

**THE AREA CODE TREE FOR APPROXIMATE NEAREST NEIGHBOUR  
SEARCH IN DENSE POINT SETS**

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THE AREA CODE TREE FOR APPROXIMATE NEAREST NEIGHBOUR SEARCH  
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# **Dedication**

I dedicate this thesis to my baby girl Sarah Zaman.

# Abstract

Location based Services (LBSs) have become very popular due to the rapid development of wireless technology and mobile devices. A LBS provides results to a user of a mobile device (e.g. smart phone, tablet) based on their location, interests, and the type of query being performed. For example, a user may want to know the location of the closest restaurant to them. Sometimes the user may also be happy with another suggestion that may not be the closest but close enough to satisfy them. This is an example of an approximate nearest neighbour search. In this thesis, we propose a spatial data structure the Area Code Tree which is a trie-type structure. The Area Code Tree stores Points of Interest (POIs) that are represented in area code format. We also present the algorithms for mapping the area code of a POI, inserting and building an Area Code Tree, and approximate nearest neighbour search. Next we evaluate the Area Code Tree for accuracy, tree construction time, and compare its search performance with the Brute Force Method. We find that the average search time for Area Code Tree in locating nearest neighbour is very low and constant regardless of the number of POIs in the index. In addition, the Area Code Tree can achieve up to 60% accuracy for locating the nearest neighbour in dense point sets. This makes the Area Code tree an excellent candidate for continuous approximate nearest neighbour search for location-based services.

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# Chapter 1

## Introduction

Due to the rapid development of wireless technology and mobile devices, Location based Services (LBSs) have become very popular. LBSs are services that return results based on certain location information and query types. The environment of a LBS is classified into three types according to the mobility of the clients and the data objects: 1) Static client and static data objects, 2) moving client and static data objects, and 3) moving client and moving data objects. In a moving client and static data objects environment, the user continuously changes their location and submits a new query to the server for obtaining updated static data information. Consider two scenarios: (i) A driver in a GPS-equipped car issues a continuous query to find the nearest gas station while driving in a city, and (ii) a tourist walking through a city center uses a location-aware mobile device to issue a continuous query for the nearest restaurant. These queries are examples of exact continuous nearest neighbour queries.

Sometimes users may also be happy to find points of interest (POIs) that may not be the closest but close enough to satisfy them. This is an example of an approximate nearest neighbour search. These queries are sent to the server through a wireless network to be processed with the result returned to the user. As the user moves around, their nearby POIs change, so the result must be updated continuously on their device. The conventional approach of providing up-to-date results to a mobile user is to submit a new query to the server after every location update. This could cause high network overhead and extra processing effort. Although repeated searching is not desirable, but it may be the only option available

when mobile devices have very limited storage capacity. Therefore, it is very important to use an efficient query processing strategy in the server that can be re-executed repeatedly.

Many strategies have been proposed for continuous nearest neighbour processing for location based services [20, 22, 21, 27, 17, 13, 15]. Some limitations of these strategies include repeated searching, caching a significant amount of data on the mobile device in order to avoid query re-submission, and the requirement to know the query trajectory in advance.

In this thesis, a spatial data structure the Area Code Tree<sup>1</sup> is proposed for storing and managing points of interest (POI) data for the purpose of continuous nearest neighbour processing. The Area Code Tree is a Trie-type structure, in which all POIs are represented by area codes. An area code is a sequence of digits that represents the relative location of a point in space. We index the area code data of a POI instead of actual spatial values (i.e. longitude and latitude) to reduce the search for a nearest neighbour to a sequence of simple numeric comparison. We also present the algorithms for mapping the area codes of a POI, inserting and building an Area Code Tree, and approximate nearest neighbour search. Finally, the Area Code Tree is evaluated for accuracy, tree construction time and its search performance is compared with the Brute Force Method which is a basic benchmark for processing nearest neighbour queries.

Twenty-one data sets that represent collections of different POIs across New Zealand are used for our evaluation. Tree construction is performed by inserting one POI area code at a time into the Area Code Tree. The accuracy of the Area Code Tree is measured by calculating the percentage of times that an accurate nearest neighbour is found. We compare the nearest neighbour search performance of the Area Code Tree with the Brute Force Method. Our evaluation results show that the Area Code Tree significantly outperforms the Brute Force Method. Regardless of the number of POIs in the index, the average search time for Area Code Tree is very low and constantly less than 10ms. Our evaluation results

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<sup>1</sup>The paper [19] based on this work won the Best Paper Award at SEDE, 2016.

also show that the Area Code Tree achieves higher accuracy as the data set increases in size. In particular, for dense POI sets, the Area Code Tree can achieve up to 60% accuracy for locating the nearest neighbour.

The remainder of this thesis proceeds as follows. Chapter 2 summarizes related works in the area of continuous nearest neighbour processing and other background that are necessary for our work. Chapter 3 presents the Area Code Tree, the algorithms for mapping area code of a POI, inserting and building the Area Code Tree, and approximate nearest neighbour search. Chapter 4 presents the strategy and results of performance evaluation of the Area Code Tree. Chapter 5 concludes the thesis and presents future research directions.

# Chapter 2

## Related Work and Background

In this chapter, we summarize related work in continuous nearest neighbour searching for location-based services. Although nearest neighbour strategies have been proposed in other contexts, they are considered outside of the scope of this work. In addition, we summarize other background that is necessary for our work.

The *k*-Nearest Neighbour (*k*NN) query is one of the most popular query types in location-based services, where a user based on their location issues a query to the service provider for the *k*-nearest objects of interest to user's current location [1]. A moving *k*-Nearest Neighbour (M*k*NN) query is a continuous *k*NN query issued by a moving object. There are two general approaches for processing M*k*NN queries:

- i. The Sampling-based method, which processes the M*k*NN query as a *k*NN query at sampled locations. This method does not provide answers between sampled locations. In order to provide a continuous answer to the query, a high sampling rate is required, which makes the method inefficient due to the frequent processing of *k*NN queries.
- ii. The safe region based-method provides an answer with a *safe region*. As long as the query point stays in the safe region, the answer remains the same. When the query point moves out of the safe region, another answer with its associated safe region is returned.

## 2.1 Early Techniques for Processing MkNN Queries

Song and Roussopoulos [20] introduce four static-branch-and-bound methods that use sampling to solve the continuous  $k$ -nearest neighbour problem. These methods retrieve redundant data entries and utilize caching, which greatly reduces the cost of query re-evaluation. To represent the movement of the query point  $q$ , the authors use a periodical sampling technique, where the total time period is divided into equal intervals, and each break between time intervals is called a “*sampled position*”. Using the information from two consecutive sampled positions, the location of the query point  $q$  is calculated using splines. At sampled position  $t$ ,  $q_t$  is the location of  $q$ ,  $n$  is the total number of sites and  $S$  is site set. At any time  $t$ , the sites are sorted in  $S$  in ascending order based on their distance to  $q_t$ . The search upper bound is  $\varepsilon_t$ , which means the distance of  $k$ th nearest neighbour to query point  $q$  will be no more than  $\varepsilon_t$ , (i.e.  $D_t(k) \leq \varepsilon_t$ ).

Song and Roussopoulos propose the following algorithms [20]: Fixed Upper Bound, Lazy Search, and Pre-fetching Search, which are best for unpredictable query object movement, and Dual Buffer Search, which is suitable for calculating the next position of the moving query object when the speed and time interval length of the object are available.

### 2.1.1 Fixed Upper Bound (FUB) Algorithm

The Fixed Upper Bound (FUB) algorithm sets the search bound to infinity if no history of previous sampled positions is available. Otherwise it can set a smaller search bound initially from the search result at the previous sampled position. At sampled position  $t$ , the location of query point is  $q_t$ . The maximum distance to the  $k$  sites to  $q_t$  is  $D_t(k)$ , which is represented in Figure 2.1 by the solid (i.e. inner) circle [20]. After the search the values of  $q_t$  and  $D_t(k)$  are recorded and used to calculate the search bound at the next sampled position. At sampled position  $t + 1$ , the new position of the query point is  $q_{t+1}$ , the distance between locations  $q_t$  and  $q_{t+1}$  is  $\delta$ , and the search bound is  $\varepsilon_{t+1} = D_t(k) + \delta$  is shown by a dashed (i.e. outer) circle in Figure 2.1.

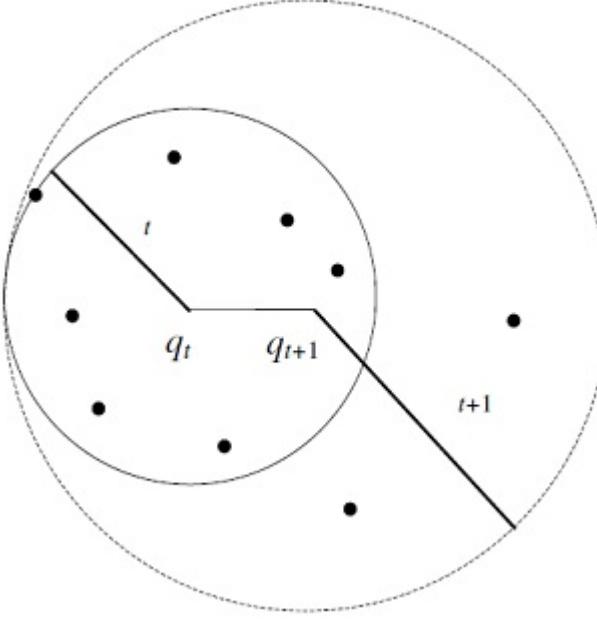


Figure 2.1:  $\varepsilon_{t+1}$  does not need to be infinite [20].

### 2.1.2 Lazy Search (LS) Algorithm

At sampled position  $t$ , the Lazy Search (LS) algorithm stores the query result set, location of the query point  $q_t$ , and the maximum distance of  $k$  sites to query point  $D_t(k)$ , and the minimum distance of sites that outside of  $k$  sites set to query point  $D_t(k+1)$  in the buffer. At sampled position  $t + 1$ , the query point moves to  $q_{t+1}$ . Next, the algorithm checks the validity of the previous query result for the new query. If the distance between  $q_t$  and  $q_{t+1}$  is no more than  $[(D_t(k+1) - D_t(k))/2]$ , the previous query result is still valid for the new query. Otherwise, the algorithm finds the sites from the previous query result set that are still valid for the new query. Since, the new query result contains some sites  $p_i$  with  $D_t(i) < D_t(k+1) - 2\delta$ , where  $\delta$  is the distance between  $q_t$  and  $q_{t+1}$ . Next, it stores the new query result set, the values of  $D_{t+1}(k)$ ,  $D_{t+1}(k+1)$ , and  $q_{t+1}$  in the buffer and waits for the next query.

### 2.1.3 Pre-Fetching Search (PFS) Algorithm

According to the Pre-Fetching Search (PFS) algorithm, at sampled position  $t = 1$ , a search is performed for the  $m$  nearest neighbours, where  $m > k$ . The location of the query point  $q_t$  along with maximum distances  $D_1(k)$  and  $D_1(m)$  are stored. From this result set,  $k$  nearest neighbours are identified. Next at sampled position  $t > 1$ , the algorithm checks the requirement for updating the result set for  $q_t$ . If the result is valid, it does not need to update the result set in the buffer. In Figure 2.2 the sites which are in the dashed circle is already stored in the buffer. Otherwise, a new static  $m$  nearest neighbour search is executed and the new result set, the location of the query point, and distance  $D_t(k)$  and  $D_t(m)$  are stored in the buffer.

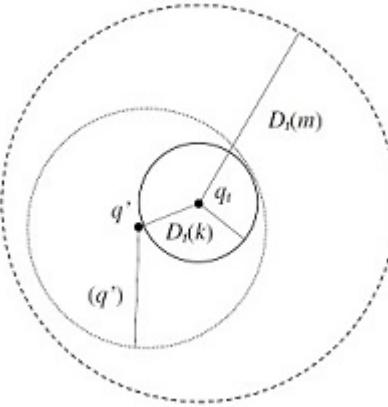


Figure 2.2: In some cases, there is no need to update the buffer [20].

### 2.1.4 Dual Buffer Search (DBS) Algorithm

According to the Dual Buffer Search (DBS) algorithm, two buffers of the same size  $k$  are maintained. At sampled position 1, a static  $k$  nearest neighbour search algorithm is executed, and the result set is stored in the first buffer. In the meantime, the next possible position of the query point  $q'$  is calculated, and the set of  $k$  nearest neighbours of  $q'$  are stored in the second buffer. At sampled position  $t > 1$ , the contents of the second buffer is moved to the first buffer, and the distance of the  $k$ -th neighbour to  $q_t$  is used as the next search bound. At the same time, according to the next possible position of moving query

point  $q'_{t+1}$ , the  $k$  nearest neighbours are sorted and stored in the second buffer. After that, a new  $k$  nearest neighbour search is executed and both buffers are updated.

### 2.1.5 Discussion and Experimental Results

The authors compare the four algorithms and perform an experimental evaluation of them [20]. FUB always executes a new search at new sampled positions with a better initial search bound, while DBS follows the same idea with a lower initial search bound. The LS and PFS are relatively expensive, since they need to collect extra information such as the distance of the  $k + 1$ -th neighbour to a query point. However, by using these two algorithms at some sampled positions, new searches do not need to be executed as the stored result set in the buffers can be used. The authors refer to these sampled positions as no search positions. The algorithms are evaluated by varying four parameters: uncertainty factor ( $r$ ), number of sites (i.e. points of interest), speed of the moving object (i.e. query point), and  $k$ .

Experiment results show that the FUB, DBS, and LS algorithms perform better than the pure static method by a factor of 3 for a large number of sites. When the moving pattern of a query point is predictable, DBS outperforms FUB. However, when the number of sites is smaller and the speed of the query point is slower, LS performs better than FUB. PFS performs better when the percentage of *no search position* is large.

