$= -150(x^2 - 6x + 9) + 6000 + 1350$
$= -150(x - 3)^2 + 7350$
The maximum revenue is $7350

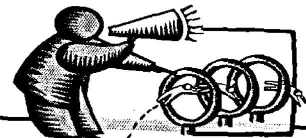
Reflect – Pair – Share – Write

Who Has Mail

Individually, students should write a letter to someone who missed class today. In the letter, students must explain how to find the characteristics from a completed square form equation. Give time for this activity. Allow some students to read their letters to the class.



Lesson Five



Factors
Prior
Learning

27
1, 3, 9, 27

14
1, 2, 7, 14

Attention: Teacher will have a hat filled with trinomials and binomials that can be factored using mental factoring. The teacher will pull one or two out and mentally factor these for the class. This process should draw on the prior knowledge of the students and they should start to remember how to mental factor. Allow those students that want to try factoring one to pick one out of the hat. Next, the teacher will take some of the quadratic equations and graph them on the LCD projector. Again, the teacher will ask a student to mentally factor the equation. The teacher will then show the students the relationship between the factors of a quadratic function and the x-intercepts of the graph of a quadratic function.

Factors
Prior
Learning

24
1, 2, 3, 4,
6, 8, 12,
24

10
5, 2, 10, 1

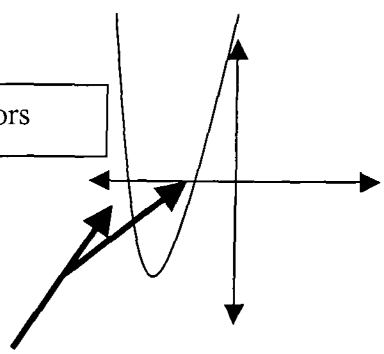
$y = x^2 + 7x + 10$

$y = (x + 5)(x + 2)$ Factors

$x + 5 = 0 \quad x + 2 = 0$

$x = -5 \qquad x = -2$

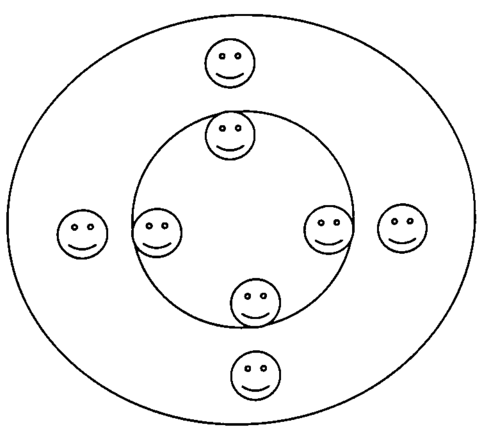
Zeros, x-intercepts, roots



GOAL: Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: factoring and graphing.

Rehearsal/Elaboration Activities:

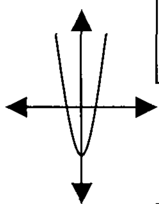
1. Inside/Outside Circles Activity: This activity involves placing students in two circles – one circle within the other. Depending on the size of your class you could do many adaptations to this. The activity can be employed with groups of six or more students (with half of the group forming the inside circle and half the outside – with each student in one circle facing a student in the other). This structure facilitates dialogue between students, as well as movement in either direction for the activity. Once the students are in position, the teacher will put a factoring question on the board. Some of the factoring questions will be trinomials or binomials that require students to use mental factoring, and decomposition or long factoring. These are both concepts learned in grades 9 and 10. The teacher will ask the students to individually think about how they might respond to the factoring question on the board for about 30 to 60 seconds. Now the student on the inside of the circle will be asked to tell the person across from them on the outside of the circle how they would attempt to solve the factoring question. When the student is finished sharing they will pass the talking over to the student on the outside and that student will share or extend the thinking of the inside person. The students need to agree on a method to solve the factoring equation then they should implement the strategy and solve the question. Then students should verify their solutions using their graphing calculator. When finished, have the outside students rotate one to the left or right. The students are now ready for the next question and interaction with a new person. Teacher will continue using different types of factoring questions that require different strategies to solve. **Variation** – Teachers give a question and the inside and outside students compete against each other to find the correct solution.



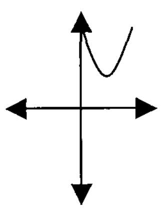
<u>Problem Solving:</u>

In groups of two, students give an example of a quadratic equation that would have the following roots according to the graphs:

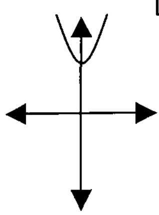
2 real roots



2 equal roots



no real roots

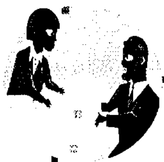


Students should be ready to defend their answers and show work to support their answers.