## 2.2 Time-Parameterized (TP) $k$ NN Queries

Tao and Papadias [22] propose a general framework for time-parameterized queries in spatio-temporal databases, and their application in  $k$ NN searching. In a dynamic environment, the results of conventional spatial queries may become invalid due to the movements of objects and queries. A time-parameterized (TP) query returns the current query result, the validity period of that result and the change of the result at the end of the validity period. It is represented as  $\langle R, T, C \rangle$ , where  $R$  is the set of objects satisfying the corresponding

spatial query,  $T$  is the validity time of  $R$  and  $C$  is the result change at  $T$ . The client can incrementally compute the next result from the sets  $R$ , and  $C$ . The authors give the following example of a TP nearest neighbour query in Figure 2.3 for the static object set. In Figure 2.3 the query point  $q$  is moving east with speed 1, and the point  $d$  is the current nearest neighbour of  $q$ . For a nearest neighbour query, the influence time of object  $o$ ,  $T_{INF}(o, q)$  is time that it becomes closer to query  $q$  than the current nearest neighbour. For example, the influence time of point  $g$  is 3, because at time unit 3,  $g$  will be closer to  $q$  than  $d$ , if the result does not change due to another object, as shown in Figure 2.4. At time unit 3,  $f$  is the nearest neighbour to  $q$ . Therefore,  $f$  (in the time-parameterized component,  $C$ ) is invalidating the current query result, since it is the point with the smallest influence time.

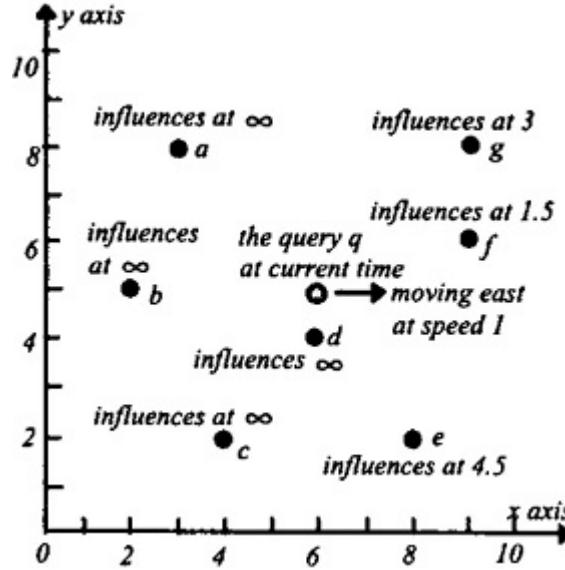


Figure 2.3: TP nearest neighbour query [22].

### 2.2.1 The TP Nearest Neighbour Query

Tao and Papadias [22] also propose a  $k$ -nearest neighbour search algorithm, which uses the R-tree [2] and is based on the branch-and-bound framework. The algorithm uses two distance metrics: (i)  $mindist$  represents the minimum distance between the query  $q$  and any other object in the subtree of entry  $E$ , and (ii)  $minmaxdist$  is the minimum distance from  $q$  within which an object in the subtree of  $E$  must be found.  $E$  is an entry in an R-tree node,

which has a subtree that contains objects.

$T_{INF}(O, q)$  is the influence time of an object  $o$  with respect to a query  $q$ . The expiry time of the current result is the minimum influence time of all objects. Therefore, the time-parameterized component of a TP query can be reduced to a nearest neighbour problem by treating  $T_{INF}(O, q)$  as the distance metric. The aim is to find the objects ( $C$ ) with the minimum  $T_{INF}(T)$ . These are the candidates that may generate the change of the result at the expiry time (by adding to or deleting from the previous answer set).  $T_{INF}$  for intermediate entries  $E$  is defined in a way similar to mindist in NN search.  $T_{INF}(E, q)$  is the minimum influence time  $T_{INF}(O, q)$  of any object  $o$  that may lie in the subtree of  $E$ . The authors define  $P_{NN}$  as the current nearest neighbour of a query point  $q$ , and  $o(t)$ ,  $q(t)$ , and  $P_{NN}(t)$  are the positions of object  $o$ , query  $q$  and  $P_{NN}$  at time  $t$  respectively.  $T_{INF}(o, q)$  has the minimum time  $t$  if it satisfies the conditions:

$$dist(o(t), q(t)) \leq dist(P_{NN}, q(t)) \text{ and } t \geq 0.$$

If no time  $t$  satisfies the inequality,  $T_{INF}(o, q)$  is set to  $\infty$ .

In case of non-leaf entries,  $T_{INF}(E, q)$  is the earliest time when one or more objects of subtree  $E$  will get closer to  $q$  than  $P_{NN}$ .  $T_{INF}(E, q)$  is the minimum time  $t$  if it satisfies the conditions:

$$dist(E(t), q(t)) \leq dist(P_{NN}, q(t)) \text{ and } t \geq 0.$$

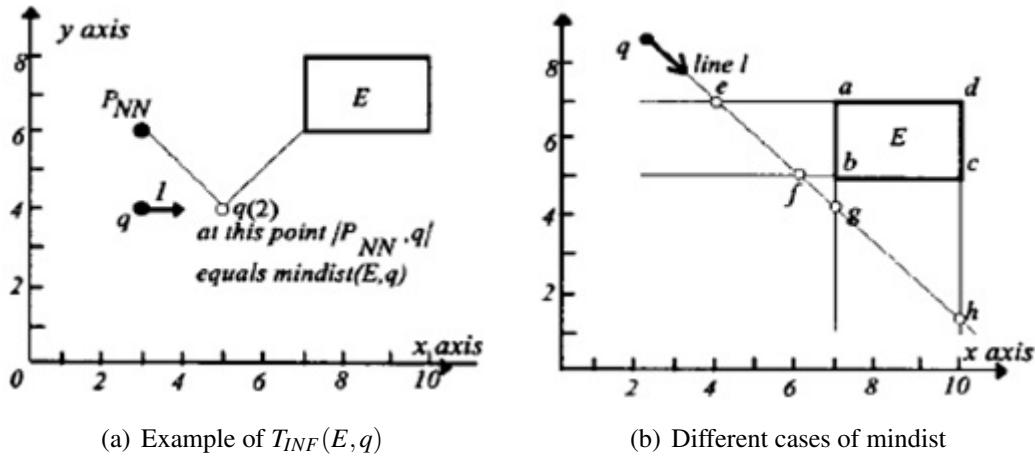


Figure 2.4: An example of the approximate mindist between  $q$  and  $E$  [22].

Figure 2.4 shows an example of the approximate mindist between  $q$  and  $E$ . To calculate the mindist between an edge of  $E$  and  $q$  the following rules are applied [21]:

- i. If the mindist is computed with respect to a corner point, the closest edges are chosen and their connected corner point would be the expected corner point.
- ii. If the mindist is computed with respect to an edge, the closest edge is chosen. The mindist is the distance between the query point and the edge.

Figure 2.4(a) shows  $T_{INF}(E, q)$  for a non-leaf entry  $E$ ;  $\text{mindist}(E, q) = \text{dist}(q, P_{NN})$  and Figure 2.4(b) shows the Minimum Bounding Rectangle (MBR) of  $E$  with corner points  $a$ ,  $b$ ,  $c$ , and  $d$ , with query point  $q$  moving on line  $l$ . When  $q$  passes the point  $e$ ,  $\text{mindist}(E, q)$  is computed with respect to corner point  $a$ . When  $q$  is on line segment between  $e$  and  $f$ , then  $\text{mindist}(E, q)$  is the distance between  $q$  and the edge  $ab$  of  $E$ .

### 2.2.2 Extension to $k$ NN Queries

The authors extend their strategy to handle a to  $k$ NN query. Given  $P_{NN1}, P_{NN2}, \dots, P_{NNk}$ , the current  $k$  nearest neighbours to  $q$ , the influence time  $T_{INF_i}$  of object  $o$  is calculated with respect to each of the  $k$  nearest neighbours.  $T_{INF}(o, q)$  is the minimum of  $T_{INF1}, T_{INF2}, \dots, T_{INFk}$ .

### 2.2.3 Experimental Result

The authors evaluate the number of disk accesses for different numbers of buffer pages. The experiment results show that, when a buffer is used, TP- $k$ NN query significantly outperforms ordinary  $k$ NN query.

## 2.3 Retrieve-Influence-Set $k$ NN (RIS- $k$ NN) strategy

Zhang *et al.* [27] propose a query processing algorithm called the retrieve-influence-set  $k$ NN (RIS- $k$ NN) to locally compute  $k$  Voronoi Diagram (kVD) cells using a spatial index. RIS- $k$ NN uses the time parameterized  $k$ NN (TP $k$ NN) query [22] to find each edge

of a kVD cell that surrounds a query point. Using these techniques fewer queries must be issued to the server, resulting in comparatively less computation and network overhead than regular spatial queries. The authors propose the following strategies to process a location-based nearest neighbour query. The server first executes a TP nearest neighbour query [20] to retrieve the nearest neighbour  $o$  of the query point  $q$  and then forms a validity region  $V(q)$  around  $q$ . Figure 2.5 gives an example. The initial validity region is formed from the overall data universe which is defined by four vertices:  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  in Figure 2.5(a). One of the vertices is selected randomly for issuing a TPNN query with the trajectory from  $q$  to the chosen vertex. When the query finds a new influence object, the validity region is updated with the intersection of perpendicular bisector of new influence object. Figure 2.5(b) represents the updated validity region with vertices  $v_4$ ,  $v_5$ , and  $v_6$ . Next, another vertex is chosen and a new TPNN query is issued which retrieves a new influence object. The validity region is updated respectively (shown in Figure 2.5(c)). If

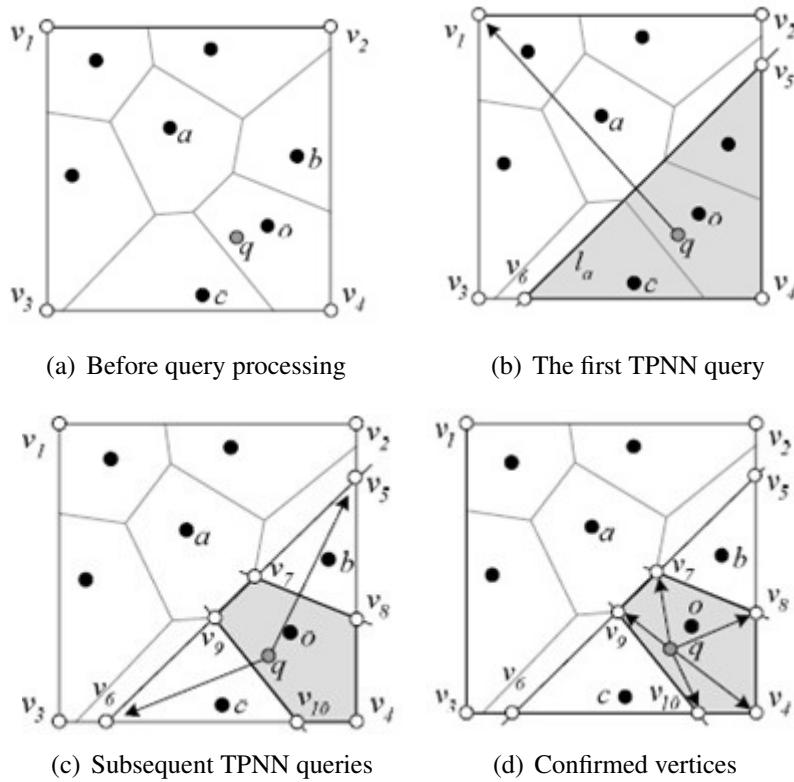


Figure 2.5: Computation of validity region for nearest neighbour [27].

new influence objects cannot be found with a vertex, the vertex is not considered again and it is confirmed. The same process is repeated until all vertices are confirmed. Finally, when all vertices are confirmed, the influence object set and the query result are returned by the server to the client. Figure 2.6 represents the algorithm to retrieve the influence object set for kNN query [27]. In the RIS-kNN algorithm, a TPkNN query is issued instead of TPNN query, which returns a pair of influence objects and one of the k nearest neighbours. At the same time, the existence of the pair is checked. If the pair has not been discovered before, then the pair is added to the influence pair set. When all vertices are confirmed, all influence objects are returned and the algorithm terminates.

```

Algorithm Retrieve_Influence_Set_kNN ( $q, S_o$ )
/*  $q$  is the query point,  $S_o$  is the set of nearest neighbors of  $q$  */
1.  $S_{inf\_p} = \emptyset$  // initialize the influence pair set
2.  $V = \{\text{universe boundary vertices}\}$  // initialize the vertex set
3. while ( $V$  contains non-confirmed vertex)
4.    $v = \text{any non-confirmed vertex in } V$ 
5.    $\langle o_{inf}, o_r \rangle = \text{TPkNN}(q, v, S_o)$  // query from  $q$  pointing to  $v$ 
6.   if ( $o_{inf} = \emptyset$  or  $\langle o_{inf}, o_r \rangle \in S_{inf\_p}$ ) confirm  $v$ 
7.   else // a new influence object  $o_{inf}$  is discovered
8.      $S_{inf\_p} = S_{inf\_p} \cup \{\langle o_{inf}, o_r \rangle\}$ 
9.     update  $V$ 
10.  $S_{inf} = \text{all } o_{inf} \text{ in } S_{inf\_p}$ 
11. return  $S_{inf}$ 
End Retrieve_Influence_Set_kNN

```

Figure 2.6: Algorithm for retrieving the influence set kNN query [27].

## 2.4 Incremental Search Grid

Park *et al.* [17] introduce a spatial index structure called the Incremental Search Grid (ISGrid) to efficiently process continuous  $k$ NN queries for static objects and moving queries. ISGrid uses a grid structure to access the leaf nodes directly. Therefore, unnecessary non-leaf nodes visiting can be avoided. In addition, this index structure uses minimum bounding rectangles (MBR) as leaf nodes to minimize the dead space that is covered by the grid. Each leaf node is linked to neighbouring leaf nodes. Figure 2.7 represents the configuration of

ISGrid which consists of 3 layers [17]:

1. The grid structure provides direct access to MBRs.
2. A set of leaf node MBRs that are connected to neighbouring nodes.
3. A set of objects for each MBR.

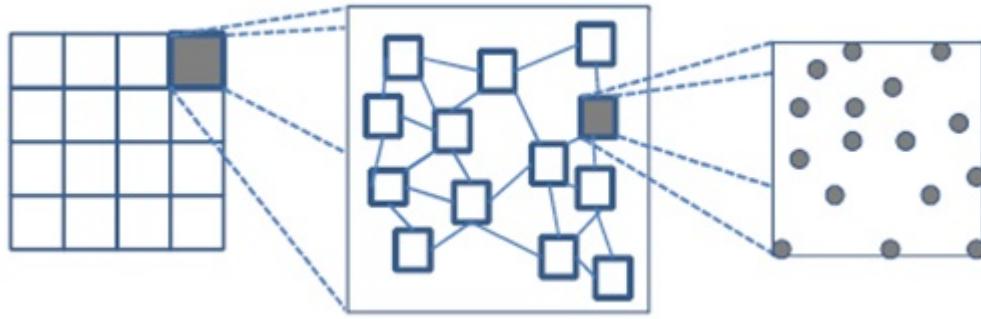


Figure 2.7: Configuration of ISGrid [17].

Therefore, ISGrid has no internal nodes. For  $k$ NN query processing using the R-tree index structure [2], visits to internal nodes are required in order to access the leaf nodes. The authors give an example of how ISGrid outperforms the R-tree in Figure 2.8. In their example, the query point  $Q$  needs to access the leaf nodes  $M10$ ,  $M16$ , and  $M22$ . In the R-tree,  $Q$  would also need to access the parent nodes  $M4$ ,  $M6$ , and  $M8$ . However, in ISGrid,  $M10$  contains the pointers to  $M16$  and  $M22$ , which are used to access these nodes. In order to avoid unnecessary visits of the neighbouring leaf nodes, a leaf node only contains pointers to leaf nodes, which are the nearest neighbours in each direction. The authors use the Voronoi regions of the neighbouring nodes adjacent the Voronoi region of the leaf node to determine the nearest neighbours of a leaf node.

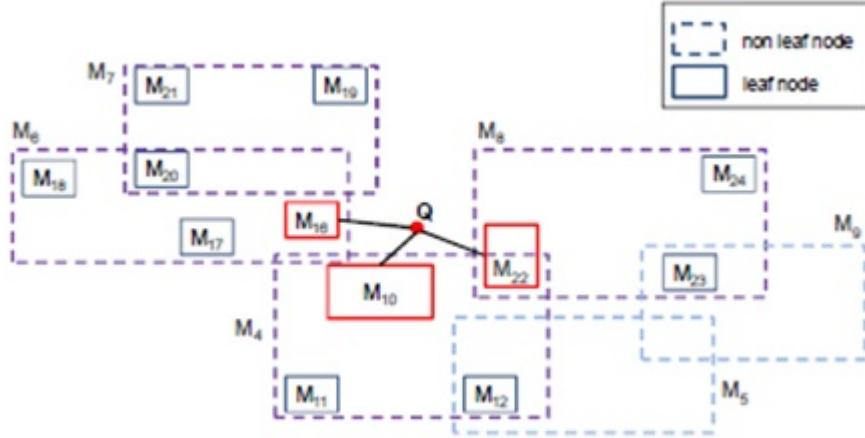


Figure 2.8: Structural problem of R-tree on processing kNN queries [17].

```

00  k-NN query(qid, k, q_point, G)
01  input – qid : identifier of query, q_point : query point,
02    k : number of required objects, G : ISGird
03  output – resultset : k-NN objects
04  {
05    Heap h;
06    LeafNode n;
07    n = FindNearestNeighborNode(G, p_point, h);
08    h.push(n);
09    Until {
10      n = h.pop();
11      resultset = resultset □ FindkNNs(n, q_point);
12      h.push(n.NNs);
13    } While ((NumOf(resultset) < k) or
14      (dist(point, k-dist ≥ dist(point, n)));
15  }

```

Figure 2.9: kNN query processing algorithm [17].

#### 2.4.1 *k* Nearest Neighbour Query Processing Algorithm

Park *et al.* [17] propose the following *k*NN query processing algorithm, which is illustrated in Figure 2.9. The algorithm first locates the cell that contains the query point, before locating the nearest overlapping leaf node for the query point. Next, the algorithm finds the next nearest leaf nodes using the information of the neighbour nodes. If the number of leaf

node is less than  $k$ , the leaf node is added to the query result. The given steps are repeated until the  $k$  nearest leaf nodes to the query point are found.

#### 2.4.2 Performance Evaluation

Park *et al.* [17] evaluate the number of disk I/Os for different values of  $k$ . The experiment result shows that ISGrid performs better than the R-tree [2], ST2B-tree [3], and ISR-tree [16] by 25%, 35%, and 10% respectively.

### 2.5 V\*-Diagram and V\*-kNN

Nutanong *et al.* [13] introduce an integrated safe-region (ISR) technique called the V\*-Diagram, and its associated algorithm, V\*-kNN, to efficiently process moving  $k$  nearest neighbour queries (MkNN). To compute the safe-region, the V\*-Diagram uses not only the knowledge of the data objects, but also the knowledge of the query location and the current search space. According to the authors, both the computation and the data retrieval of the V\*-Diagram are more economical than other existing methods.

The authors define the ISR that contains two types of regions. The first is a Fixed Rank Region, which is an order-sensitive Voronoi diagram cell, where the ranking of all objects based on their distances is fixed.

$$F(L) \cap \left( \bigcap_{i=1}^k S(q_b, z, p_i) \right)$$

Here,  $F(L)$  is the fixed rank region of  $L$ .  $L$  is a list of  $(k+x)$  objects sorted in ascending order by their distance to the query point.  $k$  is the number of requested nearest neighbours and  $x$  is the additional objects being fetched.  $q_b$  represents the current location of the moving query point.  $S(q_b, z, p_i)$  is the safe region with regard to data object  $p_i$ , and  $z$  is the  $(k+x)$ th NN to  $q_b$ .

$F(L)$  is represented by a list  $B$  of the  $(k+x-1)$  rank-adjacent bisectors,  $B_{(p_i p_{i+1})}$ , in the order corresponding to  $L$ , (for  $i = 1, 2, \dots, k+x-1$ ). To determine the correct half plane of

each bisector in  $B$ ,  $q_b$  is used as a reference point. Point  $q_b$  also indicates where the last call to BF- $k$ NN was made to retrieve the  $(k+x)$ NNs to  $q_b$ .

The authors define the safe region for a data object  $p$  as

$$S(q_b, z, p) = \{q' : \text{Dist}(q', p) + \text{Dist}(q_b, q') \leq \text{Dist}(q_b, z)\}$$

where  $q'$  is the next position of the moving query point after  $q_b$ .

The authors also define the concepts of known and reliable regions. A known region  $W(q_b, z)$  is a sphere centered at  $q_b$  with radius  $\text{Dist}(q_b, z)$ . Point object  $p$  is one of the  $(k+x-1)$ NNs of  $q_b$ . The sphere with center  $q'$  and radius  $\text{Dist}(q', x)$  is a reliable region of  $q'$ . If an object is in reliable region, it is known as reliable object. As long as  $p$  is in the  $\text{sphere}(q', \text{Dist}(q', x))$ , it is a reliable object and one of the  $(k+x)$ NNs of  $q'$ . Figure 2.10 depicts the known, reliable, and safe regions. The authors give an example of the V\*-diagram to compute an ISR (shown in Figure 2.11) for  $k = 2$  and  $x = 2$ .

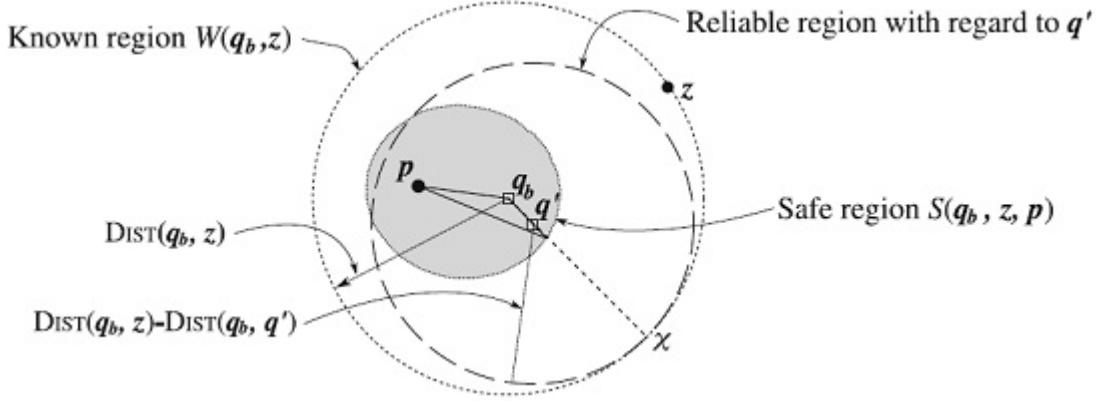


Figure 2.10: The known, reliable, and safe region [13].

Initially, the BF- $k$ NN algorithm [6] is called to retrieve 4 (i.e.  $2+2$ ) nearest neighbours to the query point,  $q_1$ . The retrieved objects are  $a, c, b$ , and  $f$ , with  $f$  being the farthest object from  $q_1$ . Based on the rank adjacent bisectors in the order corresponding to  $(a, c, b, f)$ , the fixed rank region,  $F < a, c, b, f >$  is computed. Then, the safe region  $S(q_1, f, c)$  with

regard to  $c$  is computed. Finally, the integrated safe region is computed by

$$F < a, c, b, f > \cap S(q_1, f, c)$$

which is the grey region shown in Figure 2.11. As long as  $q_1$  remains in this ISR, no objects outside of the known region,  $W(q_1, f)$  are nearer to the objects  $a$  and  $c$ , and the ranking of  $F < a, c, b, f >$  remains the same. Figure 2.11 shows the algorithm Compute-V\* to compute the initial ISR [13]. Figure 2.13 illustrates the V\*-kNN algorithm for processing MkNN queries based on the V\*-Diagram.

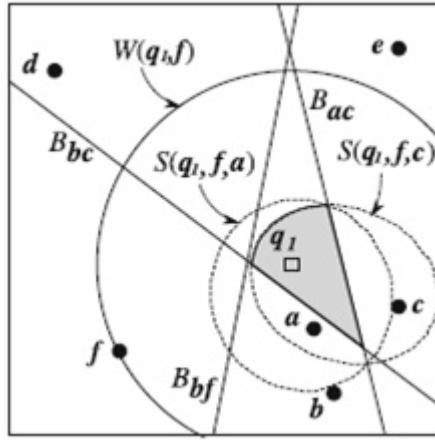


Figure 2.11: Integrated safe region example ( $k = 2, x = 2$ ) [13].

According to the algorithm, the initial ISR is computed by the algorithm Compute-V\*, where the starting location of the query point is  $q_0$  (line 2). Then, the algorithm maintains the current ISR with the movement of the query point. When the query point exits the current ISR one of two events is triggered: *RankUpdate* and *ReliabilityUpdate*. The event generation is associated with a timestamp and the corresponding query position.

A *RankUpdate* event is triggered when the query point  $q$  crosses a rank-adjacent bisector or exits the current fixed-rank region,  $F(L)$ . For this event, the ranks of the two objects corresponding to the bisector are swapped and the bisector list  $B$  is updated accordingly. If the event affects the rank of any of the  $k$ NNs, then the new  $k$ NNs are reported. In addition,

if the rank update changes  $p_k$ ,  $S_k$  also needs to be updated, where  $p_k$  is the  $k^{th}$  object in  $L$  and  $S_k$  is the safe region with regard to  $p_k$ .

---

**Algorithm 2: Compute-V\*( $q_b, k, x$ )**


---

```

1  $L \leftarrow \text{BF-}k\text{NN}(q_b, k + x)$ 
2  $z \leftarrow L.\text{Item}(k + x)$ 
3  $p_k \leftarrow L.\text{Item}(k)$ 
4  $S_k \leftarrow S(q_b, z, p_k)$ 
5  $B \leftarrow \text{CreateBisectorList}(L)$ 
6  $\text{ISR} \leftarrow \text{ConstructISR}(S_k, B, q_b)$ 
7 return ( $L, z, S_k, B, \text{ISR}$ )

```

---

Figure 2.12: The algorithm Compute-V\* [13].

---

**Algorithm 1: V\*-kNN( $q_0, k, x$ )**


---

```

1  $q_b \leftarrow q_0$ 
2  $(L, z, S_k, B, \text{ISR}) \leftarrow \text{Compute-V*}(q_b, k, x)$ 
3  $\text{ReportResult}(L.\text{Head}(k))$ 
4 while ( $\text{Event} \leftarrow \text{GetEvent}()$ ) do
5    $q \leftarrow \text{Event}.Position$ 
6   switch  $\text{Event}.Type$  do
7     case  $\text{RankUpdate}$ 
8        $\text{Bisector} \leftarrow \text{Event}.Bisector$ 
9        $L.\text{OrderSwap}(\text{Bisector.Index})$ 
10       $B.\text{Update}(L, \text{Bisector.Index})$ 
11      if  $\text{Bisector.Index} \leq k$  then
12         $\text{ReportResult}(L.\text{Head}(k))$ 
13      if  $\text{Bisector.Index} \in [k - 1, k]$  then
14         $p_k \leftarrow L.\text{Item}(k)$ 
15         $S_k \leftarrow S(q_b, z, p_k)$ 
16         $\text{ISR} \leftarrow \text{ConstructISR}(S_k, B, q)$ 
17      case  $\text{ReliabilityUpdate}$ 
18         $q_b \leftarrow q$ 
19         $(L, z, S_k, B, \text{ISR}) \leftarrow \text{Compute-V*}(q_b, k, x)$ 

```

---

Figure 2.13: The V\*-kNN algorithm [13].

A *ReliabilityUpdate* event is triggered when the query point is leaving safe region  $S_k$ . This means that the number of reliable objects is about to become less than  $k$ . Compute-V\* is called again using the current position  $q$ , to obtain  $x$  new auxiliary objects and to

make sure all  $(k + x)$  objects are reliable again. The new ISR is constructed accordingly. However, neither the current  $k$  nearest neighbours nor their ordering changes.