<u>Project Work:</u>
Video Game Design

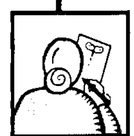
Continue working on the items below. Design three holes of a golf course that would be suitable for inclusion in a video game. Your golf course design could be presented in one of the following ways:

- Through the use of computer programs
- Through the use of a video camera
- Through a dramatized play or skit
- Through a golf brochure
- Through the construction of a hands-on model
- Through a child's storybook
- Create your own

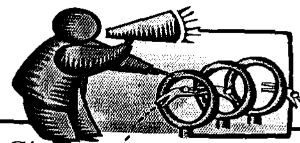


Reflect – Pair – Share – Write

Explain in your own words, what you think the relationship is between the factors of a quadratic equation and the x-intercepts, roots or zeros of the graph of a quadratic equation. Give students time to individually reflect on the question. Then students are to find a partner and share their knowledge with each other. After that students are to individually answer the question. Allow some students to share their explanations with the whole class.



Lesson Six



Attention: Give students the following quadratic equation:

$$Y = x^2 - 4x + 1$$

Ask students to factor the equation. Students will try to factor the equation using mental factoring and soon will agree that it cannot be factored. Teacher would then introduce the solution to the problem:

The Quadratic Formula

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

GOAL: Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: the quadratic formula and graphing.

Rehearsal/Elaboration Activities:

1. Walk About Activity: The teacher should spend time reviewing students' prior knowledge on a few skills involved in using the quadratic formula. One of the biggest problems students have with the quadratic formula is previously learned skills like simplifying radicals and reducing radicals to lowest terms, not with the quadratic formula itself. I find if students are given time to rehearse these skills a lot of the student error and frustration is alleviated.

The teacher models both skills for the students before the activity begins.

$$\sqrt{12} = 2\sqrt{3}$$

Simplified

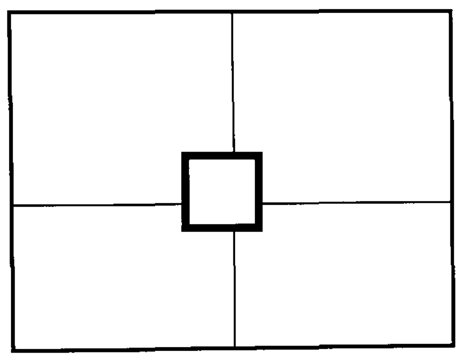
$$\frac{4 \text{ + or } - 2\sqrt{3}}{2}$$

Reduced

$$\frac{2 \text{ + or } - 1\sqrt{3}}{1}$$

In pairs, students identify themselves as A or B. The teacher will put either a simplifying radical question or a reducing radical question on the board. In pairs, the students try to solve the question. The teacher roams the room and helps out where needed. Once the students have completed the question, the teacher will ask all A's to raise their hands, then the B's get up and move to the next A. Another question is put on the board and the students will begin to exchange ideas again.

2. Place Mat Activity: Before beginning the activity, the teacher should model the steps involved in solving a quadratic equation for the x-intercepts, roots, or zeros using the quadratic formula. The teacher should emphasize the need for exact answers in the form of whole numbers, fractions, or simplified radicals. The place mat activity involves a group of four students working both alone and together around a single piece of paper to simultaneously involve all members. The paper is divided up into four pieces with a central square.



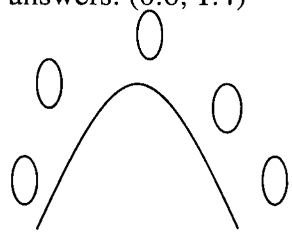
The teacher will put one question on the board. One student will put the question in the small square in the middle of the place mat. Students work out the solution in their space. When finished, they share their solution with the rest of the group. As a group, they will decide on which solving methods and solutions are correct. Students should graph the quadratic equation to help support their answers. Then some groups will share with the whole class.

Problem Solving:

A ball is thrown straight up in the air. Its height, h(m), after t seconds is given by

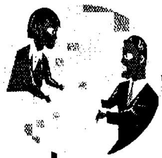
$$h = -5t^2 + 10t + 2.$$

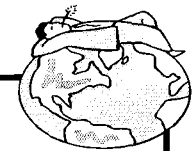
To a tenth of a second, when is the ball 6m above the ground? Explain why there are 2 answers. (0.6, 1.4)



Project Work:
Video Game Design

Write a creative golf story with characters playing the three holes of your golf course. Mathematical data should be included in the story. An explanation of why certain clubs are picked for certain golf shots depending on their x and y graph distances should be included. As well, the mathematical information in the flight table for each club should provide support to your story.





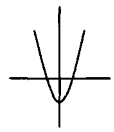
Reflect – Pair – Share – Write

Character of the roots of a Quadratic Equation

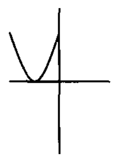
Discriminant: $\mathbf{b^2 - 4ac}$

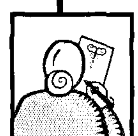
In pairs, students analyze the following information:

a) $\mathbf{b^2 - 4ac > 0}$ 2 real roots

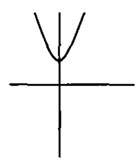


b) $\mathbf{b^2 - 4ac = 0}$ 2 equal roots



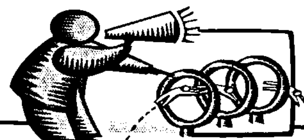


c) $\mathbf{b^2 - 4ac < 0}$ no real roots



Using this information, find three quadratic equations that were used in today's lesson that would satisfy one of the above characteristics in a, b, or c. Write why each equation satisfies the above characteristic. When finished, students should share their results with another close pair. Then results should be shared with the whole class.