### 2.5.1 Insertion and Deletion of Objects

Nutanong *et al.* [13] also propose DatasetUpdate to insert objects into, or delete objects from the dataset for  $V^*-k$ NN. Figure 2.14 depicts the algorithm. In this algorithm,  $q$  is the position of the query point and  $p$  is the object to be inserted or deleted. Initially, the algorithm checks whether to see  $p$  is in the known region,  $W(q_b, z)$ . If it is not, the update can be ignored because it cannot affect the current ISR. Otherwise, an insertion or deletion of  $p$  into or from  $L$  is performed and the list  $B$  is updated accordingly. The insertion of  $p$  needs  $q$  for computing distances between  $q$  and the objects of  $L$ , in order to find the correct insertion slot in  $L$ . The deletion of  $p$  from  $L$  requires a simple lookup operation. After the bisector update, the ISR and  $L$  could be in one of the following three cases:

- i. For deletion, the number of objects in  $L$  becomes smaller than  $k$ . In this case,  $q_b$  is set to  $q$  and the algorithm Compute- $V^*$  is called to retrieve more objects and compute the new *ISR* accordingly.
- ii. The number of objects in  $L$  is still greater than  $k$  but the update affects the  $k$ NN set. Updates to  $p_k$  and  $S_k$  are required. If  $q$  is not inside the new  $S_k$ , Compute- $V^*$  is called. Otherwise, the *ISR* is updated to reflect the changes in  $B$  and  $S_k$ . Since the  $k$ NNs have changed, the new result is reported to the user.
- iii. The update has no effect on  $k$ NN set, but affects one of the auxiliary objects. Only the *ISR* is updated to reflect the change in  $B$ .

### 2.5.2 MkNN with Dynamically Changing k Values

$V^*-k$ NN can handle changing  $k$  values because of its incremental nature. Figure 2.15 represents the algorithm KUpdate to shows how  $V^*-k$ NN handles dynamically changing  $k$  values. The algorithm takes two inputs:  $q$  as the current location of the query point and the

new  $k$  value. At first, it checks that the new  $k$  is greater than the length of  $L$ . If it is yes,  $q_b$  is set to  $q$  and Compute-V\* is called. Otherwise,  $p_k$  and  $S_k$  are updated for the new  $k$ . If  $q$  is not inside the new  $S_k$ ,  $q_b$  is set to  $q$  and Compute-V\* is called. Otherwise, only the ISR has to be updated to incorporate the new  $S_k$ . Finally, the new  $k$ NNs are reported.

---

**Algorithm 3:** DatasetUpdate( $q, p, Operation$ )

```

1 if  $p \in W(q_b, z)$  then
2   if  $Operation = Insertion$  then
3      $L \leftarrow \text{Insert}(L, p, q)$ 
4   else
5      $L \leftarrow \text{Delete}(L, p)$ 
6    $B.\text{Update}(L)$ 
7   if  $k > L.\text{Length}()$  then
8      $q_b \leftarrow q$ 
9      $(L, z, S_k, B, ISR) \leftarrow \text{Compute-V}^*(q_b, k, x)$ 
10    ReportResult( $L.\text{Head}(k)$ )
11  else if  $\text{DIST}(q, p) \leq \text{DIST}(q, p_k)$  then
12     $p_k \leftarrow L.\text{Item}(k)$ 
13     $S_k \leftarrow S(q_b, z, p_k)$ 
14    if  $q \notin S_k$  then
15       $q_b \leftarrow q$ 
16       $(L, z, S_k, B, ISR) \leftarrow \text{Compute-V}^*(q_b, k, x)$ 
17    else
18       $ISR \leftarrow \text{ConstructISR}(S_k, B, q)$ 
19      ReportResult( $L.\text{Head}(k)$ )
20  else
21     $ISR \leftarrow \text{ConstructISR}(S_k, B, q)$ 

```

---

Figure 2.14: The algorithm DatasetUpdate [13].

---

**Algorithm 4:** KUpdate( $q, k$ )

```

1 if  $k > L.\text{Length}()$  then
2    $q_b \leftarrow q$ 
3    $(L, z, S_k, B, ISR) \leftarrow \text{Compute-V}^*(q_b, k, x)$ 
4 else
5    $p_k \leftarrow L.\text{Item}(k)$ 
6    $S_k \leftarrow S(q, z, p_k)$ 
7   if  $q \notin S_k$  then
8      $q_b \leftarrow q$ 
9      $(L, z, S_k, B, ISR) \leftarrow \text{Compute-V}^*(q_b, k, x)$ 
10  else
11     $ISR \leftarrow \text{ConstructISR}(S_k, B, q)$ 
12 ReportResult ( $L.\text{Head}(k)$ )

```

---

Figure 2.15: The algorithm KUpdate [13].

### 2.5.3 Experimental Result

The V\*- $k$ NN algorithm is evaluated and compared with the RIS- $k$ NN [27]. The authors vary the following parameters in their experiments: (i) the number of auxiliary objects  $x$ , (ii) the cache size  $c$ , (iii) the trajectory length  $l$ , (iv) the cardinality  $n$  of the dataset, (v) the value of  $k$ , (vi) the step size  $s$ , and (vii) the probability  $\delta$  that the  $k$  value is altered by 1. The authors used both synthetic and real datasets in their experiments. The authors evaluate the impact of  $x$  on the response time and the number of page accesses. Results show that: (1) the response time starts to increase when  $x$  has approximately 15 objects, and (2) the page-access cost increases as  $x$  increases. The authors also rate an increase in CPU for increasing  $t$ . Next, the impact of  $c$  on the performances of V\*- $k$ NN and RIS- $k$ NN is evaluated. The experimental results show that V\*- $k$ NN outperforms the RIS- $k$ NN in terms of both the total response time and the number of page accesses. For both methods, the page access cost decreases and the total response time remains stable as the buffer size increases. In addition, the authors experimented with the effects of (i) trajectory length ( $l$ ), (ii) the number of objects in the dataset ( $n$ ), (iii) the value of  $k$ , (iv) the step size ( $s$ ), and (v) the probability that the  $k$  value is altered, on the performances of V\*- $k$ NN and RIS- $k$ NN. The experimental results show that V\*- $k$ NN consistently outperforms RIS- $k$ NN in terms of the total response time, the I/O cost and the communication cost.

## 2.6 The mqr-tree

Moreau and Osborn [10] propose a two-dimensional spatial access method the mqr-tree which allows spatial objects to be organized in a two-dimensional node based on their spatial relationships. The mqr-tree utilizes a node organization, set of spatial relationship rules and insertion strategy in order to gain significant improvements in overlap and over-coverage. In addition, zero overlap is achieved when the mqr-tree is used to index point data.

### 2.6.1 Node Organization and Validity

In the mqr-tree, a node contains 5 locations - northeast (NE), northwest (NW), southwest (SW), southeast (SE) and centre (EQ). Each location contains a pointer which points to either an object or a subtree. The node layout is shown in Figure 2.16. A node must have at least two locations and not more than five locations. It is possible to have a node containing pointers to both objects and subtrees. The origin of the node is the centroid of the node MBR. The node MBR contains all objects, and any subtrees in the node. As objects are added to and removed from the node, the centre of the node will change.

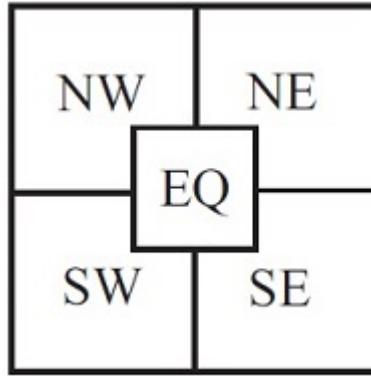


Figure 2.16: Node layout [10].

In every node of the mqr-tree, the relative placement of both objects and subregions are determined by using the centroids of their MBRs. The objects that are referenced from the centre location have the same centroid as the centroid of the node MBR for the node. The placement of all other objects and subregions that are referenced from the other locations (NW, SE, SW, NE) are determined with respect to the centroid of the node MBR. Figure 2.17 depicts the spatial relationships, where A refers to the centroid of a new object, and B refers to the centroid of the node MBR.

### 2.6.2 Insertion Strategy

First, the root node MBR is adjusted to include a new object and the appropriate location is identified relative to the centroid of the node MBR for inserting the reference of the new object. If the location is empty, the object reference is inserted. Otherwise, if the location contains an object reference, a new leaf node is created for the insertion of the new and existing object reference. Otherwise, a proper location is found in the subtree. Insertion or deletion of an object results in one of the following three events: (i) the centroid of the node remains the same, (ii) the centroid moves and the area of the node MBR expands (shown in Figure 2.18(a)), or (iii) the centroid of the node moves and the area of the node MBR shrinks (shown in Figure 2.18(b)). If the centroid of the node moves, some objects might not be in their proper node location. Therefore, these objects need to be reinserted into the proper relative location.

$A_x = B_x$	$A_x > B_x$	$A_y = B_y$	$A_y > B_y$	Placement
0	0	0	0	SW
0	0	1	0	SW
0	0	0	1	NW
1	0	0	1	NW
0	1	0	0	SE
1	0	0	0	SE
0	1	0	1	NE
0	1	1	0	NE
1	0	1	0	EQ

Figure 2.17: Relative orientation of A with respect to B [10].

### 2.6.3 Experiment Result

The experiments compare the mqr-tree insertion strategy with the R-tree [10]. The results shows the mqr-tree achieves significant improvements in overlap and overcoverage. In addition, a comparison of region searching shows that the mqr-tree uses a lower number of disk accesses in many cases.

## 2.7 The Trie Data Structure

A Trie is a special data structure used to store groups of strings that can be visualized like a graph [24]. It consists of nodes and edges. Strings are stored in a top to bottom manner on the basis of their prefix. If two string have a common prefix, they will have the same ancestor in the tree. All prefixes of length 1 are stored at level 1, all prefixes of length 2 are sorted at level 2 and so on. A Trie consists of a special node called the root node. This node has no incoming edges. The Trie is used to do prefix based search.

In Figure 2.19, every character is a Trie node. For example, the root node has children a, b and t. Similarly, ‘a’ at the next level is having only one child ‘n’.

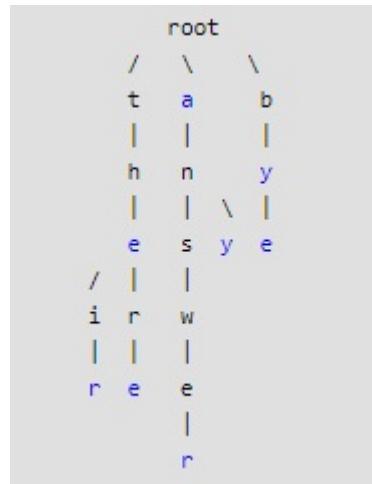
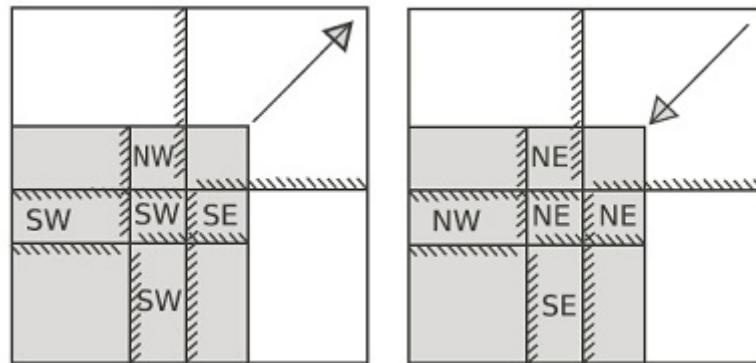


Figure 2.19: Trie data structure [24].



(a) NorthEast node MBR expansion (b) SouthWest node MBR contraction

Figure 2.18: Insertion Strategy [10].

### 2.7.1 Prefix

The prefix of a string is  $n$  letters, where  $n < |String|$  that makes the beginning of a string [24]. For example, the word ‘abacaba’ has the following prefixes: a, ab, aba, abac, abaca, abacab.

### 2.7.2 Insertion and Search in Trie

The insertion of any string into a Trie starts from the root node. All prefixes of length one are direct children of the root node. In addition, all prefixes of length 2 become children of the nodes existing at level one. The pseudo code for insertion of a string into a Trie is shown in Figure 2.20.

While searching, start from the root node. We keep on reading characters of given key and according to indices of characters, travel from root node to leaf node. The search can terminate due to the end of string. If the value of last node is non-zero then the key exists in Trie. The key refers to the word that exists in Trie. The search can also terminate due to the lack of key nodes in trie without examining all of the characters. The pseudo code to check whether a single word exists in a dictionary of words or not is shown in Figure 2.21.

```
void insert(String s)
{
    for(every char in string s)
    {
        if(child node belonging to current char is null)
        {
            child node=new Node();
        }
        current_node=child_node;
    }
}
```

Figure 2.20: Insertion of a string into Trie [24].

```

boolean check(String s)
{
    for(every char in String s)
    {
        if(child node is null)
        {
            return false;
        }
    }
    return true;
}

```

Figure 2.21: Searching a single word in Trie [24].

## 2.8 Limitations of Related Works

To solve the continuous  $k$ NN query, Song and Roussopoulos [20] extend the search for  $m$ NN objects ( $m > k$ ). According to their strategy, user can be assured that  $k$ NN is contained by the  $m$ NN result if the distance it moved from query point  $q$  is less than  $\{Dist(m, q) - Dist(k, q)\}/2$ , i.e. the half distance difference between  $k$ -th NN and  $m$ -th NN with respect to new query point  $q'$ . This attempts to keep the result current while the query point moves around and a new query call is not necessary to the server. However, this is ineffective to reduce query reevaluation. For example in Figure 2.22 based on  $q$ , a 2NN is executed as a 4NN and  $\delta$  is the distance between the second NN and the fourth NN. A new query point  $q'$  is located more than a distance of  $\delta/2$  away from  $q$  and hence needs reevaluation. Although the 4NN result still covers 2NN objects to  $q'$ , this implies that their strategy cannot effectively reduce the number of queries. Choosing a proper value of  $m$  is a significant problem of this strategy. A high value of  $m$  will increase the network overhead and the storage requirements at the client, while a low value does not reduce the number of queries.

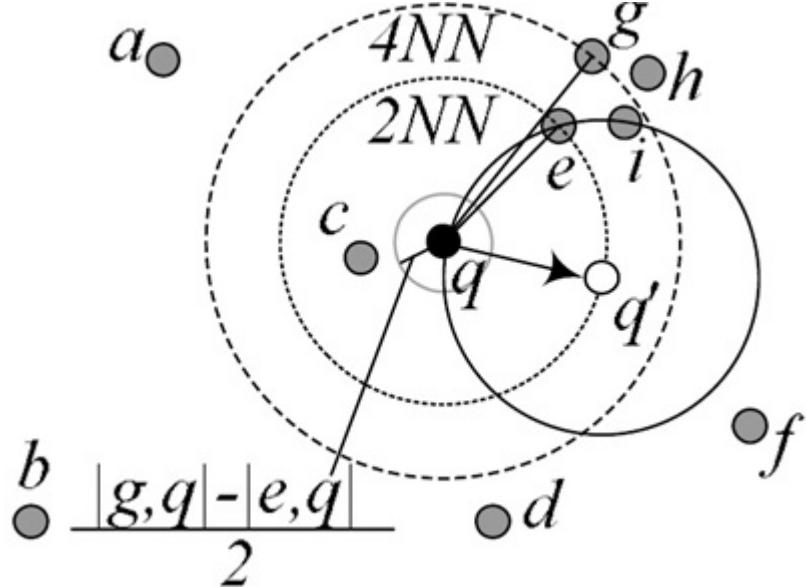


Figure 2.22: mNN query.

Tao et al. [22] utilizes the R-tree [2] to speed up the repeated searching needed for their continuous kNN strategy. Their strategy process a continuous NN query on R-tree according to the movement of a client. The method estimates the future location of the client and provides the continuous query result by pre-computing the result on the estimated path where the velocity of the client is known and constant during the lifespan of the query. This assumption may not work for many applications where the query velocities are continuously updated as the users change their speed or direction of movement.

Zhang *et al.* [27] derive the retrieve-influence-set kNN (RIS-kNN) algorithm in order to reduce the number of repeated searches and the cost of precomputation. However, it is still expensive because RIS-kNN performs multiple TPkNN queries [22] and each TPkNN query involves a costly tree traversal. In addition, RIS-kNN only works for a static number of nearest neighbours. Changing  $k$  to a larger value requires a recalculation of the kVD cell. If an update affects the kNN result or the influence set, the current kVD cell must be recalculated.

Our Area Code Tree provides the ability to perform repeated searching that is efficient, but at the cost of accuracy. The Area Code tree stores the POI area codes instead of co-

ordinates of Minimum Bounding Rectangles. This provides a simple numeric comparison for quickly identifying a nearest neighbour to the query point. The Area Code Tree is a Trie type structure. In the Trie, insert and search algorithms both cost  $O(K)$ , where  $k$  is length of key. The key refers to the string that is inserted or searched in trie. The memory requirement of the trie is  $O(k*N)$  where  $N$  is number of keys in trie. From our experiment results we find that the average search time for the Area Code Tree is very low and constant-time regardless of the number of POIs in the index. The Area Code Tree needs to be constructed only once for searching many times in static data sets. In addition, for an insertion the cost is also very low and a complete rebuild of the tree is not required. For denser data sets, the Area Code Tree achieves higher accuracy up to 60% for locating the approximate nearest neighbour. This makes the Area code Tree an excellent candidate for continuous approximate nearest neighbour search for location based services. We present the Area Code Tree in the following chapter.

# Chapter 3

## Area Code Tree

In this chapter, we introduce the Area Code Tree<sup>2</sup> for continuous approximate nearest neighbour search for location based services. The Area Code Tree is a Trie-type structure that stores points of interest (POIs) that are represented in area code format. An area code is a sequence of digits that represent the relative location of a point, such as a POI, in space. Therefore, indexing area code data of a POI instead of actual spatial values such as longitude and latitude, reduces the nearest neighbour search to a sequence of simple numeric comparison. In addition, we also present the algorithms for mapping the area code of a POI, inserting and building an Area Code Tree, and approximate nearest neighbour search. The corresponding pseudocode for each algorithm can be found in Appendix A.

### 3.1 Mapping Area Code of a Point of Interest (POI)

A POI is represented as  $(P_x, P_y)$ . A Minimum Bounding Rectangle (MBR) defines a region of space that is occupied by POIs. It is represented with different coordinates: 1) the lower left coordinate  $(MINP_x, MINP_y)$ , 2) the upper right coordinate  $(MAXP_x, MAXP_y)$ , and 3) its center  $(CM_x, CM_y)$ . The area code is obtained by recursively partitioning the space into quadrants. At each level of partitioning, a POI obtains a digit depending on which quadrant it resides in. The quadrant are numbered as follows: SW(1), SE(2), NW(3), and NE(4). The space partitioning continues until either: the POI  $(P_x, P_y)$  and the center of the region  $(CM_x, CM_y)$  are equal, or the lower left  $(MINP_x, MINP_y)$  and upper right

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<sup>2</sup>The paper [19] based on this work won the Best Paper Award at SEDE, 2016.

$(MAXP_x, MAXP_y)$  coordinates of the region are the same.

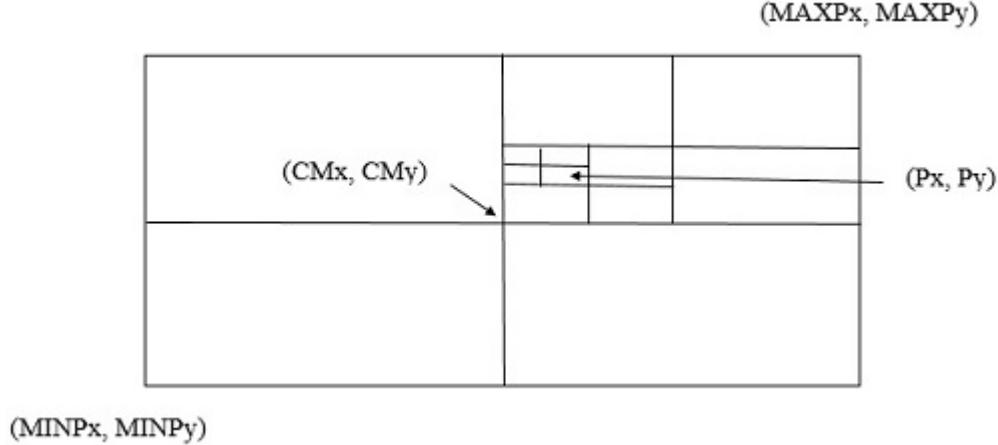


Figure 3.1: Area code mapping of a POI.

- Case 1:  $P_x < CM_x$  and  $P_y < CM_y$ . The POI is located in SW quadrant and 1 is assigned to the POI area code. The coordinates of the region are updated as follows: The lower left coordinate remains  $(MINP_x, MINP_y)$ , while the upper right coordinate is updated to  $(CM_x, CM_y)$ , the center point of the previous region (i.e.  $MAXP_x = CM_x$  and  $MAXP_y = CM_y$ .)
- Case 2:  $P_x > CM_x$  and  $P_y < CM_y$ . The POI is located in SE quadrant and 2 is assigned to the POI area code. The coordinates of the region are updated as follows: the new lower left corner of the region is  $(CM_x, MINP_y)$  (i.e.  $MINP_x = CM_x$  and  $MINP_y = MINP_y$ ), while the new upper right corner of the region is  $(MAXP_x, CM_y)$  (i.e.  $MAXP_x = MAXP_x$  and  $MAXP_y = CM_y$ ).
- Case 3:  $P_x < CM_x$  and  $P_y > CM_y$ . The POI is located in NW quadrant and 3 is assigned to the POI area code. The coordinates of the region are updated as follows: the new lower left coordinate is  $(MINP_x, CM_y)$  (i.e.  $MINP_x = MINP_x$  and  $MINP_y = CM_y$ ). The new upper right coordinate of the region is  $(CM_x, MAXP_y)$  (i.e.  $MAXP_x = CM_x$  and  $MAXP_y = MAXP_y$ ).
- Case 4:  $P_x > CM_x$  and  $P_y > CM_y$ . The POI is located in NE quadrant and 4 is assigned

to the POI area code. The coordinates of the region are updated as follows: the lower left coordinate of the region is  $(CM_x, CM_y)$  (i.e.  $MINP_x = CM_x$  and  $MINP_y = CM_y$ ), while the upper right coordinate of the updated region remains  $(MAXP_x, MAXP_y)$ .

Figure 3.1 depicts an example of space partitioning and mapping. Beginning with the top-most partition, the POI resides in NE quadrant. It is followed by the SW quadrant at the next level, then the NW quadrant, and finally the SE quadrant. Therefore, the area code of the POI in the diagram is 4132.

## 3.2 Area Code Tree and Node Structure

The Area Code Tree is a trie-type structure. It consists of nodes and edges. Each node can consist of a maximum of 4 child nodes. Edges connect each parent node to its child nodes. Each node represents either a prefix or an area code of a POI. The root node does not have any incoming edges. The nodes that are direct children of the root node represent prefixes of length 1, while the nodes that are 2 edges of distance from the root node represent prefixes of length 2, and so on. In other words, a node that is  $k$  edges of distance of the root has an associated prefix of length  $k$ . The digits of a POI area code are stored in a top to bottom manner on the basis of their prefix in the Area Code Tree. For example: a POI area code 4132 has the prefixes 4, 41, and 413.

## 3.3 Area Code Tree Construction

Once the area code of a POI is determined, it is inserted into the Area Code Tree in the following manner. Beginning at the root node, we take the most significant digit of the POI area code and check for the existence of a prefix of a child node for that digit. If no such prefix exists, then a new child node is created for that digit and the remainder of area code is placed in the node. If a prefix exists for that digit, then the search for an insertion path continues in the corresponding child node. The search for an insertion location continues with the next digit in the area code of the POI. After a new leaf node is created, there may

exist subsequent area code digits that two or more POIs share in common. In this case, further nodes are created until each POI has its own leaf node.

Table 3.1 gives some example area codes that are inserted into the area code tree shown in Figures 3.2 and 3.3. Beginning with POI A, the most significant digit of POI A's area code is 1. The root node has no child node with prefix 1. A new child node is created to store POI A's area code. Next, we insert POI B. The most significant digit of that area code is 3. No child node of the root node has a prefix 3. Therefore, a new child node is created to contain POI B. Figures 3.2(a) and 3.2(b) depicts the Area Code Tree. There are two child nodes of the root: one is POI A (prefix 1, Figure 3.2(b)) and the other is POI B (prefix 3, Figure 3.2(b)).

Table 3.1: Sample Area Codes

POI	Area Code
A	12134
B	32321
C	12141
D	32114
E	21324

Next we insert POI C. This is depicted across the two Figures 3.2(c) and 3.2(d). The most significant digit of the POI C's area code is 1. There is an existing child node with prefix 1 of the root node. Therefore, the search proceeds in that child node. Since the child node is a leaf node, a new leaf node is created that corresponds to the second digit of POI C area code, which is 2. This is shown in Figure 3.2(c). However, both POIs that are now referenced by the new leaf node A and C have the same third area code digit, which is 1. Therefore, another node that corresponds to the common third digit is created. From here, two leaf nodes are created using the fourth area code digits for POIs A and C respectively.

The resulting tree is depicted in Figure 3.2(d). Similarly, POIs D (Figure 3.2(e)) and E (Figure 3.3(a)) are inserted into the Area Code Tree. The final tree is depicted in Figure 3.3(a).