Lesson Seven



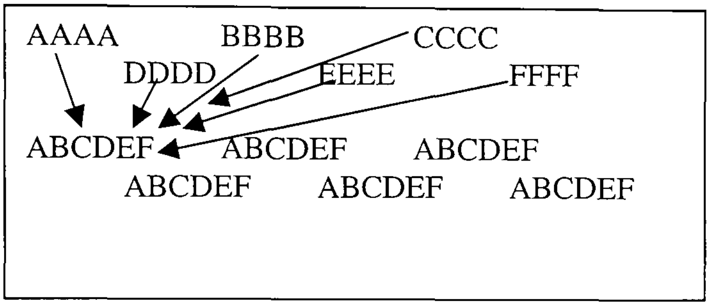
Attention: KWL Activity: Divide students into their KWL activity groups from the beginning of the unit. Students are to take down their KWL charts on the classroom walls and complete filling in the last column L - What we learned. This will provide an opportunity for students to organize and write down what they feel they have learned throughout the unit. Allow time for groups to discuss their ideas, as well get groups to present to the whole class.

What we Know	What we want to Know	What we Learned

GOAL: Actively review the concepts connected to quadratic functions and equations.

Rehearsal/Elaboration Activities:

1. Teams – Games – Tournament (TGT): Divide students into groups of four or five. These groups will be the student teams. Give students the first five minutes to do two things: make a team name, and make team symbols for team members to wear to identify themselves. They then break into tournament groups where one student from each team group gets together. These students will compete against each other.



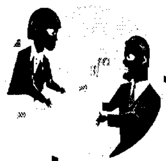
Tournament groups then answer a number of questions. The questions are placed on cards with answers on the back (attached). Points will be given as follows: if you do not get the question correct – 0 points; one student in the tournament group gets the question correct and no one else does – 2 points; if more than one person in the tournament group gets the question correct then those who got the question right – 1 point. When they have completed the questions or the time is up, they return to their home team and add up their individual tournament scores. The group with the most points receives an incentive. Winning team present a team cheer at the end.

Problem Solving:

In pairs, students make up a problem-solving question, swap, then solve.

Project Work:
Video Game Design

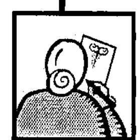
Groups present their projects.



Reflect – Pair – Share – Write

Students make a list of their strengths and weaknesses in this unit. Share and discuss your lists with another student. Together, students are to try and think of strategies that will help them cope with their weaknesses.

Strengths	Weaknesses



Tournament Card #1

The revenue generated at a fitness club can be expressed as a function of the membership fee by $R = -0.8f^2 + 480f$ where R is the revenue and f is the membership fee.

What is the maximum revenue this fitness club can obtain from membership fees to the nearest dollar? ______________ (72 000)

Tournament Card #2

The revenue generated at a fitness club can be expressed as a function of the membership fee by $R = -0.8f^2 + 480f$ where R is the revenue and f is the membership fee.

What membership fee yields the maximum revenue? ______________ (300)

Tournament Card #3

Find the exact roots of the quadratic equation $3x^2 - 8x - 5 = 0$.

____

$$\frac{(4 + \text{or} - \sqrt{31}}{3}$$

Tournament Card #4

Solve the equation $-0.2d^2 + 2.5d + 8 = 0$ for d, the horizontal distance from the kicker to the landing point, to the nearest metre.

_______________________ (15)

Tournament Card #5

Change $y = 3x^2 - 12x + 17$ into completed square form. State the vertex, axis of symmetry, maximum or minimum graph, and domain and range.

____________________ $(y = 3(x - 2)^2 + 5)$
____________________ (2, 5)
____________________ (x = 2)
____________________ (minimum)
____________________ (x E R, Y $\geq$ 5)

Tournament Card #6

Describe the nature of the roots of the following quadratic function using algebra and your graphing calculator to support your answer.

$$\mathbf{2x^2 + 3x - 7 = 0}$$

_______________________ (2 real roots)

Summary

This chapter applied brain-based principles to classroom practice to produce a practical brain-based resource for pure mathematics teachers. The resource demonstrates how to teach the pure mathematics 20 unit on quadratic equations and functions with the brain in mind. The resource is built on the following underlying learning principles: the importance that each lesson recognizes the different learning styles present in the class; the importance of learning pure mathematics with meaning and understanding; and the importance that each lesson uses brain-based teaching and learning strategies. The framework of each lesson in the resource considers the following brain strategies: previous learned knowledge, attention and learning, emotion and learning, movement and learning, rehearsal and learning, memory and learning, elaboration and learning, and collaboration and learning. Each lesson includes activities that call for attention, rehearsal, elaboration, problem solving, project wok in real life situations, and reflection, sharing, writing. All of these ideas connect together to produce a teacher resource that changes the presentation of the pure math 20 unit on quadratic equations and functions to its students in order to teach pure mathematics with meaning and understanding.

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