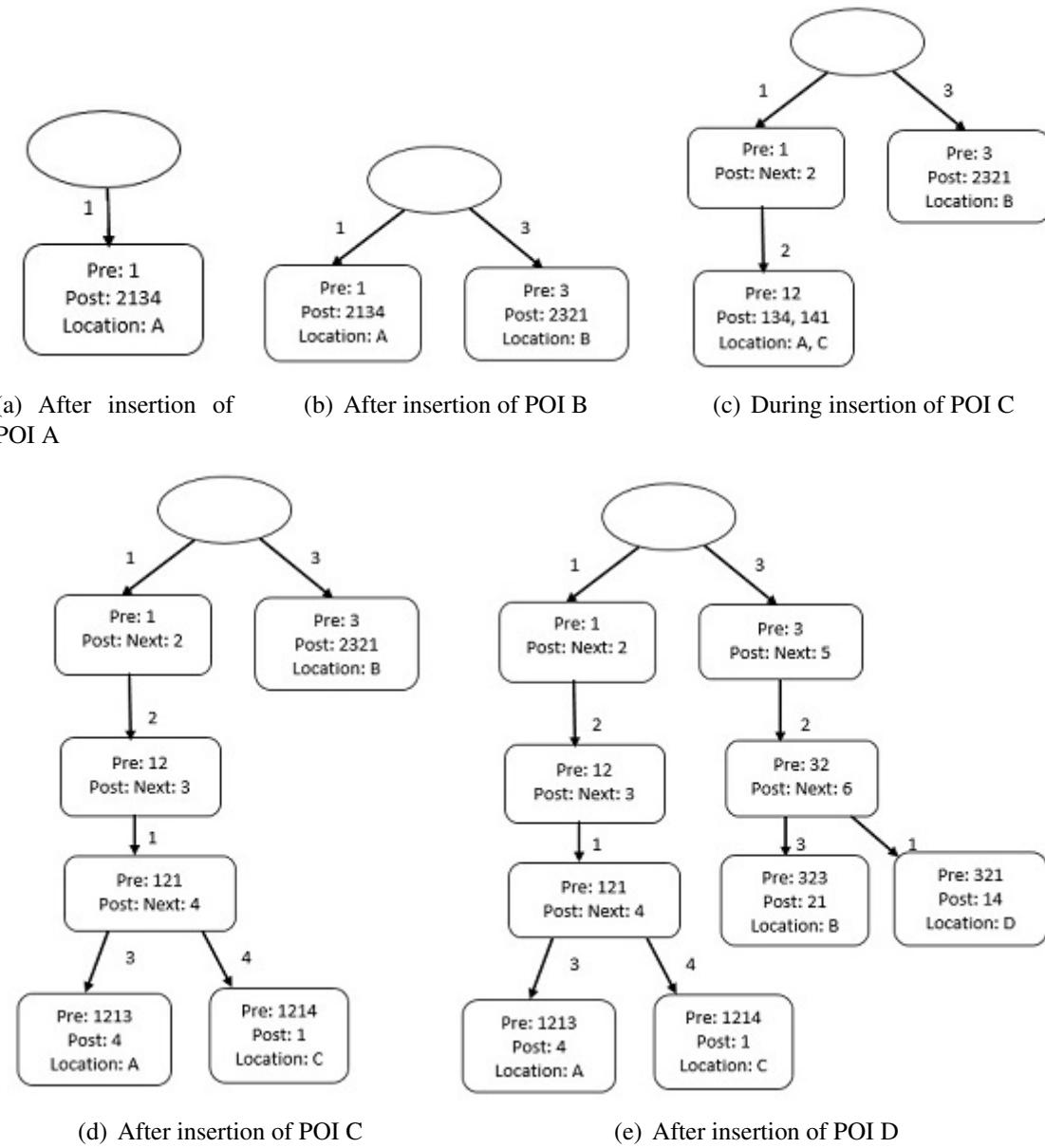
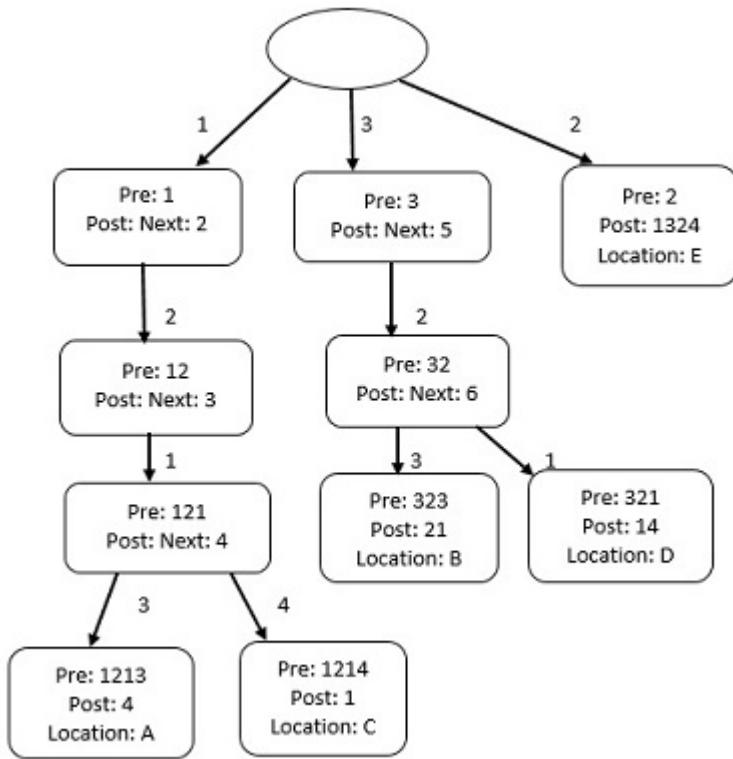


Figure 3.2: Index construction via insertion-Part 1.



(a) After insertion of POI E

Figure 3.3: Part 2 of index construction via insertion.

### 3.4 Approximate Nearest Neighbour Search

First the area code of the query point needs to be calculated according to the strategy of mapping area code of a POI. Because the POIs in the Area Code Tree are inserted using area code digits, the task of performing a nearest neighbour search is reduced to numeric comparisons. Begin with the most significant digit of the query area code, a path is followed down the area code tree while digits in the query match the digits in the tree nodes. If a match does not exist at a particular level, the closest match is taken. Although this may be possible at any level of the Area Code Tree. However it is very unlikely to occur in an index that is managing many POIs and more likely to happen at the leaf levels. The source code is shown in Appendix A.3. For example, we have query area code of 12134 and an Area Code Tree in Figure 3.3(a), the search proceeds along the path identified by 1, then

2, then 1, then 3, and finally reached the node for location A. if a match does not exist at a particular level, the closest match is taken. For example, for a query area code 32113 the closest match is location D. Since that path matches the first four digits of query area code and the closest last digit is 4.

We introduce the Area Code Tree for continuous approximate nearest neighbour search for location based services in this chapter. We also present its mapping, insertion, and approximate nearest neighbour search algorithms. The strategy and results of performance evaluation of the Area Code Tree will be presented in the next chapter.

# Chapter 4

## Experimental Evaluation

The methodology and results of our performance evaluation of the Area Code Tree are presented in this chapter. We compare the Area Code Tree’s search performance with the Brute Force method. We also evaluate the Area Code Tree for tree construction time and the accuracy for locating an approximate nearest neighbour. By inserting POI area code one at a time, we construct the Area Code Tree. We measure the accuracy by calculating the percentage of times that an exact nearest neighbour is found by the Area Code Tree.

### 4.1 The Brute Force Search

The Brute Force Search is a very general problem solving technique that consists of finding all possible candidates for the solution and determines whether each candidate satisfies the problem’s statement [23]. Here, we compute all the distances between the user location and the POIs. Next we sort the computed distances and select the POI with the smallest distance and compute the nearest neighbour search time. These steps are repeated for all query points (i.e. user locations).

### 4.2 Experimental Setup

For our experiments, we use the XAMPP cross-platform web server which consists of an Apache HTTP Server, a MySQL database, and the PHP programming language. We use Apache as the local web server. We store POI area codes using the MySQL database. We use the PHP programming language to map the POIs’ area codes, calculate the construction

time of the Area Code Tree, and calculate the search time of an exact or approximate nearest neighbour query.

### 4.3 Data Sets

Twenty-one data sets of different POIs from across New Zealand are used for our evaluation. Ten of those data sets of random POIs are taken from different locations of the North Island of New Zealand. The smallest contains 1000 points and the largest contains 10000 points. One of these data sets contains 1000 points is shown in Appendix B.1. Another ten data sets of random POIs are drawn from different locations of the Waikato Region of New Zealand (part of the North Island). The Waikato data sets are denser than the North Island data sets, which allows us to evaluate the Area Code Tree in denser data sets. For our evaluation, the remaining one data set is a trajectory which contains 10 user's locations shown in Appendix B.2. This trajectory contains the sequence of coordinates (i.e. locations) along a user's route in the Waikato region. These locations serve as the nearest neighbour queries. The first twenty data sets were generated by combining some actual locations in New Zealand (North Island and Waikato) with other randomly generated locations. The user location trajectory was generated using tracking S/W<sup>3</sup>.

### 4.4 Experiments

Altogether, 200 nearest neighbour evaluations are performed in the following manner. First an Area Code Tree is created for each of the 20 POI sets above. For each tree, we perform ten nearest neighbour searches using the user location set. We also perform the same searches using the Brute Force method.

We measure the following performance criteria:

1. For each Area Code Tree construction, we measure the overall tree construction time and the average insertion time for each POI area code in Area Code Tree.

---

<sup>3</sup>These locations were recorded by Dr. Annika Hinze in the Waikato region of New Zealand.

2. For the search comparison, we measure the average nearest neighbour search time in milliseconds.
3. The accuracy of the Area Code Tree is measured by calculating the percentage of POIs found by the Area Code Tree that matched those found by the Brute Force search.

#### 4.4.1 Results

The results of our comparison using the Waikato and North Island data sets are represented by Figures 4.1 and 4.2 respectively. For both figures, the x-axis represents thousands of POIs (i.e. 1 represents 1000 POIs, up to 10 for 10,000 POIs), while the y-axis represents the average search time in seconds. For both groups of POI sets, the search time for the Area Code Tree is significantly less than the Brute Force method. Regardless of the density of the dataset and the number of POI area codes in the index, the average search time for the Area Code Tree is less than 10ms. On the other hand, the average search time for Brute Force method increases steadily from less than 10ms to almost 50ms for searching 10,000 area codes.

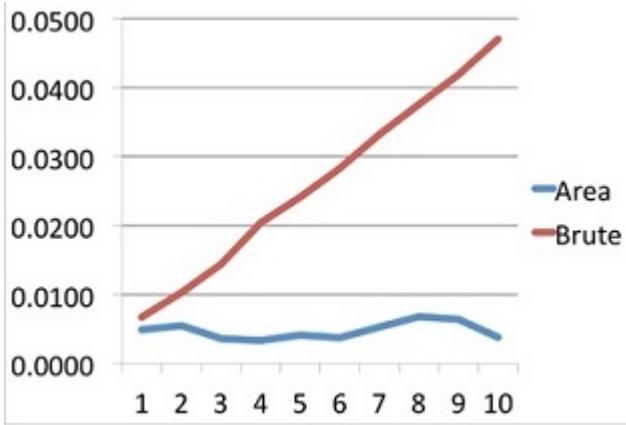


Figure 4.1: Waikato POIs.

The results of the evaluation for accuracy are represented by Figure 4.3. The Area Code Tree can achieve between 40% and 60% accuracy in locating the nearest neighbour for

dense POI sets (i.e. Waikato). For the less dense point sets (i.e. North Island), the accuracy was lower between 0% and 40%.

Finally, Table 4.1 shows the construction times of the Area Code Tree for different POI sets. Both the overall tree construction time (O/A Time) and the average insertion time of a POI (Avg. Time) are recorded. The table shows that as the data set size increases, the construction time of an Area Code Tree increases significantly as well. For the North Island datasets, the construction time is just under a minute for 1000 POIs, and for 10,000 POIs, it is just over 8 minutes. However, for the Waikato Region datasets, the construction times range from less than a minute to over 10 minutes.

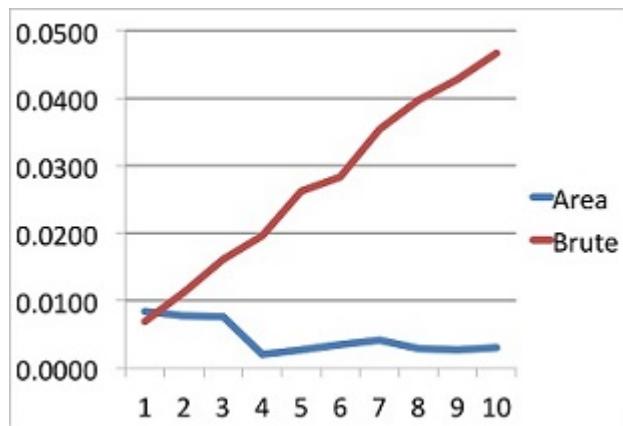


Figure 4.2: North Island POIs.

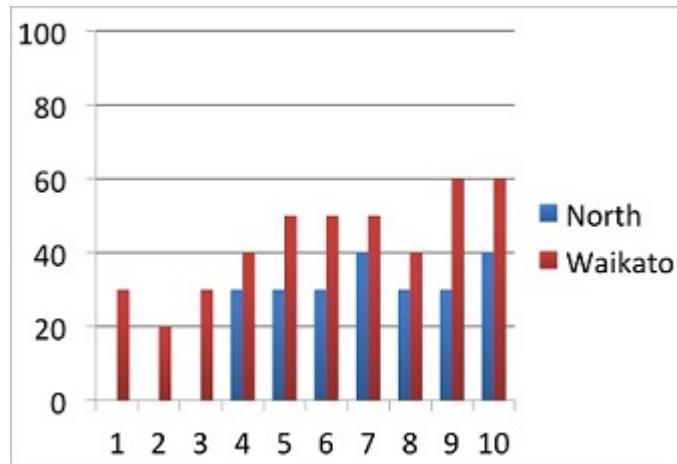


Figure 4.3: Accuracy of Area Code Tree.

Table 4.1: Sample Area Codes

Data Sets	POIs	Tree Construction (Sec.)	Avg. insertion (Sec.)
North Island	1000	46.16	0.046
	2000	79.60	0.040
	3000	106.61	0.036
	4000	128.21	0.032
	5000	166.32	0.033
	6000	219.94	0.037
	7000	281.70	0.040
	8000	334.07	0.042
	9000	437.50	0.049
	10000	490.96	0.049
Waikato	1000	18.63	0.019
	2000	39.71	0.020
	3000	91.74	0.030
	4000	159.69	0.040
	5000	227.17	0.045
	6000	303.42	0.051
	7000	409.23	0.058
	8000	481.41	0.060
	9000	586.86	0.065
	10000	686.63	0.069

## 4.5 Discussion

As a result, we conclude that the Area Code Tree provides higher accuracy in nearest neighbour searching as the data set size increases in size. Although, it appears that the overall tree construction times are high, there need two things to be noted. First of all, the Area Code Tree needs to be constructed only once for searching many times in static data sets. Secondly, for an insertion, it takes under 50ms on average for less denser data sets and a complete rebuild of the tree is not required.

# Chapter 5

## Conclusion

In this thesis, we propose a spatial data structure the Area Code Tree for storing and managing points of interest (POIs). We also present the algorithms for mapping the area code of a POI, inserting and building an Area Code Tree, and an approximate nearest neighbour search. In addition, we evaluate the Area Code Tree for accuracy, tree construction time, and compare its search performance with the Brute Force method. Our Area Code Tree provides the ability to perform efficient repeated searching for continuous nearest neighbours but at the cost of accuracy. We observe from our experiment results that Area Code Tree performs significantly better than the Brute Force method for an approximate nearest neighbour search. We also found that the search time for the Area Code Tree is very low and constantly less than 10ms regardless of the number of POIs in the index. Most importantly, the Area Code Tree can achieve up to 60% accuracy for locating the nearest neighbour in dense point sets. This makes the Area Code Tree an excellent candidate for continuous approximate nearest neighbour search for location based services.

### 5.1 Future Work

In the future, we will evaluate the search performance of the Area Code Tree by comparing to other spatial access methods and strategies for continuous nearest neighbour search such as Time-Parameterized  $k$ NN (TP- $k$ NN) [22] and Retrieve-Influence-Set  $k$ NN (RIS- $k$ NN) [27] strategies. Time-Parameterized  $k$ NN strategy (TP- $k$ NN) utilizes the R-tree to stores spacial objects using their coordinates of Minimum Bounding Rectangles. There-

fore, the nearest neighbour search using this strategy involves repeated access of R-tree index which results a large number of disk accesses. Although Retrieve-Influence-Set  $k$ NN strategy (RIS- $k$ NN) reduces the number of repeated searches and the cost of precomputation for continuous nearest neighbours searching but it is still expensive. Because RIS- $k$ NN strategy performs multiple TP- $k$ NN queries to estimate all possible client's movement path. On the other hand, our Area Code Tree stores spacial objects (i.e. POI) using their area codes instead of their coordinates of Minimum Bounding Rectangles. This provides the Area Code Tree the ability to quickly identify nearest neighbours to the query point by doing simple numeric comparisons. We also extend the Area Code Tree for K nearest neighbour search and find that approximate k-nearest neighbour searching is accomplished in very low and constant time. In future, we will evaluate the search performance of the Area Code Tree for locating k-nearest neighbours with other special access methods. Furthermore, we will use other types of area codes, such as telephone area codes, and postal/zip codes in the Area Code Tree. Our experiment results show that the Area Code Tree has promise for different applications in continuous query processing.

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# Appendix A

## Source Code

### A.1 Area Code Mapping of a POI

```
<?php
require("config.inc.php");
set_time_limit(5000);

$x_maximum=179.0;
$x_minimum=0;
$y_maximum=9.0;
$y_minimum=0.0;
$x_maximum=($x_maximum * 100000)/2;
$x_minimum=($x_minimum * 100000)/2;
$y_maximum=($y_maximum * 100000)/2;
$y_minimum=($y_minimum * 100000)/2;
// reterieve points fom database
$query = "Select place ,x_1 ,x_2 ,y_1 ,y_2 FROM databasetable";

if ($rows)
    foreach ($rows as $row) {
        $place_id=$row["place"];

        $point_x1=($row ["x_1"]*100000)/2;
        $point_x2=($row ["x_2"]*100000)/2;
        $point_y1=($row ["y_1"]*100000)/2;
        $point_y2=($row ["y_2"]*100000)/2;

        $main_lx=$x_minimum;
        $main_hx=$x_maximum;
        $main_ly=$y_minimum;
        $main_hy=$y_maximum;
        $center_x=floor (( $point_x1+$point_x2 )/2);
        $center_y=floor (( $point_y1+$point_y2 )/2);

        $direction ="";
```

```

while(true){
$main_center_x=floor(( $main_lx+$main_hx )/2);
$main_center_y=floor(( $main_ly+$main_hy )/2);
// area code calculation
if( $center_x >= $main_center_x && $center_y >= $main_center_y){
    $direction.="4";
    $new_main_lx=$main_center_x;
    $new_main_ly=$main_center_y;
    $new_main_hx=$main_hx;
    $new_main_hy=$main_hy;
}

else if($center_x <= $main_center_x && $center_y <= $main_center_y){
    $direction.="1";
    $new_main_lx=$main_lx;
    $new_main_ly=$main_ly;
    $new_main_hx=$main_center_x;
    $new_main_hy=$main_center_y;
}

else if($center_x >= $main_center_x && $center_y <= $main_center_y){
    $direction.="2";
    $new_main_lx=$main_center_x;
    $new_main_ly=$main_ly;
    $new_main_hx=$main_hx;
    $new_main_hy=$main_center_y;
}

else if($center_x <= $main_center_x && $center_y >= $main_center_y){
    $direction.="3";
    $new_main_lx=$main_lx;
    $new_main_ly=$main_center_y;
    $new_main_hx=$main_center_x;
    $new_main_hy=$main_hy;
}

else if($main_center_x==$center_x && $main_center_y==$center_y){
    $direction.="5";
    break;
}

if( floor($main_hx)==floor($new_main_hx) &&
    floor($main_hy)==floor($new_main_hy) &&
    floor($main_lx)==floor($new_main_lx) &&
    floor($main_ly)==floor($new_main_ly)){
    break;
}

```

```

$main_hx=floor($new_main_hx);
$main_hy=floor($new_main_hy);
$main_lx=floor($new_main_lx);
$main_ly=floor($new_main_ly);

$query="UPDATE databasetable area_code=:area
WHERE place=:id ";
$query_params=array(
    ':area'=>$direction ,
    ':id'=>$place_id
);
try {
    $stmt=$db->prepare($query );
    $result=$stmt->execute($query_params );
}
catch (PDOException $ex) {

}
}
?>

```

## A.2 Inserting and Building an Area Code Tree

```

<?php
require("config.inc.php");
set_time_limit(50000);
$child_data_format="\\"1\"-{}\\"2\"-{}\\"3\"-{}\\"4\"-{}\\"5\"-{}";
// get child node value for corresponding row
function child_value_calculation($row_id,$db){
$table=$GLOBALS['table_name'];
$query="select child from " . $table ."
where row=:row_id";
$query_params=array(':row_id'=>$row_id);
try {
    $stmt=$db->prepare($query );
    $result=$stmt->execute($query_params );
}
catch (PDOException $ex) {
    echo $ex->getMessage();
}
        $row = $stmt->fetch();
        return $row["child"];
}
// get what is inside any {} for a specific $pre<-prefix value
function get_value_in_string($pre,$value){

```

```

$count=0;
    $index=0;
    $double_quote = " ";
    // checking the number of parenthesis
for ($i=0;$i<strlen($value);$i++){
    if ($value[$i]==""){
        $count++;
    }
    if ($count==((2*$pre)-1))
        break;
}
$position="";
    for ($i=$i+5;$i<strlen($value);$i++){
        if ($value[$i]=='}')
            break;
        else $position.= $value[$i];
    }
    return $position;
}
// get the postfix value
function postfix_value($value){
$post_value="";
    if (strlen($value)==0)
        return $post_value;
    if ($value[0]=='P')
        for ($i=2;$value[$i]!=',';$i++)
            $post_value.=$value[$i];
    else return $post_value;
    return $post_value;
}
// get location information
function locaiton_value($value){
$location="";
if (strlen($value)==0)
return $location;
if ($value[0]=='P'){
// go through the string until , is found
for ($i=2;$value[$i]!=',';$i++);
// next character is L, so next all the values after :
if ($value[$i+1]=='L'){
        for ($i=$i+3;$i<strlen($value);$i++)
            $location.=$value[$i];
    }
    return $location;
}

```

```
else return $location;
if ($value[0]=='N'){
    $location_exist=0;
    for ($i=2;$i<strlen($value);$i++)
        if ($value[$i]==','){
            $location_exist=1;
            break;
        }
    if ($location_exist==1)
        for ($i=$i+3;$i<strlen($value);$i++)
            $location.=$value[$i];
    }
    return $location;
}
// check the next row
function next_row_value($value){
$next_row="";
if (strlen($value)==0)
return $next_row;
if ($value[0]=='N'){
    for ($i=2;$i<strlen($value);$i++){
        if ($value[$i]<48 && $value[$i]>57)
            break;
        $next_row.=$value[$i];
    }
}
else return $next_row;
return $next_row;
}
// build the child node value
// this function appends a location after row if necessary
function location_append_after_Row($pre,$value,$data){
$prefix=chr($pre+48);
$child_data="";
for ($i=0;$i<strlen($value);$i++){
    if (( $value[$i]==$prefix)&&($value[$i-1]=='))')
        break;
    else $child_data=$child_data.$value[$i];
}
for (;$i<strlen($value);$i++){
    if ($value[$i]=='}')
        break;
    else $child_data=$child_data.$value[$i];
}
$child_data.=$data;
```

```
for(; $i<strlen($value); $i++)  
$child_data=$child_data . $value[$i];  
return $child_data;  
}  
function child_data_value($pre, $value, $data){  
$prefix=chr($pre+48);  
$child_data="";  
for($i=0; $i<strlen($value); $i++){  
if(( $value[$i]==$prefix)&&($value[$i-1]=='''))  
break;  
else $child_data=$child_data . $value[$i];  
}  
$child_data=$child_data . $prefix . "\\" . "-" . $data;  
for(; $i<strlen($value); $i++)  
if($value[$i]=='}')  
break;  
for(; $i<strlen($value); $i++)  
$child_data=$child_data . $value[$i];  
return $child_data;  
}  
// insert new row  
function insert_new_row($db, $row_id, $prefix_data){  
$child_data_format ="\\" . "1" . "-" . "{}" . "\\" . "2" . "-" . "{}" . "\\" . "3" . "-" . "{}" . "\\" . "4" . "-" . "{}" . "\\" . "5" . "-" . "{}";  
$table= $GLOBALS['table_name'];  
$query="insert into ".$table." (parent, child, pre)  
values (:parent, :child, :prefix)";  
$query_params=array(':parent'=>$row_id,  
':child'=>$child_data_format, ':prefix'=>$prefix_data);  
try {  
    $stmt=$db->prepare($query);  
    $result=$stmt->execute($query_params);  
}  
catch (PDOException $ex) {  
    echo $ex->getMessage();  
}  
}  
function get_latest_id($db){  
$table=$GLOBALS['table_name'];  
$query="SELECT LAST_INSERT_ID() as row from ".$table;  
try{  
    $stmt=$db->prepare($query);  
    $result=$stmt->execute();  
}  
catch(PDOException $ex){  
    echo $ex->getMessage();
```

```
        }
        $row=$stmt->fetch();
return $row[”row”];
}
function Update_with_nextRow_location_value
($db,$row_id,$pre,$value,$data){
$node_info=location_append_after_Row($pre,$value,$data);
$table=$GLOBALS[’table_name’];
$query=”update ”. $table .” set child=:data where row=:row_id ”;
$query_params=array(’:data’=>$node_info,’:row_id’=>$row_id);
try {
$stmt= $db->prepare($query);
$result = $stmt->execute($query_params);
}
catch (PDOException $ex) {
echo $ex->getMessage();
}
}
// update when any field is empty
// $db-> database link
// $row_id -> row link
// $pre -> prefix
// $value -> child value
function Update_with_prefix_location_value
($db,$row_id,$pre,$value,$post_with_location){
$node_info=child_data_value($pre,$value,$post_with_location);
$table=$GLOBALS[’table_name’];
$query=”update ”. $table .” set child=:data
where row=:row_id ”;
$query_params=array(’:data’=>$node_info,’:row_id’=>$row_id);
try{
$stmt=$db->prepare($query);
$result=$stmt->execute($query_params);
}
catch (PDOException $ex){
echo $ex->getMessage();

}
}
// first id => starting row number
// $area => area code
function process_data($first_id,$Area,$location,$db){
// last created row number
$row_number=get_latest_id($db);
```

```
// first character of area code
if(strlen($Area)>0)
$pre = ord($Area[0])-48;
else $pre='';
// remaining character of area code
if(strlen($Area)>1)
$post=substr($Area,1); // new_value
else $post="";
$row_id=intval($first_id,0);
$prefix_data="";
while(1){
    // child value in database
    $value=child_value_calculation($row_id,$db);
$inner_value="";
$postfix="";
$location_data="";
$next_row="";
// getting the value for corresponding prefix
switch($pre){
    case 1:
        $inner_value=get_value_in_string(1,$value);
        break;
    case 2:
        $inner_value=get_value_in_string(2,$value);
        break;
    case 3:
        $inner_value=get_value_in_string(3,$value);
        break;
    case 4:
        $inner_value=get_value_in_string(4,$value);
        break;
    case 5:
        $inner_value=get_value_in_string(5,$value);
        break;
}
$postfix=postfix_value($inner_value);
$location_data=locaiton_value($inner_value);
$next_row=next_row_value($inner_value);
if(strlen($inner_value)==0){
    $post_with_location = "P:". $post.",L:". $location;
    Update_with_prefix_location_value
    ($db,$row_id,$pre,$value,$post_with_location);
    break;
}
else {
```

```

if (strlen ($postfix)>0&&strlen( $location_data)>0){
// create new row and get the row number
    $prefix_data.= $pre;
    insert_new_row ($db , $row_id , $prefix_data );
    $row_number= get_latest_id ($db );
    $data = "N:". $row_number;
    // update previous node data
    Update_with_prefix_location_value
    ($db , $row_id , $pre , $value , $data );
// get info about new row
    $row_id=intval($row_number ,0);
    // collect two location info
    $location_1_prefix=$post[0];
    $location_2_prefix=$postfix[0];
    $location_1_place=$location;
    $location_2_place= $location_data;
    $location_1_postfix=substr($post ,1);
    $location_2_postfix=substr($postfix ,1);
    $both_empty=0;
// if prefix of new location and old location are same
// create a new row
while($location_1_prefix==$location_2_prefix){
    $prefix_data.= $location_1_prefix ;
    insert_new_row ($db , $row_id , $prefix_data );
    $row_number= get_latest_id ($db );
    $value=child_value_calculation ($row_id , $db );
    $data = "N:". $row_number;
    Update_with_prefix_location_value
    ($db , $row_id , $location_1_prefix , $value , $data );
    $row_id=intval($row_number ,0);
    if (strlen ($location_1_postfix)==1&&
        strlen ($location_2_postfix)==1){
        $both_empty=1;
        $location_1_prefix=$location_1_postfix [0];
        $location_2_prefix=$location_2_postfix [0];
        $location_1_postfix="";
        $location_2_postfix="";
        break;
    }
    $location_1_prefix=$location_1_postfix [0];
    $location_2_prefix=$location_2_postfix [0];
    $location_1_postfix=substr($location_1_postfix ,1);
    $location_2_postfix=substr($location_2_postfix ,1);
}
if ($both_empty==0){
    $value=child_value_calculation ($row_id , $db );
}

```

```

        $data = "P:". $location_1_postfix .",L:". $location_1_place ;
        Update_with_prefix_location_value
        ($db , $row_id , $location_1_prefix , $value , $data );
        $value=child_value_calculation($row_id , $db );
        $data = "P:". $location_2_postfix .",L:". $location_2_place ;
        Update_with_prefix_location_value
        ($db , $row_id , $location_2_prefix , $value , $data );
    }
    else {
        $value=child_value_calculation($row_id , $db );
        $data="P:". $location_1_postfix .",
            L:". $location_1_place .",
            L:". $location_2_place ;
        Update_with_prefix_location_value
        ($db , $row_id , $location_2_prefix , $value , $data );
    }
    break ;
}
else if ( strlen($next_row)>0){
    $prefix_data.=$pre ;
    $previous_prefix=$pre ;
    if ( strlen($post)>0)
        $pre = ord($post[0])-48;
        else $pre=' ';
        if ( strlen($post)>1)
            $post = substr($post , 1);
            else {$post="";
            $data=",L:". $location ;
            Update_with_nextRow_location_value(
            $db , $row_id , $pre , $value , $data );
        }
        $row_id= intval($next_row ,0);
    }
}
}
}

function get_PlaceAreaCode($id , $db){
    $table=$GLOBALS[ ' area_table_name ' ];
    $query="select place , area_code from " . $table .
    " where Place_id = :row_id ";
    $query_params = array (':row_id' => $id );
    try {
        $stmt=$db->prepare($query );
        $result=$stmt->execute($query_params );
    }
}

```

```

        catch (PDOException $ex) {
            echo $ex->getMessage();
        }
        $row=$stmt->fetch();
        $place_name=$row[ ' place ' ];
        $area=$row[ ' area_code ' ];
    return array($place_name , $area);
}
if ( isset($_POST[ "mySubmit" ]) ){
    $time_start = microtime(true);
    insert_new_row($db ,0 ,"" );
    $first_id=get_latest_id($db );
    $query="select place , area_code FROM ". $area_table_name ;
    try {
        $stmt = $db->prepare($query );
        $result = $stmt->execute();
    }
    catch (PDOException $ex) {
    }
    $rows=$stmt->fetchall();
    if ($rows)
        foreach ($rows as $row) {
            $new_area_code=$row[ ' area_code ' ];
            $location=$row[ ' place ' ];
            process_data
                ($first_id , $new_area_code , $location , $db );
        }
    $time_end = microtime(true);
    $time = $time_end - $time_start;
    echo "Times required to build area code for Table
". $area_table_name ." are ". $time ." seconds";
}
?>

```

### A.3 Approximate Nearest Neighbour Search

```

<?php
require("config . inc . php ");
set_time_limit(50000);
$table_name="databasetable";
function area_code_calculation($latitude , $longitude ){
$main_lx=(0*100000)/2;
$main_hx=(179.0*100000)/2;
$main_ly=(0.0*100000)/2;
$main_hy=(9.0*100000)/2;
$center_x=($latitude *100000)/2;

```

```

$center_y=($longitude *100000)/2;
$direction="";
while(true){
    $main_center_x=floor(( $main_lx+$main_hx )/2);
    $main_center_y=floor(( $main_ly+$main_hy )/2);
    if($center_x >=$main_center_x && $center_y >=$main_center_y){
        $direction.= "4";
        $new_main_lx=$main_center_x;
        $new_main_ly=$main_center_y;
        $new_main_hx=$main_hx;
        $new_main_hy=$main_hy;
    }
    // area code calculation
    else if($center_x <=$main_center_x && $center_y <=$main_center_y){
        $direction.= "1";
        $new_main_lx=$main_lx;
        $new_main_ly=$main_ly;
        $new_main_hx=$main_center_x;
        $new_main_hy=$main_center_y;
    }
    else if($center_x >=$main_center_x && $center_y <=$main_center_y){
        $direction.= "2";
        $new_main_lx=$main_center_x;
        $new_main_ly=$main_ly;
        $new_main_hx=$main_hx;
        $new_main_hy=$main_center_y;
    }
    else if($center_x <=$main_center_x && $center_y >=$main_center_y){
        $direction.= "3";
        $new_main_lx=$main_lx;
        $new_main_ly=$main_center_y;
        $new_main_hx=$main_center_x;
        $new_main_hy=$main_hy;
    }
    else if($main_center_x==$center_x && $main_center_y==$center_y){
        $direction.= "5";
        break;
    }
    if( floor($main_hx)==floor($new_main_hx) &&
        floor($main_hy)==floor($new_main_hy) &&
        floor($main_lx)==floor($new_main_lx) &&
        floor($main_ly) == floor($new_main_ly))
    {
        break;
    }
}

```

```
$main_hx=floor($new_main_hx);
$main_hy=floor($new_main_hy);
$main_lx=floor($new_main_lx);
$main_ly=floor($new_main_ly);
}
return $direction;
}
function get_value_in_string($pre,$value){
$count=0;
$index=0;
$double_quote="''";
// checking the number of parenthesis
for($i=0;$i<strlen($value);$i++){
if($value[$i]=="''"){
$count++;
}
if($count==((2*$pre)-1))
break;
}
$position="";
for($i=$i+5;$i<strlen($value);$i++){
if($value[$i]=='}')
break;
else $position.=$value[$i];
}
return $position;
}
function postfix_value($value){
$post_value="";
if(strlen($value)==0)
return $post_value;
if($value[0]=='P')
for($i=2;$value[$i]!=',';$i++)
$post_value.=$value[$i];
else return $post_value;
return $post_value;
}
// get location information
function locaiton_value($value){
$location="";
if(strlen($value)==0)
return $location;
for($i=0;$i<strlen($value);$i++)
if($value[$i]=='L')
break;
```

```

$count_start=0;
for($i=$i+2;$i<strlen($value);$i++)
if($value[$i]>='0'&& $value[$i]<='9'){
    $location.=$value[$i];
    $count_start=1;
}
else {
    if($count_start==1)
    break;}
}

return $location;
}
// check the next row
function next_row_value($value){
$next_row="";
if(strlen($value)==0)
return $next_row;
if($value[0]=='N'){
for($i=2;$i<strlen($value);$i++){
    if($value[$i]==',')
    break;
    $next_row.=$value[$i];
}
}
else return $next_row;
}
function child_value_calculation($row_id,$db){
$table=$GLOBALS['table_name'];
$query="select child,parent,pre from ".$table."
where row=:row_id";
$query_params=array(':row_id'=>$row_id);
try {
$stmt=$db->prepare($query);
$result=$stmt->execute($query_params);
}
catch (PDOException $ex) {
echo $ex->getMessage();
}
$row=$stmt->fetch();
return array($row["child"],$row["parent"],$row["pre"]);
}
$file=fopen("path-points.txt","r");
$path_no=0;
echo "File Name: ".$table_name."<br/><br/>";

```

```

// Output a line of the file until the end is reached
while(! feof($file)){
$path_no++;
fscanf($file,"%lf%lf", $latitude , $longitude );
echo "Path ".$path_no." : ".$latitude ." ". $longitude ."  
>";
$time_start=microtime(true);
$direction=area_code_calculation($latitude , $longitude );
$location_found=0;
$prefix=$direction [0];
$postfix_value=substr($direction ,1);
$row_id=1;
echo "Nearby Path Name:" ;
while(1){
$row_info=child_value_calculation($row_id , $db );
$value=$row_info [0];
$parent=$row_info [1];
$pref=$row_info [2];
$inner_value="";
$postfix="";
$location="";
$next_row="";
$inner_value=get_value_in_string($prefix , $value );
if (strlen($inner_value)==0){
for ($i=1;$i <=5;$i++){
    if ($prefix==$i)
        continue ;
    else {
        $inner_value=get_value_in_string($i , $value );
        if (strlen($inner_value)>0){
            break ;
        }
    }
}
$postfix=postfix_value($inner_value );// what is in database
$location=locaiton_value($inner_value );// what is in database
$next_row=next_row_value($inner_value );
if (strlen($location)>0){
$location_found++;
echo $location ." ";
for ($i=1;$i <=5;$i++){
if (intval($prefix ,0)==$i)
continue ;
else {
$inner_value=get_value_in_string($i , $value );

```

```
if( strlen( $inner_value )>0){
$prefix=$i;
break;
}
}
}
}
if( $location_found==0){
$postfix_value[0];
$postfix_value=substr( $postfix_value ,1); }
if( strlen( $next_row)>0){
$row_id=intval( $next_row ,0);
continue;
}
if( $location_found >=2|| strlen( $postfix_value )==0)
break;
}
$time_end = microtime(true);
$time = $time_end - $time_start;
echo "Total time for Path ".$path_no." is ".$time." seconds";
}
fclose($file );
?>
```

# Appendix B

## Sample Data Sets

### B.1 Static Data set of 1000 POIs across New Zealand

174.78000	:	0.00000	:	174.78000	:	0.00000	:	0	:	1
174.06000	:	0.02000	:	174.06000	:	0.02000	:	0	:	2
176.63000	:	0.02000	:	176.63000	:	0.02000	:	0	:	3
177.43000	:	0.02000	:	177.43000	:	0.02000	:	0	:	4
173.71000	:	0.03000	:	173.71000	:	0.03000	:	0	:	5
176.89000	:	0.03000	:	176.89000	:	0.03000	:	0	:	6
177.28000	:	0.03000	:	177.28000	:	0.03000	:	0	:	7
175.51000	:	0.04000	:	175.51000	:	0.04000	:	0	:	8
176.29000	:	0.04000	:	176.29000	:	0.04000	:	0	:	9
176.45000	:	0.04000	:	176.45000	:	0.04000	:	0	:	10
173.54000	:	0.06000	:	173.54000	:	0.06000	:	0	:	11
175.47000	:	0.06000	:	175.47000	:	0.06000	:	0	:	12
174.12000	:	0.07000	:	174.12000	:	0.07000	:	0	:	13
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177.05000	: 5.29000	: 177.05000	: 5.29000	: 0	: 836
174.84000	: 5.30000	: 174.84000	: 5.30000	: 0	: 837
175.30000	: 5.30000	: 175.30000	: 5.30000	: 0	: 838
176.69000	: 5.30000	: 176.69000	: 5.30000	: 0	: 839
177.14000	: 5.30000	: 177.14000	: 5.30000	: 0	: 840
177.98000	: 5.30000	: 177.98000	: 5.30000	: 0	: 841
175.72000	: 5.31000	: 175.72000	: 5.31000	: 0	: 842





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174.07000 : 5.90000 : 174.07000 : 5.90000 : 0 : 934  
177.88000 : 5.90000 : 177.88000 : 5.90000 : 0 : 935  
173.66000 : 5.91000 : 173.66000 : 5.91000 : 0 : 936  
175.50000 : 5.91000 : 175.50000 : 5.91000 : 0 : 937  
176.24000 : 5.91000 : 176.24000 : 5.91000 : 0 : 938  
176.70000 : 5.91000 : 176.70000 : 5.91000 : 0 : 939  
177.78000 : 5.91000 : 177.78000 : 5.91000 : 0 : 940  
173.24000 : 5.93000 : 173.24000 : 5.93000 : 0 : 941  
173.88000 : 5.93000 : 173.88000 : 5.93000 : 0 : 942  
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178.27000 : 5.94000 : 178.27000 : 5.94000 : 0 : 946  
173.78000 : 5.96000 : 173.78000 : 5.96000 : 0 : 947  
174.12000 : 5.96000 : 174.12000 : 5.96000 : 0 : 948  
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174.54000 : 6.00000 : 174.54000 : 6.00000 : 0 : 952  
174.05000 : 6.01000 : 174.05000 : 6.01000 : 0 : 953  
174.12000 : 6.01000 : 174.12000 : 6.01000 : 0 : 954  
174.79000 : 6.01000 : 174.79000 : 6.01000 : 0 : 955  
176.99000 : 6.01000 : 176.99000 : 6.01000 : 0 : 956  
174.94000 : 6.02000 : 174.94000 : 6.02000 : 0 : 957  
175.49000 : 6.02000 : 175.49000 : 6.02000 : 0 : 958  
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176.36000 : 6.08000 : 176.36000 : 6.08000 : 0 : 965  
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176.98000 : 6.09000 : 176.98000 : 6.09000 : 0 : 967  
173.17000 : 6.11000 : 173.17000 : 6.11000 : 0 : 968  
175.83000 : 6.11000 : 175.83000 : 6.11000 : 0 : 969  
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176.53000 : 6.14000 : 176.53000 : 6.14000 : 0 : 972  
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173.27000 : 6.16000 : 173.27000 : 6.16000 : 0 : 975  
175.49000 : 6.16000 : 175.49000 : 6.16000 : 0 : 976  
173.19000 : 6.17000 : 173.19000 : 6.17000 : 0 : 977

173.78000 : 6.18000 : 173.78000 : 6.18000 : 0 : 978  
175.04000 : 6.18000 : 175.04000 : 6.18000 : 0 : 979  
177.07000 : 6.18000 : 177.07000 : 6.18000 : 0 : 980  
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173.17000 : 6.20000 : 173.17000 : 6.20000 : 0 : 982  
174.05000 : 6.20000 : 174.05000 : 6.20000 : 0 : 983  
177.30000 : 6.20000 : 177.30000 : 6.20000 : 0 : 984  
174.67000 : 6.21000 : 174.67000 : 6.21000 : 0 : 985  
175.07000 : 6.21000 : 175.07000 : 6.21000 : 0 : 986  
175.70000 : 6.21000 : 175.70000 : 6.21000 : 0 : 987  
176.64000 : 6.21000 : 176.64000 : 6.21000 : 0 : 988  
173.25000 : 6.23000 : 173.25000 : 6.23000 : 0 : 989  
173.70000 : 6.23000 : 173.70000 : 6.23000 : 0 : 990  
174.48000 : 6.23000 : 174.48000 : 6.23000 : 0 : 991  
175.04000 : 6.25000 : 175.04000 : 6.25000 : 0 : 992  
175.89000 : 6.25000 : 175.89000 : 6.25000 : 0 : 993  
177.16000 : 6.25000 : 177.16000 : 6.25000 : 0 : 994  
174.49000 : 6.26000 : 174.49000 : 6.26000 : 0 : 995  
175.51000 : 6.26000 : 175.51000 : 6.26000 : 0 : 996  
174.99000 : 6.27000 : 174.99000 : 6.27000 : 0 : 997  
177.51000 : 6.27000 : 177.51000 : 6.27000 : 0 : 998  
173.43000 : 6.28000 : 173.43000 : 6.28000 : 0 : 999  
173.47000 : 6.30000 : 173.47000 : 6.30000 : 0 : 1000

## B.2 10 User Locations along their Trajectory

175.31742 3.48012  
175.33814 3.46961  
175.35851 3.45078  
175.36101 3.44832  
175.38889 3.42815  
175.39926 3.41810  
175.41133 3.40457  
175.42336 3.39956  
175.42591 3.39762  
175.44345 3.39085